Heterogeneity and the Welfare Cost of Inflation[1]

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Abstract

We study a heterogeneous economy where agents hold cash to insure against consumption risk and differ in either their matching profiles, discount factors or labor disutilities. For some forms of heterogeneity, different agent types hold different money balances in equilibrium. This heterogeneity generally disappears if nominal interest rates are zero. We then calibrate the model for the U.S. economy and use it to quantify the welfare cost of inflation. Results are affected by the distribution of agents' types and the kind of heterogeneity considered. With heterogeneity in discounting or disutility, different agents face different - though always positive - welfare costs of inflation. When agents differ in their matching profiles instead, positive inflation can be welfare increasing for agents with low consumption risk.

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1 Introduction

A considerable amount of theoretical work, based on disparate modeling approaches, supports the notion that allocative efficiency in a monetary economy hinges on a deterministic deflationary policy known as the 'Friedman rule', yet in practice deflationary policies are not implemented and moderate inflation is widely tolerated.

This discrepancy has motivated a vast literature aimed at quantifying the welfare loss due to positive inflation and recent work in this field has developed within three strands of literature. The first is based on the representative agent framework of Lagos and Wright (2005), which displays explicit micro foundations for money but does not account for heterogeneity in wealth. The second is based on the search model computed by Molico (2006), in which random matching induces heterogeneity of money balances that, however, is analytically intractable. The third is represented by several papers that study monetary economies with heterogeneous agents, but in which money has a rather "descriptive" role. Among them, Erosa and Ventura (2002) and Albanesi (2007), focus on models that account for income inequality; Doepke and Schneider (2006) study the effect of unanticipated inflation on agents with nominal asset positions that differ in terms of liquidity or maturity; Attanasio et al. (2002) quantify the welfare cost of inflation for agents that use ATM cards and agents who don't.

Our work intends to complement these different approaches and investigate how the very nature of the heterogeneity considered affects the redistributive role of anticipated inflation. We do so by retaining the explicit micro foundations found in Lagos and Wright (2005), while enriching the model by introducing forms of heterogeneity that generate tractable equilibrium distributions of wealth. Specifically, we construct a model where agents hold cash to insure against consumption risk, trade in large competitive markets, and are ex-ante heterogeneous in either their matching profiles, discount factors or labor disutility. Analytical results show that different agent types hold different money balances in competitive equilibrium, unless they differ in terms of labor disutility. This heterogeneity generally disappears if nominal interest rates are zero.

We then calibrate the model to the U.S. economy and use it to quantify the welfare cost of inflation. For the representative agent, we find that ten percent inflation is worth less than one percent of consumption, in line with previous studies such as Lucas (1981), Fischer (1981) and Cooley and Hansen (1989). Results, however, are affected by the distribution of agents' types and the kind of heterogeneity considered. With heterogeneity in discounting and disutility from labor, different agents face different - but always positive - welfare costs of inflation. This is due to an intratemporal labor transfer from the centralized to the decentralized market in the case of different disutility of labor. With heterogeneity in discounting instead, the reason is heterogeneity in money holdings. The welfare cost of inflation is shown to be non monotonic in the disount factor due to a trade-off between intensive and extensive margin induced by changes in the shadow interest rate.

Heterogeneity in consumption risk appears to have a somewhat different impact, as we find that positive inflation can in fact be welfare-increasing for a segment of the population. These are agents who have low money balances because their expected marginal benefit from holding cash is lower than the rest of the population. For these agents, inflation generates a beneficial redistribution of wealth because lump-sum money transfers more than offset their inflation-tax burden.

The remainder of the paper is organized as follows. Section 2 presents the model economy. Section 3 introduces the definition of efficiency considered throughout the paper. Section 4 derives the stationary monetary equilibrium allocations. Section 5 discusses the quantitative analysis for the case of a representative agent and for the different types of heterogeneity studied. Section 6 concludes.

2 The model

We describe a spatially separated economy in which money has an explicit medium of exchange function and there is no role for private credit. The model builds on Boel and Camera (2006), which is based on Aliprantis et al. (2004), Kocherlakota (1998), Lagos and Wright (2005) - henceforth LW - and Townsend (1980). Time is discrete, starts with date 1 and the horizon is infinite. There is a population $X = \mathbb{N}$ of heterogeneous infinitely-lived agents who want to consume perishable goods and discount only even to odd dates. Thus, as in LW, we work with trading cycles indexed by $t = 1, 2, ...$ each including an odd and an even date. As in Townsend (1980) there are infinitely many spatially separated trading groups, each of which defines a competitive market. On each date, every market includes infinitely many anonymous agents who have never met before. Thus, in each trading cycle each agent visits two anonymous markets, denoted 'one' in the odd date and 'two' in the even. Trading groups are formed by a matching process that repeatedly partitions the population into disjoint sets of agents. For details, see Boel and Camera (2006).

2.1 Preferences and technologies

Dates differ in terms of agents' preferences and economic activities, as in LW. Odd dates are characterized by idiosyncratic trading risk as an arbitrary agent either works but does not wish to consume, or consumes but cannot work, or is idle, i.e., he neither wishes to consume nor is able to work. We call these 'trading frictions.' Everyone can work and consume on even dates.

Agents draw i.i.d. trading shocks at the start of each odd date. Those who are not idle either wish to consume or are able to produce, states that are assumed to be equally likely and mutually exclusive. Hence, on odd dates each agent faces idiosyncratic consumption risk, but on even dates everyone can produce and consume. Agents are heterogeneous in the first market. We assume two types of agents $j = H, L$ in proportions ρ and $1 - \rho$. Agents can alternatively differ either in their trading shocks, discounting or the productivity of labor.

On each date, a single perishable good can be produced. Sellers can supply any positive amount of labor and can access a technology that transforms each unit of labor into one unit of consumption goods. As in LW, it is assumed that preferences on even dates are quasilinear $U(q_j) - x_j$ for every agent type j. The first term denotes the utility from $q_j \geq 0$ consumption and the second term is disutility from supplying $x_j \geq 0$ labor. Odd date preferences are as follows. A consumer derives utility $u(c)$ from consuming $c \geq 0$ of someone else's production. A producer of any type j suffers $\phi_j(y)$ disutility from supplying labor to produce y goods. The functions u, ϕ and U are twice continuously differentiable with $u' > 0$, $\phi' > 0$, $U' > 0$, $u'' < 0$, $\phi' \geq 0$ and $U'' < 0$. Also, $u(0) = U(0) = \phi(0) = 0$ and let c^* and q^* be the solution to $u'(c^*) = \phi'(c^*)$ and $U'(q^*) = 1$ respectively.

As is standard in monetary models, we assume limited enforcement and limited commitment. This simply means that agents have exclusive rights to their assets and endowments, and their actions cannot be subject to retribution, so that trading plans must be compatible with individual incentives. This together with the market frictions assumed above implies an essential role for money (see Huggett and Krasa (1996) and Kocherlakota (1998)) since on odd dates trade is *quid pro quo* but consumers cannot produce. Thus, a consumption shock on odd dates corresponds to a need for currency.

We assume a government exists that is the sole supplier of fiat currency, of which there is an initial stock $\bar{M} > 0$. We let the money stock evolve deterministically at gross rate π by means of lump-sum cash transfers at the beginning of even dates.

3 Efficient allocations

Consider the allocation selected by a benevolent planner who maximizes the agents' lifetime utilities treating agents identically. The planner is subject to the same physical and informational constraints faced by the agents and therefore can observe neither types nor identities. So, the planner can just propose a type-independent consumption plan in each trading cycle without having the ability to transfer resources across agents over time. Equivalently, the planner maximizes expected utility of the arbitrary agent on each date. The solution to such a sequence of static problems is called a constrained-efficient allocation. It corresponds to the outcome arising in each market if traders can coordinate and commit to a plan ex-ante before realizing their individual shocks, and it is stationary across trading cycles. For details, see the Appendix .

4 Stationary monetary allocations

We focus on stationary monetary outcomes in which consumption is invariant across trading cycles and the sequence of nominal prices evolve so that the money stock has constant positive real value.

Due to stationarity, we simplify notation omitting t subscripts and use a prime superscript to identify next-cycle variables, when necessary. Accordingly, we let p_1 and p_2 denote the nominal price of goods on odd and even dates of an arbitrary trading cycle t. In addition, we find it convenient to work with real variables normalizing all nominal variables by p_2, so that market one trades occur at real price $p = \frac{p_1}{p_2}$. In this manner, the timing of events during cycle t for an agent of type j can be discussed as follows.

The arbitrary agent of type j enters cycle t with real money holdings $m_j \geq 0$, carried over from the preceding cycle. Subsequently, trade occurs and after market one closes the agent enters market two on the even date with real money holdings $m_{j,k}$ where $k = n, s, b$ denotes the trading shock experienced in market one. Here, n identifies an agent who was idle, while b and s identify a buyer and a seller, respectively.

Money holdings evolve within the cycle as follows:

$$\begin{aligned} m_{j,b} &= m_j - pc_j \\ m_{j,s} &= m_j + py_j \\ m_{j,n} &= m_j \end{aligned} \tag{1}$$

That is, buyers deplete balances by pc_j while sellers increase them by py_j. Cash left over is used to trade in market two, when the real price is one, q_j is consumption bought and $x_{j,k}$ is production sold by an agent who experienced shock k (the notation q_j is without loss in generality, as we later show). In market two, agents also choose their savings. Let $m'_j \geq 0$ denote the real values of the agent's money holdings at the start of next trading cycle (multiply by p'_2 or $\frac{p'_2}{p_2}$ to get the current nominal or real values).

In a stationary monetary economy real money holdings must be positive and constant. Therefore, in the remainder of the paper we focus on outcomes such that $m'_j = m_j$. If M

is cash at the start of a cycle and $M' = \pi M$ is cash available in market two, then

$$\frac{p_2'}{p_2} = \frac{M'}{M} = \pi, \tag{2}$$

i.e., in a stationary economy aggregate real balances are constant so the inflation rate equals the rate of growth of money. This rate is controlled by means of per-capita lump-sum transfers τ in market two, so the government budget constraint is

$$\tau = [\rho m_H + (1-\rho) m_L](\pi - 1) \tag{3}$$

Stationarity and money **market clearing** imply that real balances available in market one must equal the money stock at each date, i.e.

$$\tfrac{M}{p_2} = \quad \rho m_H + (1-\rho) m_L. \tag{4}$$

4.1 Even dates

Given the recursive nature of the problem, we use a dynamic programming approach to describe the problem faced by the representative agent of type j at any date. We let $V_j(m_j)$ be the expected lifetime utility of this agent when he starts the trading cycle with m_j before trading shocks are realized. We also let $W_j(m_{j,k})$ be the expected lifetime utility from entering an even date with $m_{j,k}$. For the moment, suppose $\beta_j = \beta$ for $j = H, L$.

The agent's budget constraint at the start of an even date is:

$$x_{j,k} = q_j + \pi m_j' - (m_{j,k} + \tau) \tag{5}$$

The resources available to the agent in market two partly depend on the realization of the trading shock k, as he has $m_{j,k}$ real balances carried over from market one. Other resources are $x_{j,k}$ receipts from current sales of goods and the lump-sum real balances transfer τ.[2] These resources can be used to finance current consumption q_j, or simply to carry $\pi m_j'$ real money balances into tomorrow's markets (short-selling is not allowed). The factor $\pi = \frac{p_2'}{p_2}$ multiplies m_j' because the budget constraint lists current real values.

[2]Notice that $x_{j,k} \geq 0$ so we must verify that this is true for all k in equilibrium.

Notice that savings depend on the expected rate of return on cash since agents can save only with money and cannot lend to each other.

The agent's problem at the start of an even date can be represented as follows:

$$W_j(m_{j,k}) = \max_{q_j, m'_j \geq 0} \{U(q_j) - q_j - \pi m'_j + m_{j,k} + \tau + \beta V_j(m'_j)\} \tag{6}$$

It follows that in a stationary monetary economy

$$\frac{\partial W_j(\omega_{j,k})}{\partial m_{j,k}} = 1 \quad \text{for } j = H, L. \tag{7}$$

The result hinges on the linearity of production disutility and the use of competitive pricing, linear in the quantity sold. It follows that the marginal value of money must simply reflect the price of real balances, which is one. The economic implication is the marginal valuations of real money balances and bonds in market two are identical and do not hinge on the agent's type j, money holdings $m_{j,k}$ or trading shock k.

The model allows us to disentangle the agents' portfolio choices from their trading histories since

$$W_j(m_{j,k}) = \quad W_j(0) + m_{j,k}, \tag{8}$$

i.e., the agent's expected value from having portfolio $m_{j,k}$ at the start of an even date is the expected value from having no wealth $W_j(0)$, letting $m_j = 0$, plus the current real value of wealth $m_{j,k}$. This implies agents of identical type exit an even date with identical money holdings m'_j, independent of their trading histories, much as in [22]. However, different types might choose different money holdings, as we demonstrate next.

Start by observing that by (6) we have

$$q_j = q^* \;\; \text{for } j = H, L. \tag{9}$$

That is, everyone consumes the same amount q^* independent of his money holdings. The reason is agents in market two can produce any amount at constant marginal cost.

Given (9) we write

$$W_j(m_{j,k}) = U(q^*) - q^* + m_{j,k} + \tau + \max_{m'_j \geq 0} [-\pi m'_j + \beta V_j(m'_j)]. \tag{10}$$

The central implication is the agents' lifetime utility and the efficiency of the decentralized monetary solution will hinge on the trades that take place in market one. Since these depend on the availability and the liquidity of financial resources, then we expect that efficiency will impinge on the agents' portfolio decisions m'_j. This is studied next.

Given that we are focusing on monetary outcomes, i.e. $m'_j > 0$, we must have:

$$1 = \frac{\beta}{\pi} \times \frac{\partial V_j(m'_j)}{\partial m'_j} \tag{11}$$

Recalling that one unit of real balances buys one unit of consumption, the left hand sides of the expressions simply define the marginal cost of assets. The right hand sides define the expected marginal benefit from holding money discounted according to time preferences and inflation. Since agents are heterogenous, it follows that the expected benefit of holding money balances will generally differ across types j. To see how, we must study trades on odd dates.

4.2 Odd dates

Consider an agent of type j with m_j at the start of an odd date. For a moment, consider that this agent may differ from other types in more than one dimension, trading shocks, preferences or discount factors. The expected lifetime utility of agent j entering a period with m_j, in this general case, must satisfy

$$\begin{aligned} V_j(m_j) = \quad & \max_{c_j \in [0, \frac{m_j}{p}]} \tfrac{\alpha_j}{2}[u(c_j) + W_j(m_{j,b})] \\ & + \max_{y_j} \tfrac{\alpha_j}{2}[-\phi_j(y_j) + W_j(m_{j,s})] + (1-\alpha_j)W_j(m_{j,n}) \end{aligned} \tag{12}$$

The agent maximizes his expected utility by choosing consumption of specialty goods c_j as a buyer and his production y_j as a seller.

Clearly, in an optimum a seller's choice of production must satisfy

$$-\phi'_j(y_j) + \frac{\partial W_j(m_{j,s})}{\partial m_{j,s}} \frac{\partial m_{j,s}}{\partial y_j} = 0. \tag{13}$$

Since $\frac{\partial W_j(m_{j,s})}{\partial m_{j,s}} = 1$ from (7), $\frac{\partial m_{j,s}}{\partial y_j} = p$ from (1), then

$$\phi'_j(y_j) = p. \tag{14}$$

Now consider a buyer. Let $\lambda_j \geq 0$ be the multiplier on the budget constraint. Recall from (1) that $m_{j,b}$ depends on c_j. Then, the first order condition for the buyer's problem is

$$u'(c_j) + \frac{\partial W_j(m_{j,b})}{\partial m_{j,b}} \frac{\partial m_{j,b}}{\partial c_j} - \lambda_j = 0.$$

Since $\frac{\partial W_j(m_{j,b})}{\partial m_{j,b}} = 1$ from (7), $\frac{\partial m_{j,b}}{\partial c_j} = -p$ from (1) and $p = \phi_j'(y_j)$ from (14), the first order condition becomes

$$u'(c_j) = p + \lambda_j. \tag{15}$$

If the constraint is not binding, then $\lambda_j = 0$ and $u'(c_j) = \phi_j'(y_j)$. The buyer consumes c^* and spends the efficient amount of cash $m^* = c^*$. If the constraint is binding, then $\lambda_j > 0$ and $u'(c_j) > \phi_j'(y_j)$. The buyer j consumes $c_j < c^*$ and spends $c_j < m^*$. Thus, c_j will never exceed c^*. That is,

$$c_j = \min(m_j, c^*). \tag{16}$$

To find the optimal cash holdings of an agent j we must calculate the expected marginal value of holding money, $\frac{\partial V_j(m_j)}{\partial m_j}$. To do so we use (1) and (8) in $V_j(m_j)$ to obtain

$$V_j(m_j) = \quad m_j + \tfrac{\alpha_j}{2}[(u(c_j) - pc_j) + (py_j - \phi_j(y_j))] + W_j(0) \tag{17}$$

where c_j satisfies (16).

The expected lifetime utility at the start of a period depends on the agent's real wealth m_j and two additional elements. First, the expected surplus from trade in market one. With probability $\alpha_j/2$ the agent is a buyer and spends pc_j of his wealth on consumption and gets net utility $u(c_j) - pc_j$. With probability $\alpha_j/2$ the agent is a seller, he earns py_j from selling y_j and derives a disutility $\phi_j(y_j)$ from production. Second, there is the continuation payoff $W_j(0)$.

Equation (17) makes it simple to calculate the equilibrium marginal value of money. Specifically,

$$\frac{\partial V_j(m_j)}{\partial m_j} = \quad 1 + \tfrac{\alpha_j}{2}[u'(c_j) - p]\frac{\partial c_j}{\partial m_j} \tag{18}$$

where $\frac{\partial c_j}{\partial m_j} = \frac{1}{p}$ if the agent is liquidity constrained, and zero otherwise. It follows that $V_j(m_j)$ is strictly concave in cash holdings if buyer j is liquidity constrained, and linear otherwise:

$$\frac{V_j(m_j)}{\partial m_j} = \begin{cases} 1 + \frac{\alpha_j}{2p}[u'(c_j) - p] & \text{if } m_j < m^* \\ 1 & \text{otherwise} \end{cases} \tag{19}$$

For a cash-constrained buyer, the marginal value of money depends on the marginal utility of consumption and it is decreasing in m_j, since $u''(c_j) < 0$. If instead an agent is not cash constrained, the marginal value of money is constant and equal to one, the real value of one unit of money.

We are now ready to examine different types of heterogeneity

4.3 Heterogeneity in trading shocks

In this section we let agents differ only in the frequency of trading shocks on odd dates. Type H and type L agents have respectively a probability α_H and α_L of meeting a counterpart, with $0 < \alpha_L < \alpha_H \leq 1$. Sellers have identical linear (unit) costs of production in the first market, i.e. $\phi_j(y_j) = y_j$. So we set $\phi_j(y) = y$ in (12).

We start by finding the relative prices in the two markets. Using (13), (1) and (7) we must have

$$p = 1. \tag{20}$$

Indeed, if $p > 1$ then production would be infinite on odd dates, whereas if $p < 1$ there would be no production on odd dates. Thus, in a stationary monetary economy (20) must hold. Goods market clearing on odd dates then implies

$$y = \rho\alpha_H c_H + (1-\rho)\alpha_L c_L \tag{21}$$

Using (19) and (11), given an outcome with $m_j > 0$ the agent's optimal portfolio choice must satisfy

$$\frac{\pi}{\beta} = 1 + \frac{\alpha_j}{2}[u'(c_j) - 1] \tag{22}$$

The first and the second component of (22) are standard: the discount factor β and the real yield on cash $\frac{1}{\pi}$. The third component, which is non-standard, is $\frac{\alpha_j}{2}[u'(c_j) - 1]$, a non-negative value since $u'(c_j) \geq 1$ from (16). This can be interpreted as the expected liquidity premium from having cash available in market one and it arises because money is needed to trade in that market. This premium grows with the severity of the cash constraint and the likelihood of a consumption shock.

Since the net nominal interest rate is equal to $\frac{\pi}{\beta} - 1$[3], then we can rewrite (22) as

$$i = \frac{\alpha_j}{2}[u'(c_j) - 1] \tag{23}$$

We will use (23) extensively in our quantitative analysis.

Goods market clearing on even dates requires

$$\begin{aligned} q^* = \quad & (1-\rho)[\tfrac{\alpha_L}{2}(x_{L,s} + x_{L,b}) + (1-\alpha_L)x_{L,n}] \\ & +\rho[\tfrac{\alpha_H}{2}(x_{H,s} + x_{H,b}) + (1-\alpha_H)x_{H,n}]. \end{aligned} \tag{24}$$

We can now provide a definition of equilibrium.

Definition 1 *Given an initial money stock $\bar{M} > 0$ and a government policy specified by (π, τ), a competitive stationary monetary equilibrium is a time-invariant list of real quantities (c_j, y, q, x_{jk}, m_j) and cycle-dependent prices $(p_{1,t}, p_{2,t})$ that solve the agent's problems (6) and (12), satisfy (20), the government budget constraint (3), and market clearing (4), (21), and (24).*

Policy affects the return of money holdings and therefore agents' money holdings choices. Since cash available affect agents' ability to consume, policy decisions affect the efficiency of the allocation. The next result immediately follows.

Lemma 1 *Let $0 < \alpha_L < \alpha_H \leq 1$. In any stationary equilibrium we must have $\pi \geq \beta$.*

Proof By way of contradiction, suppose a monetary equilibrium exists with $\pi < \beta$. From (22) we need $\pi \geq \beta + \beta(\alpha_j/2)[u'(c_j) - 1] \geq \beta$. This is in contradiction with $\pi < \beta$.■

The lesson here is that the rate of return on money $\frac{1}{\pi}$ cannot be excessive in a stationary monetary equilibrium. Precisely, the upper bound for the return on money corresponds to the pure rate of time preference $\frac{1}{\beta}$, i.e., the shadow interest rate. Intuitively,

[3]This comes from pricing an illiquid bond. See Boel and Camera (2006), LW and Berentsen, Camera and Waller (2005) for details.

if $\frac{1}{\pi} > \frac{1}{\beta}$ then cash pays such a good return that a patient agent would want to keep accumulating money, which cannot be a stationary equilibrium. Thus, in what follows we investigate whether there is any $\pi \geq \beta$ that sustains the efficient allocation.

Lemma 2 *Consider $\pi > \beta$. A unique stationary monetary equilibrium exists and money holdings are heterogeneous, $0 < m_L < m_H < m^*$. As $\pi \rightarrow \beta$ we have $c_H \rightarrow c^*$ and $c_L \rightarrow c^*$.*

Proof From (22) we know that given an outcome with $m_H > 0$ and $m_L > 0$ we must have:

$$\pi = \beta\left\{1 + \tfrac{\alpha_H}{2}\left[u'\left(c_H\right) - 1\right]\right\}$$

$$\pi = \beta\left\{1 + \tfrac{\alpha_L}{2}\left[u'\left(c_L\right) - 1\right]\right\}$$

Note that $\pi \geq \beta$ is necessary. If $\pi > \beta$ then $c_L < c_H < c^*$ and $m_L < m_H < m^*$. As $\pi \rightarrow \beta$ then $u'\left(c_H\right) \rightarrow 1$ and $u'\left(c_L\right) \rightarrow 1$. Therefore, $m_H \rightarrow m^*$ and $m_L \rightarrow m^*$. Thus, the Friedman rule can achieve the efficient allocation. Existence follows from inspection of the individual optimality and market clearing conditions.■

Lemma 2 has several implications. First, only one steady state equilibrium exists. Second, type H agents take more money out of the second market than type L agents. As a consequence, the distribution of money has two mass points, with type H agents holding more money than type L agents. What is the intuition for this? Type H agents trade more frequently in the first market. Therefore, they bring more money to insure themselves against liquidity shocks. Last, both type H and type L agents are liquidity constrained in the first market so that trades are not efficient, i.e. $c_j < c^*$ for $j = L, H$. As $\pi \rightarrow \beta$ all money holdings converge to m^* as agents become indifferent between having a dollar today or one tomorrow. In this case, trade-frequency considerations do not enter saving decisions.

4.4 Heterogeneity in time preferences

In this section we consider agents differing with respect to their time preferences β_j. Type H and type L agents discount the future at rate β_H and β_L respectively, with

$0 < \beta_L < \beta_H < 1$. Agents have identical probability of trade, i.e. $\alpha_j = \alpha$, and sellers have identical linear (unit) costs of production in the first market, i.e. $\phi_j(y_j) = y_j$. The equilibrium in this economy can be characterized as follows (for full derivations, see [11]). The expected lifetime utility of agent j entering a period with m_j must satisfy (12) given $y_j = y$ for all j.

From the seller's problem, we know $p = 1$, and from the buyer's problem we have that $c_j = \min(m_j, c^*)$. Moreover, goods market clearing requires

$$y_j = y = \rho c_H + (1-\rho)c_L \quad \text{for} \;\; j = H, L \tag{25}$$

Given an outcome with $m_j > 0$ the agent's optimal portfolio choice must satisfy

$$\pi = \beta_j \left\{1 + \tfrac{\alpha}{2}[u'(c_j) - 1]\right\} \tag{26}$$

The intuition for (26) is analogous to the one for (22). Since the nominal interest rate for type j agent is $i_j = \frac{\pi}{\beta_j}$ (26) can be written as

$$i_j = \tfrac{\alpha}{2}[u'(c_j) - 1] \;\; . \tag{27}$$

We will use (27) extensively in our quantitative analysis. Goods market clearing on even dates requires

$$\begin{aligned} q^* = \;\; & (1-\rho)[\tfrac{\alpha}{2}(x_{L,s} + x_{L,b}) + (1-\alpha)x_{L,n}] \\ & +\rho[\tfrac{\alpha}{2}(x_{H,s} + x_{H,b}) + (1-\alpha)x_{H,n}]. \end{aligned} \tag{28}$$

The definition of equilibrium follows the layout of Definition 1 with the obvious modification. That is, we must account for equations (25) and (28).

As proved in [11], in any stationary monetary equilibrium we must have $\pi \geq \beta_H$. The lesson is that the rate of return on money $\frac{1}{\pi}$ cannot be excessive in a stationary monetary equilibrium. Precisely, the upper bound for the return on money corresponds to the lowest pure rate of time preference $\frac{1}{\beta_H}$, i.e., the shadow interest rate. Intuitively, if $\frac{1}{\pi} > \frac{1}{\beta_H}$ then cash pays such a good return that a patient agent would want to keep accumulating money, which cannot be a stationary equilibrium. The implication is policy makers are constrained in their ability to give cash a return that is sufficiently attractive for everyone. Thus, inefficiencies are to be expected when saving can only take the form of cash.

Indeed, in [11] we prove that the allocation is inefficient for all $\pi \geq \beta_H$. The intuition is that since discounting disparities in equilibrium impose the restriction $\pi \geq \beta_H$, the more impatient agents tend to under-insure because their shadow interest rate $\frac{1}{\beta_L}$ exceeds the rate of return on money $\frac{1}{\pi}$. This leaves them liquidity constrained in market one, which creates an inefficiency. Of course, setting $\pi \to \beta_H$ leads to a second best (since $m_H \to m^*$).

4.5 Heterogeneity in disutility from production

In this section we consider agents with different and non-linear marginal disutilities from labor on odd dates. Production of y output generates disutility $\phi_j(y)$ with $j \in \{L, H\}$ and $\phi'_L(y) > \phi'_H(y)$. The expected lifetime utility of agent j entering a period with m_j must satisfy (12) given $\alpha_j = \alpha$.

Using (1) and (7), we have that prices must satisfy

$$p = \phi'_j(y_j). \tag{29}$$

Indeed, if $p > \phi'_j(y_j)$ then production would be infinite on odd dates, whereas if $p > \phi'_j(y_j)$ there would be no production on odd dates. Note that (29) implies that $\phi'_H(y_H) = \phi'_L(y_L)$ and therefore $y_L < y_H$ since $\phi'_H(y_H) > \phi'_L(y_L)$. Since prices are taken as given and agents of type L face a higher disutility from production on odd dates, their production is lower.

Goods market clearing on odd dates implies:

$$\rho y_H + (1 - \rho) y_L = \rho c_H + (1 - \rho) c_L \tag{30}$$

Given an outcome with $m_j > 0$, the agent's optimal portfolio choice must satisfy

$$\pi = \beta \left\{ 1 + \tfrac{\alpha}{2p} [u'(c_j) - p] \right\} \ . \tag{31}$$

The intuition for (31) is analogous to the one for (22).

Since the net nominal interest rate is equal to $\frac{\pi}{\beta} - 1$, then we can rewrite (31) as $i = \frac{\alpha}{2p}[u'(c_j) - p]$. Since equation (31) has to hold for all j, it's clear that $c_j = c$ and therefore (31) becomes

$$i = \frac{\alpha}{2p}[u'(c) - p] \quad . \tag{32}$$

Therefore, using (29) equation (32) becomes

$$i = \frac{\alpha}{2\phi_j'(y_j)}[u'(c) - \phi_j'(y_j)] \quad . \tag{33}$$

We will use (33) extensively in our quantitative analysis. Goods market clearing on even dates requires

$$\begin{aligned} q^* = \quad & (1-\rho)[\tfrac{\alpha}{2}(x_{L,s} + x_{L,b}) + (1-\alpha)x_{L,n}] \\ & +\rho[\tfrac{\alpha}{2}(x_{H,s} + x_{H,b}) + (1-\alpha)x_{H,n}]. \end{aligned} \tag{34}$$

Note that since $m_j = c_j = c$ and $y_H > y_L$ from (29), equation (5) implies that $x_{Hs} < x_{Ls}$, i.e. the labor effort of type L agents in market 2 has to be higher than the one of type H agents to make up for lower sales in market 1. The definition of equilibrium follows the layout of Definition 1 with the obvious modifications. That is, we must account for equations (29), (30) and (34).

In this setting, once again, we get a result identical to that in Lemma 1, i.e., in any stationary equilibrium we must have $\pi \geq \beta$. Therefore, we prove the following result.

Lemma 3 *Consider $\pi > \beta$. A unique stationary monetary equilibrium exists and money holdings are inefficient, i.e. $m < m^*$. As $\pi \to \beta$ we have $c \to c^*$ where c^* corresponds to the allocation chosen by a social planner not subject to informational frictions.*

Proof. From (32) we have

$$i = \frac{\alpha}{2p}[u'(c) - p]$$

Moreover, $\pi \geq \beta$ is necessary. If $\pi > \beta$ then $i > 0$, implying $c < c^*$ and $m < m^*$. As $\pi \to \beta$ then $u'(c) \to p$, implying that $c \to c^*$ and $m \to m^*$. Thus, the Friedman rule can achieve the efficient allocation. Note that the Friedman rule is able to achieve the unconstrained efficient allocation, i.e. the allocation that would be chosen by a social planner not subject to informational frictions. Existence easily follows from inspection of the individual optimality and market clearing conditions.■

Lemma 3 has several implications. First, only one steady state equilibrium exists. Second, type H agents take the same amount of real balances as type L agents out of the second market. This is because agents here are only heterogeneous in their marginal disutilities from production, which are not influenced by the amount of real money balances agents own. Therefore, both type H and type L agents carry the same real money balances in the first market. Last, both type H and type L agents are liquidity constrained in the first market so that trades are not efficient, i.e. $c_j < c^*$ for $j = L, H$. As $\pi \to \beta$ all money holdings converge to m^* as agents become indifferent between having a dollar today or one tomorrow. Fourth, the Friedman rule allows agents to reach the allocation that would be chosen by a social planner with no informational restrictions.

5 Quantitative analysis

In this section we parameterize the model and evaluate its implications in terms of welfare cost of inflation. We start by focusing on a representative agent economy, as this allows us to determine the value of the preference parameters that are common across agents, as well as the average value of the parameters with respect to which agents differ. In later sections we introduce heterogeneity components to the analysis.

As long as we consider a representative agent model, we let $\alpha_j = \alpha$, $\beta_j = \beta$ and $\phi_j(y) = \phi(y)$ for $j = H, L$. It is straightforward to show that in this case the relative price p must satisfy:

$$p = \phi'(y), \tag{35}$$

and given $m > 0$ the agent's optimal portfolio choice must satisfy

$$i = \frac{\alpha}{2\phi'(y)}[u'(c) - \phi'(y)]. \tag{36}$$

Moreover, from (16) we must have $pc = m$.

We consider the following functional forms: in the first market, $u(c) = c^{1-a}/(1-a)$, with $a > 0$ and $\phi(y) = \frac{y^\delta}{\delta}$ with $\delta \geq 1$; in the second market, $U(q) = A\ln(q)$, which implies

$q^* = A$. We consider a yearly model, mainly to facilitate comparison with LW and Lucas (2000). The annual rate of time preference is $r = 0.04$. The vector of parameters to identify is therefore $\Theta = (\alpha, \beta, a, A, \delta)$.

Two parameters can be assigned numbers to quite straightforwardly. Specifically, we set $a = 1$[4] (actually 1.001) and $\beta = 0.96$.

The remaining parameters require some more thought. For the parameter δ we consider two cases. In the first one, $\delta = 1$ and the disutility of labor is linear, i.e. $\phi(y) = y$ as in Sections 4.3 and 4.4. In the second case, the disutility of labor is convex as in Section 4.5. As shown in the appendix, the elasticity of the disutility of labor with respect to the labor effort is equal to δ and the elasticity of the labor supply with respect to p is equal to $1 - \frac{1}{\delta}$. Therefore, we can set δ to match the average elasticity of the labor supply with respect to wage in the U.S. economy. However, there is no consensus in the literature over this parameter and estimates vary depending on the group one focuses on (e.g. men and women). We follow the estimates provided by Blau and Kahn (2005) and set δ to match the average elasticity of labor supply for men and women in the period 1980-2000. This turns out to be equal to 0.29, thus implying $\delta = 4.45$.

The parameter α is set so that the theoretical elasticity of money demand matches the estimated elasticity reported by Aruoba et al. (2007), i.e. $\varepsilon = -0.226$. In the appendix we show that $\varepsilon_{\text{money demand}} = -\frac{2i}{(2i+\alpha)a}$ for both the case of linear and convex disutility. Convex and linear disutility of labor imply $\alpha = 0.28$ and $\alpha = 0.38$ respectively.

Last, we determine A to fit $L = M/PY$, where P is the nominal price level and Y is real output. As suggested in LW and Lucas (2000), L can be interpreted as money demand because real balances M/P are proportional to real spending Y, with a factor of proportionality $L(i)$ that depends on the nominal interest rate i. For the empirical L, we consider U.S. data for the sample period 1929-2005 and we measure i by the short term commercial paper rate, M by M_1, P by the GDP deflator and Y by real GDP.[5] Then,

[4]See Raj (2006) for a recent study that estimates the coefficient of risk aversion. The mean for a is ≈ 1.

[5]For 1929-75, the short term commercial paper rate is from Friedman and Schwartz (1982, Table 4.8, Column 6). For 1976-1996, it is from the *Economic Report of the President* (1996, Table B-69). For 1997-

we construct L in the model. In the case of a representative agent, nominal output is $p_1 \frac{\alpha}{2} c$ in the first market and $p_2 q^* = p_2 A$ in the second one. Therefore, overall nominal output is $PY = p_1 \frac{\alpha}{2} c + p_2 A$. From (4), we know that in equilibrium the nominal money stock is $M = p_2 m$, so that normalizing everything by p_2 in the model L becomes:

$$L = \frac{m}{\frac{\alpha}{2} pc + A} \tag{37}$$

In the Appendix we derive the theoretical L as a function of the parameters in the cases of linear and convex disutility of labor. Given the values assigned to α, β, a, and δ we can pin down A by minimizing the difference between the theoretical and the empirical L.

In the case of linear disutility of labor we find $\Theta = (0.38, 0.96, 1.00, 3.18, 1.00)$, whereas when disutility is convex we have $\Theta = (0.28, 0.96, 1.00, 1.20, 4.45)$. The parameter vector Θ and the specified functional forms are then used to quantify the welfare cost of inflation, with a procedure analogous to the one used in LW. Specifically, the welfare cost of inflation is defined as the fraction of consumption people would require as compensation to make them indifferent between a steady state with inflation rate $\gamma = \pi - 1$ and an identical steady state with zero inflation.

For the representative agent, the expected utility w_π given $\pi > 1$ is the sum of the surplus from trade in the first and the second market, i.e.,

$$(1-\beta) w_\pi = \frac{\alpha}{2} [u(c_\pi) - c_\pi] + U(q^*) - q^*, \tag{38}$$

where c_π is equilibrium consumption on odd dates given a rate of growth of the money stock π. If we reduce π to 1 so that $\gamma = 0$, and we reduce consumption in both markets by $\bar{\Delta}_1$, the expected utility becomes:

$$(1-\beta) w_1 = \frac{\alpha}{2} [u(\bar{\Delta}_1 c_1) - c_1] + U(\bar{\Delta}_1 q^*) - q^* \tag{39}$$

2005, it is the Financial Commercial Paper with maturity 3-month, from the *Federal Reserve Statistical Release*.

The money supply is M1 in billions of dollars, December of each year, not seasonally adjusted. For 1929-58, it is from Friedman and Schwartz (1982, pp. 708-718, Column 7). For 1959-2005, it is from the Federal Reserve Bank of St. Louis *FRED Database*.

For the period 1929-2005, nominal GDP is from the *National Income and Product Accounts of the U.S.*.

The welfare cost of having positive instead of zero inflation is the value $\Delta_1 = 1 - \bar{\Delta}_1$ such that $w_\pi = w_1$, i.e. agents would give up Δ_1 percent of consumption to have zero inflation. Similarly, if we reduce π to β (the Friedman rule) and consumption by $\bar{\Delta}_\beta$, the expected utility becomes:

$$(1-\beta)w_\beta = \frac{\alpha}{2}[u(c^*\bar{\Delta}_\beta) - c^*] + U(\bar{\Delta}_\beta q^*) - q^* \tag{40}$$

The welfare cost of having positive inflation instead of implementing the Friedman rule is the value $\Delta_\beta = 1 - \bar{\Delta}_\beta$ such that $w_\pi = w_\beta$, i.e. agents would give up Δ_β percent of consumption to be at the Friedman rule.

We find that for the representative agent ten percent inflation is worth less than 1% of consumption: $\Delta_1 = 0.66\%$ and $\Delta_\beta = 0.76\%$ with linear disutility and $\Delta_1 = 0.70\%$ and $\Delta_\beta = 0.79\%$ with convex disutility. Now we proceed to study how different kinds of heterogeneity affect the welfare cost of inflation.

5.1 Heterogeneity in trading shocks

In this section we investigate how heterogeneity in trading shocks affects the welfare cost of inflation, given the environment described in Section 4.3.

From (3), (10), (17) and Lemma 2 the expected utility $w_{j\pi}$ for an agent of type j with $j = H, L$ given $\pi > 1$ is:

$$\begin{aligned}(1-\beta)w_{j\pi} = \; & \tfrac{\alpha_j}{2}[u(c_{j\pi}) - c_{j\pi}] + U(q^*) - q^* \\ & + m_{j\pi}(1-\pi) + (\rho m_{H\pi} + (1-\rho)m_{L\pi})(\pi - 1)\end{aligned} \tag{41}$$

where $c_{j\pi}$ and $m_{j\pi}$ denote consumption and money holdings respectively for an agent of type j given π. The first and the second term in (41) are analogous to the ones in (38), but there are also two new terms. Specifically, $m_{j\pi}(1-\pi)$ is the inflation tax levied on an agent of type j, whereas $(\rho m_{H\pi} + (1-\rho)m_{L\pi})(\pi - 1)$ is the government's lump-sum transfer received by the same agent and it is a weighted average of money holdings of type H and type L agents. Therefore, we can interpret $m_{j\pi}(1-\pi)+(\rho m_{H\pi}+(1-\rho)m_{L\pi})(\pi-1)$ as an overall inflation subsidy from type H to type L agents. Note that this term does not appear in (38) because in the case of a representative agent the inflation tax $m_\pi(1-\pi)$ is exactly offset by the lump-sum transfer $m_\pi(\pi-1)$.

If we reduce π to 1 and consumption by $\bar{\Delta}_{j1}$ the expected utility of an agent j becomes:

$$(1-\beta)w_{j1} = \frac{\alpha_j}{2}[u(\bar{\Delta}_{j1}c_{j1}) - c_{j1}] + U(\bar{\Delta}_{j1}q^*) - q^* \tag{42}$$

If $\pi = 1$ the inflation rate is zero and therefore agents are not subject to the inflation tax, i.e. $m_{j1}(1-\pi) = 0$. Since the money stock is not growing, government lump-sum transfers are also equal to zero, i.e. $(\rho m_{H1} + (1-\rho)m_{L1})(\pi - 1) = 0$. The welfare cost of having positive instead of zero inflation for an agent j is the value $\Delta_{j1} = 1 - \bar{\Delta}_{j1}$ such that $w_{j\pi} = w_{j1}$, i.e. he would give up Δ_{j1} percent of consumption to have zero inflation.

If instead we reduce π to β (the Friedman rule) and consumption by $\bar{\Delta}_{j\beta}$, the expected utility becomes:

$$(1-\beta)w_{j\beta} = \frac{\alpha_j}{2}[u(c^*\bar{\Delta}_{j\beta}) - c^*] + U(q^*\bar{\Delta}_{j\beta}) - q^* \tag{43}$$

We know from Lemma 2 that if $\pi = \beta$, m_j converges to the efficient quantity m^* for $j = H, L$ and therefore the inflation tax $m^*(1-\pi)$ is exactly offset by the lump-sum transfer $(\rho m^* + (1-\rho)m^*)(\pi - 1)$. So, the third and the fourth elements in (41) don't appear in (43) either. The welfare cost of having positive inflation instead of the Friedman rule for an agent j is the value $\Delta_{j\beta} = 1 - \bar{\Delta}_{j\beta}$ such that $w_{j\pi} = w_{j\beta}$, i.e. he would give up $\Delta_{j\beta}$ percent of consumption to be at the Friedman rule.

In order to measure the effects of heterogeneity in trading shocks on the welfare cost of inflation, we proceed as follows. First, since we are considering agents with identical preferences we keep $(a,\ A) = (0.59, 2.05)$ from the representative agent case. Second, we use a mean preserving spread on α, i.e. we maintain the average friction α constant by considering linear combinations of α_H and α_L such that $\rho\alpha_H + (1-\rho)\alpha_L = 0.21$. In particular, we consider an economy with the same proportion of type H and type L agents, i.e. $\rho = 0.5$ and $0.5\alpha_H + 0.5\alpha_L = 0.21$. As determining α_L necessarily pins down α_H, we can conduct our analysis in terms of α_L.

We find that the welfare cost of inflation increases with the probability of trade. Therefore, as long as $\alpha_H > \alpha_L$ the welfare cost of inflation is higher for type H agents.

This is not surprising and it happens because if $\alpha_H > \alpha_L$ type H agents have higher money balances than type L agents to finance their higher consumption needs.

What is surprising, instead, is that positive inflation can even be welfare-increasing for agents with very low consumption risk, i.e. α_L close to zero. The reason this happens is that since $0.5\alpha_H + 0.5\alpha_L = 0.21$, when α_L is close to zero then α_H is close to its upper bound. Therefore, type L agents save with very little as their probability of consumption is very low. Type H agents, instead, have a relatively high probability of consumption and therefore they bring a lot of money into the first market to support their consumption needs. As a consequence, type H agents are forced to pay an inflation tax $m_{H\pi}(1-\pi)$ that is much higher than the lump-sum transfers they receive, i.e. $m_{H\pi}(1-\pi) + (0.5m_{H\pi} + 0.5m_{L\pi})(\pi - 1) < 0$. Conversely, the government lump-sum transfers received by type L agents are much higher than the inflation tax they have to pay, i.e. $m_{L\pi}(1-\pi) + (0.5m_{H\pi} + 0.5m_{L\pi})(\pi - 1) > 0$ and type L agents can be made better off by inflation. This can be seen in Figures 1 and 2 where we plot $\bar{\Delta}_{j1}$ and $\bar{\Delta}_{j\beta}$ against the probability of trade for type L agents.

5.2 Heterogeneity in time preferences

In this section we study how heterogeneity in time preferences affects the welfare cost of inflation, given the environment described in Section 4.4.

In this case, the expected utility for an agent of type j with $j = H, L$ given $\pi > 1$ is:

$$\begin{aligned}(1-\beta_j)w_{j\pi} = \;& \tfrac{\alpha}{2}[u(c_{j\pi}) - c_{j\pi}] + U(q^*) - q^* \\ & + m_{j\pi}(1-\pi) + (\rho m_{H\pi} + (1-\rho)m_{L\pi})(\pi - 1)\end{aligned} \tag{44}$$

Note that as opposed to (41) now agents face identical trading shocks and differ with respect to their discount factors. If we reduce π to 1 and consumption by $\bar{\Delta}_{j1}$ the expected utility of agent j becomes

$$(1-\beta_j)w_{j1} = \frac{\alpha}{2}[u(\bar{\Delta}_{j1}c_{j1}) - c_{j1}] + U(\bar{\Delta}_{j1}q^*) - q^*, \tag{45}$$

where both inflation tax and government lump-sum transfers are equal to zero. If we reduce π to β_H[6] and consumption by $\bar{\Delta}_{H\beta_H}$, the expected utility for type H agents becomes:

$$\begin{aligned}(1-\beta_H)w_{H\beta_H} = \;& \tfrac{\alpha}{2}[u(\bar{\Delta}_{H\beta_H}c^*) - c^*] + U(\bar{\Delta}_{H\beta_H}q^*) - q^* \\ & + \bar{\Delta}_{H\beta_H}m^*(1-\beta_H) + (\rho\bar{\Delta}_{H\beta_H}m^* + (1-\rho)\bar{\Delta}_{L\beta_H}m_{L\beta_H})(\beta_H - 1)\end{aligned} \tag{46}$$

Note that if $\pi = \beta_H$ patient agents achieve the efficient allocation whereas impatient ones are liquidity constrained, i.e. $m_{L\beta_H} < m^*$ and $c_{L\beta_H} < c^*$. Note also that $m^*(1-\beta_H) > 0$ as the value of money holdings increases with deflation, and $(\rho m^* + (1-\rho)m_{L\beta_H})(\beta_H - 1) < 0$ as the government needs to withdraw money from the system in order to implement a deflationary policy.

Similarly, if we reduce π to β_H and consumption by $\bar{\Delta}_{L\beta_H}$ the expected utility for type L agents becomes:

$$\begin{aligned}(1-\beta_L)w_{L\beta_H} = \;& \tfrac{\alpha}{2}[u(\bar{\Delta}_{L\beta_H}c_{L\beta_H}) - c_{L\beta_H}] + U(\bar{\Delta}_{L\beta_H}q^*) - q^* \\ & + \bar{\Delta}_{L\beta_H}m_{L\beta_H}(1-\beta_H) + (\rho\bar{\Delta}_{H\beta_H}m^* + (1-\rho)\bar{\Delta}_{L\beta_H}m_{L\beta_H})(\beta_H - 1)\end{aligned} \tag{47}$$

The definitions of Δ_{j1} and $\Delta_{j\beta_H}$ are standard.

In order to quantify the effects of heterogeneity in time preferences on the welfare cost of inflation, we proceed as follows. First, we keep the calibrated parameters (α, a, A) $= (0.21, 0.59, 2.05)$ from the representative agent. Then, we use a mean preserving spread on β, i.e. we keep the average discount factor constant at 0.96 and look at different combinations of β_H and β_L such that $\rho\beta_H + (1-\rho)\beta_L = 0.96$. Specifically, we consider an economy with the same proportion of type H and type L agents, i.e. $\rho = 0.5$. We keep β_H and β_L within intervals that are considered reasonable according to the studies surveyed in Frederick et al. (2002). As determining β_L necessarily pins down β_H, we can conduct our analysis in terms of β_L.

We find that as long as $\beta_L < \beta_H$ the welfare cost of inflation is higher for type H agents as they hold higher money balances. We also find that the welfare cost of inflation

[6] We know from Boel and Camera (2006) that discounting disparities in equilibrium impose the restriction $\pi \geq \beta_H$.

for impatient agents increases with β_L. This is due to the fact that a higher β_L implies a lower shadow interest rate $\frac{\pi}{\beta_L}$ and therefore higher money holdings m_L and a higher inflation tax $m_{L\pi}(1-\pi)$.

The result is somewhat different for the case of type H agents, as we find that their welfare cost of inflation is non-monotonic in β_L. The intuition is the following. Given that our analysis is conducted under the assumption of a constant average discount rate, when β_L increases then β_H must necessarily decrease. This leads to a higher shadow interest rate $\frac{\pi}{\beta_H}$ for type H agents, thus implying a higher opportunity cost of holding money and lower money holdings m_H. On the intensive margin, this leads to lower consumption c_H and therefore a utility loss. On the extensive margin, instead, consumption c_H increases because the inflation subsidy from type H to type L agents $m_{H\pi}(1-\pi)+(\rho m_{H\pi}+(1-\rho)m_{L\pi})(\pi-1)$ decreases. We find that for high values of β_H the intensive margin dominates the extensive one, hence the increasing welfare cost of inflation. As β_H decreases, the extensive margin prevails and the welfare cost of inflation starts decreasing for type H agents. Figures 3 and 4 illustrate our results.

5.3 Heterogeneity in disutility from production

In this section we investigate how heterogeneity in disutility from production affects the welfare cost of inflation, given the environment described in Section 4.5.

We proceed as follows. First, since agents have identical preferences and face identical trading shocks we keep $\alpha = 0.38$, $\beta = 0.96$, $a = 1.00$, and $A = 3.18$ as in the case of a representative agent with convex disutility from labor. Second, as agents face different disutilities from labor, we let $\phi_j(y) = \frac{y^{\delta_j}}{\delta_j}$ with $j = H, L$ and $\delta_H < \delta_L$. In our quantitative analysis, we use a mean preserving spread on δ by considering linear combinations of δ_H and δ_L such that $\rho\delta_H + (1-\rho)\delta_L = 4.45$, i.e. we keep the elasticity of labor of the representative agent constant. Specifically, we consider an economy with the same proportion of type H and type L agents, i.e. $\rho = 0.5$ and therefore $0.5\delta_H + 0.5\delta_L = 4.45$. We then use our parameters to determine the welfare cost of inflation. As determining δ_L necessarily pins down δ_H, we can conduct our analysis in terms of δ_L.

The expected utility for an agent of type j with $j = H, L$ given $\pi > 1$ in this case is:

$$(1-\beta)w_{j\pi} = \frac{\alpha}{2}[u(c_\pi) - pc_\pi] + \frac{\alpha}{2}[py_{j\pi} - \phi_j(y_{j\pi})] + U(q^*) - q^* \quad (48)$$

The first and the second term in (48) represent surplus from trade in the first market for a buyer and a seller respectively. Note that the seller's surplus was zero in (38), (41) and (44) as (unit) production costs were linear. The third term is the surplus from trade in the second market. Note that neither inflation tax nor government transfers appear in (48). This is because agents of type L and H have the same money holdings and therefore the inflation tax is exactly offset by government transfers.

If we reduce π to 1 and consumption and income by $\bar{\Delta}_{j1}$ the expected utility of an agent j becomes

$$(1-\beta)w_{j1} = \frac{\alpha}{2}[u(\bar{\Delta}_{j1}c_1) - p\bar{\Delta}_{j1}c_1] + \frac{\alpha}{2}[p\bar{\Delta}_{j1}y_{j1} - \phi_j(y_{j1})] + U(\bar{\Delta}_{j1}q^*) - q^*. \quad (49)$$

In this case we have to reduce not only consumption c_1 on odd dates and q^* on even dates, but also the sellers' income on odd dates py_{j1}. Moreover, we need to reduce the cost of consumption on odd dates pc_1 as consumption itself decreases. Similarly, if we reduce π to β (the Friedman rule) and consumption and income by $\bar{\Delta}_{j\beta}$, the expected utility becomes

$$(1-\beta)w_{j\beta} = \frac{\alpha}{2}[u(\bar{\Delta}_{j\beta}c^*) - p\bar{\Delta}_{j\beta}c^*] + \frac{\alpha}{2}[p\bar{\Delta}_{j\beta}y_{j\beta} - \phi_j(y_{j\beta})] + U(\bar{\Delta}_{jF}q^*) - q^*. \quad (50)$$

The definitions of Δ_{j1} and $\Delta_{j\beta}$ are standard.

We find that the welfare cost of inflation is higher for agents of type H, i.e. agents who face a lower disutility from production. This might seem surprising at first, as type H and type L agents hold the same money balances in equilibrium. However, inflation affects agents in two ways here. First, as soon as we diverge from the Friedman rule agents are liquidity constrained and suffer a utility loss. Such loss affects them identically, as they hold identical money balances. Second, the surplus from trade in market one is higher for agents with a low disutility of production (type H). Thus, they have an incentive to substitute labor effort from market 2 to market 1 in every period. As soon as we diverge from the Friedman rule, underproduction occurs in market 1 and type H agents are affected the most by it, thus the higher welfare cost of inflation. Figures 5 and 6 illustrate our results.

6 Final remarks

We construct a monetary economy where agents hold cash to insure against consumption risk, trade in large competitive markets, and are ex-ante heterogeneous in either their matching profiles or discount factors. We demonstrate that, for some forms of heterogeneity, different agent types hold different money balances in competitive equilibrium.

We then calibrate the model for the U.S. economy and use it to quantify the welfare cost of inflation. Even though at the aggregate level our estimates confirm the results of previous work, we find that the distributional effects of inflation can be important. Specifically, we find that positive inflation can be welfare increasing for agents with low consumption risk. These agents save with little cash and enjoy low consumption, but they pay a low inflation tax and receive high lump-sum transfers. As the opposite is true for agents with a high consumption risk, we observe a redistribution of wealth via the inflation tax. In the cases of heterogeneity in discounting and disutility from production, different agents face different - but always positive - welfare costs of inflation.

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Appendix

The constrained-efficient allocation

When the social planner is subject to the same spatial and informational frictions of agents, the planning problem corresponds to a sequence of static maximization problems subject to the technological constraints.

Recall that we are assuming that the planner weighs each agent identically and that the planner cannot recognize agents' types. On each date agents have identical preferences ex-ante and there is an identical proportion of buyers and sellers. Moreover, on each odd date agents that are active can produce or consume with equal probability and the planner maximizes expected utility of a representative agent, subject to technological feasibility. Recall that in the case of heterogeneity in trading shocks or time preferences, agents have identical and linear disutility from production. Hence, the planner's problem is:

$$\begin{aligned} &\max_{c,y} \quad \tfrac{1}{2}[u(c) - y] \\ &\text{s.t.} \qquad c = y \end{aligned}$$

When instead agents differ in their disutilities from production, the social planner problem becomes

$$\begin{aligned} &\max_{c,y} \quad \tfrac{1}{2}\rho[u(c) - \phi_H(y)] + \tfrac{1}{2}(1-\rho)[u(c) - \phi_L(y)] \\ &\text{s.t.} \qquad c = y \end{aligned}$$

On each even date the problem to be solved is the same for all three cases of heterogeneity:

$$\begin{aligned} &\max_{q,x} \quad U(q) - x \\ &\text{s.t.} \qquad q = x \end{aligned}$$

Hence, the constrained-efficient allocation is stationary across trading cycles, i.e. $c_j = y_j = c^*$ and $q_j = x_j = q^*$ for each type j in each cycle t.

Type shocks

We now consider the case in which agents receive a type shock in each period. Specifically, an agent is of type H with probability ρ and of type L with probability $1-\rho$. Everything else in the economy is identical to the case in which agents' types are fixed.

Heterogeneity in trading shocks

Agents receive a type shock at the beginning of each odd date. Before knowing the type shock he'll receive, each agent has a probability ρ of being of type H and a probability $1-\rho$ of being of type L. As in the case of fixed types, type H and type L agents have respectively a probability α_H and α_L of meeting a counterpart, with $0<\alpha_L<\alpha_H\leq 1$.

At the start of an even date, the agent's problem can be represented as follows:

$$W_j(m_{j,k}) = \max_{q_j,m'\geq 0}\{U(q_j)-q_j-\pi m'+m_{j,k}+\tau+\beta EV(m')\} \tag{51}$$

and (7), (8) and (9) still hold. Note that at the end of the second market agents do not know their type for the following cycle and therefore they all choose the same money holdings m' based on the expected continuation utility $EV(m')$:

$$EV(m) = \rho V_H(m) + (1-\rho)V_L(m) \tag{52}$$

Given that we are focusing on monetary outcomes, i.e. $m'>0$, we must have

$$1 = \tfrac{\beta}{\pi}\times\tfrac{\partial EV(m')}{\partial m'} \tag{53}$$

The intuition for (53) is analogous to the one for (11).

After an agent realizes his type shock j, his expected lifetime utility of entering a period with m must satisfy:

$$\begin{aligned} V_j(m) = \quad & \max\nolimits_{c_j\in[0,\frac{m}{p}]}\tfrac{\alpha_j}{2}[u(c_j)+W_j(m_{j,b})] \\ & +\max\nolimits_{y_j}\tfrac{\alpha_j}{2}[-\phi_j(y_j)+W_j(m_{j,s})]+(1-\alpha_j)W_j(m_{j,n}) \end{aligned} \tag{54}$$

The seller's problem is analogous to the case of fixed types and (14) still holds. Goods market clearing for odd dates is identical to (21).

The buyer's problem is similar to the case of fixed types, except for the fact that money holdings m do not depend on the agent's type, i.e. $m_{j,b} = m - pc_j$, so that (15) and (16) still hold.

To find the optimal cash holdings of an agent j we must calculate the expected marginal value of holding money, $\frac{\partial EV(m)}{\partial m}$:

$$\frac{\partial EV(m)}{\partial m} = \rho \frac{\partial EV_H(m)}{\partial m} + (1-\rho) \frac{\partial EV_L(m)}{\partial m} \tag{55}$$

where $\frac{\partial V_j(m)}{\partial m}$ satisfies (19) for $j = H, L$.

Now we can calculate the equilibrium marginal value of money. Specifically,

$$\frac{\partial EV(m)}{\partial m} = \rho[1 + \tfrac{\alpha_H}{2}(u'(c_H) - 1)\tfrac{\partial c_H}{\partial m}] + (1-\rho)[1 + \tfrac{\alpha_L}{2}(u'(c_L) - 1)\tfrac{\partial c_L}{\partial m}]$$

where $\frac{\partial c_j}{\partial m} = 1$ for $j = H, L$ if the agent is liquidity constrained, and zero otherwise. It follows that $EV(m)$ is strictly concave in cash holdings if at least a buyer is liquidity constrained, and linear otherwise:

$$\frac{\partial EV(m)}{\partial m} = \begin{cases} \rho[1 + \frac{\alpha_H}{2}(u'(c_H) - 1)] + (1-\rho)[1 + \frac{\alpha_L}{2}(u'(c_L) - 1)] & \text{if } m < m^* \\ 1 & \text{otherwise} \end{cases} \tag{56}$$

Given an outcome with $m > 0$, the agent's optimal portfolio choice must satisfy

$$1 = \tfrac{\beta}{\pi}\left\{\rho[1 + \tfrac{\alpha_H}{2}(u'(c_H) - 1)] + (1-\rho)[1 + \tfrac{\alpha_L}{2}(u'(c_L) - 1)]\right\} \tag{57}$$

the intuition for which is analogous to the one for (22).

Goods market clearing on even dates is identical to (24). Given that all agents have identical money holdings, (4) becomes

$$\frac{M}{p_2} = m. \tag{58}$$

The definition of equilibrium is analogous to the layout of Definition 1 with the obvious modification. That is, we must account for equations (51), (52), (54), and (58).

In this environment, we again obtain a result identical to Lemma 1, i.e. in a stationary monetary equilibrium we must have we must $\pi \geq \beta$. Therefore, we prove the following result.

Lemma 4 *If $\pi > \beta$, then $c_L < c^*$ and $c_H < c^*$. If $\pi \to \beta$, then $c_L \to c^*$ and $c_H \to c^*$.*

Proof. From (57) we know that if $m > 0$ then $\frac{\pi-\beta}{\beta} = \frac{\alpha_H}{2}\rho(u'(c_H)-1)+\frac{\alpha_L}{2}(1-\rho)(u'(c_L)-1)$. Note that $\pi \geq \beta$ is necessary from Lemma 1. If $\pi > \beta$, then $\frac{\alpha_H}{2}\rho(u'(c_H)-1)+\frac{\alpha_L}{2}(1-\rho)(u'(c_L)-1) > 0$. Since $m_H = m_L = m$, then $u'(c_H) > 1$ and $u'(c_L) > 1$. This implies $c_H < c^*$ and $c_L < c^*$.

As $\pi \to \beta$ then $u'(c_H) \to 1$ and $u'(c_L) \to 1$. Therefore, $m \to m^*$, $c_H \to c^*$ and $c_L \to c^*$ and neither type of agent is cash constrained. Thus, the Friedman rule can achieve the efficient allocation. Existence easily follows from inspection of the individual optimality and market clearing conditions.■

Lemma 4 has several implications. First, away from the Friedman rule all agents are cash-constrained. Remember that in this environment all agents carry the same money holdings m since they don't know their type for the following periods. Second, as $\pi \to \beta$ money holdings m converge to m^* and therefore neither type L nor type H agents are cash constrained, i.e. $c_H \to c^*$ and $c_L \to c^*$.

Heterogeneity in time preferences

Agents receive a type shock at the end of each even date, after choosing their money holdings for the next period. Before knowing the type shock he'll receive, each agent has a probability ρ of being of type H and a probability $1-\rho$ of being of type L. As in the case of fixed types, type H and type L agents discount the future at rate β_H and β_L respectively, with $0 < \beta_L < \beta_H < 1$.The agent's problem at the start of an even date is:

$$W_j(m_{j,k}) = \max_{q_j, m' \geq 0} \{U(q_j) - q_j - \pi m' + m_{j,k} + \tau + \bar{\beta} EV(m')\} \tag{59}$$

where $\bar{\beta} = \rho\beta_H + (1-\rho)\beta_L$ and (7), (8) and (9) still hold. Now we can calculate the equilibrium marginal value of money. Specifically, (56), holds with $\alpha_H = \alpha_L = \alpha$. Given an outcome with $m > 0$, the agent's optimal portfolio choice must satisfy

$$1 = \frac{\bar{\beta}}{\pi}\left\{\rho[1 + \tfrac{\alpha}{2}(u'(c_H) - 1)] + (1-\rho)[1 + \tfrac{\alpha}{2}(u'(c_L) - 1)]\right\} \tag{60}$$

for which the intuition is analogous to the one for (26).

Goods market clearing on even dates is identical to (28). Given that all agents have identical money holdings, (58) still holds. The definition of equilibrium is analogous to the layout of Definition 1 with the obvious modification. That is, we must account for equations (51), (54), (52) and (58).

In this environment, we again obtain a result identical to Lemma 1, i.e. in a stationary monetary equilibrium we must have we must $\pi \geq \beta$. Therefore, we prove the following result.

Lemma 5 *Consider $\pi > \bar{\beta}$. Then $c_L = c_H < c^*$. If $\pi \to \bar{\beta}$, then $c_L \to c_L^*$ and $c_H \to c_H^*$.*

Proof. From (60) we know that if $m > 0$ then $\frac{\pi - \bar{\beta}}{\bar{\beta}} = [\rho[1 + \frac{\alpha}{2}(u'(c_H) - 1)] + (1-\rho)[1 + \frac{\alpha}{2}(u'(c_L) - 1)]]$. Note that $\pi \geq \bar{\beta}$ is necessary. If $\pi > \bar{\beta}$ then we know that $u'(c_H) > 1$ and $u'(c_L) > 1$ and both types of agents are cash constrained. Since $m_H = m_L = m$, then $c_H = c_L < c^*$. As $\pi \to \bar{\beta}$ then $u'(c_H) \to 1$ and $u'(c_L) \to 1$. Therefore, $m \to m^*$, $c_H \to c^*$ and $c_L \to c^*$ and neither type of agent is cash constrained. Thus, the Friedman rule can achieve the efficient allocation. Existence easily follows from inspection of the individual optimality and market clearing conditions.■

Lemma 5 has several implications. Remember that in this environment all agents carry the same money holdings since the choice for m' takes place before agents know their type. First, if $\pi > \bar{\beta}$ then both types of agents are cash constrained. That's because $\pi > \bar{\beta}$ implies $\frac{1}{\pi} < \frac{1}{\bar{\beta}}$ and therefore the real return on money is lower than the expected desired return of agents. This implies that agents don't carry enough money and they find themselves cash-constrained in the following period. If $\pi \to \bar{\beta}$ money holdings converge to m^* and therefore neither type L nor type H agents are cash constrained, i.e. $c_H \to c^*$ and $c_L \to c^*$. What is interesting is that in order to achieve the efficient allocation the real return on money doesn't need to match the desired return of agents after they know their type. Since agents choose money holdings before knowing their type, what matters is that the real return on money matches the expected desired return of agents. This ensures that the opportunity cost of holding money is eliminated and agents choose $m = m^*$.

Heterogeneity in disutility from production

Agents receive a type shock at the beginning of each odd date. Before knowing the type shock he'll receive, each agent has a probability ρ of being of type H and a probability $1-\rho$ of being of type L. As in the case of fixed types, production of y output generates disutility $\phi_j(y)$ with $j \in \{L, H\}$ and $\phi'_L(y) > \phi'_H(y)$. As heterogeneity in this economy only affects marginal production disutility, money holdings choices will not be affected by the uncertainty faced by agents. That is, all agents still hold the same m at the beginning of every period as in the case of fixed types. Therefore, Lemma 1 and Lemma 3 still hold in this environment, i.e. allocations are inefficient away from the Friedman rule and efficient when the Friedman rule is implemented.

Elasticity of disutility of labor and labor supply

In what follows we derive the relationship between elasticity of disutility of labor and elasticity of labor supply with respect to the relative price. In our model the disutility of labor is $\phi(y) = \frac{y^\delta}{\delta}$ so the elasticity is

$$\varepsilon_{\text{disutility}} = \frac{d\phi(y)/\phi(y)}{dy/y} = \frac{d\phi(y)}{dy} \times \frac{y}{\phi(y)} = \frac{y^{\delta-1}y}{y^\delta}\delta = \delta.$$

Alternatively, recall that $\varepsilon_{\text{disutility}} = \frac{d\ln\phi(y)}{d\ln y}$, since $d\ln\phi(y) = \frac{d\phi(y)}{\phi(y)}$, $d(\ln y) = \frac{dy}{y}$. We have that the differential

$$d\ln\phi(y) = d(\ln\frac{y^\delta}{\delta}) = d(\delta\ln y - \ln\delta) = \frac{\delta}{y}dy$$

and so we have $\varepsilon_{\text{disutility}} = \delta$.

From the FOC of the producer (35) we have that $\phi'(y) = p$. Therefore, the labor supply $y(p)$ satisfies

$$y^{\delta-1} = p \Rightarrow y(p) = p^{\frac{1}{\delta-1}}.$$

Using the same procedure as above the elasticity is

$$\varepsilon_{\text{labor supply}} = \frac{dy(p)/y(p)}{dp/p} = \frac{d\ln y(p)}{d\ln p}$$

Since $d\ln p = \frac{1}{p}dp$ and the differential

$$d\ln y(p) = d(\ln p^{\frac{1}{\delta-1}}) = d(\frac{1}{\delta-1}\ln p) = \frac{1}{\delta-1} \times \frac{1}{p}dp$$

we have

$$\varepsilon_{\text{labor supply}} = \frac{d \ln y(p)}{d \ln p} = \frac{1}{\delta - 1}$$

Elasticity of money demand

From (16) we must have $pc = m$, so the Euler equation (36) for the representative agent becomes

$$F(\tfrac{m}{p}, i) = \quad \tfrac{\alpha}{2\phi'(y)}[u'(\tfrac{m}{p}) - \phi'(y)] - i = 0.$$

Using the implicit function theorem we have

$$\frac{\partial m/p}{\partial i} = -\frac{\partial F(\frac{m}{p}, i)/\partial i}{\partial F(\frac{m}{p}, i)/\partial(m/p)} = \quad -\frac{-1}{\frac{\alpha}{2\phi'(y)}u''(\frac{m}{p})} = \frac{2\phi'(y)}{\alpha u''(\frac{m}{p})}$$

Given $pc = m$ and the market clearing condition $c = y$, the elasticity becomes

$$\varepsilon_{\text{money demand}} = \tfrac{2p\phi'(y)}{\alpha u''(c)} \times \tfrac{i}{pc} = \tfrac{2i\phi'(y)}{\alpha c u''(c)} \tag{61}$$

In the case of linear disutility of labor, $\phi'(y) = 1$ and therefore (61) becomes $\frac{2i}{\alpha c u''(c)}$. Moreover, from (36) we know that $u'(c) = \frac{2i+\alpha}{\alpha}$. Given the functional form $u(c) = \frac{c^{1-a}}{1-a}$, we have that

$$c = (\frac{\alpha}{2i + \alpha})^{\frac{1}{a}} \tag{62}$$

and (61) becomes

$$\varepsilon_{\text{money demand}} = -\frac{2i}{a(2i + \alpha)}$$

In the case of convex disutility of labor, $\phi'(y) = y^{\delta-1}$. Given the market clearing condition $y = c$, (61) becomes $\frac{2ic^{\delta-1}}{\alpha c u''(c)}$. Moreover, from (36) we know that $\frac{u'(c)}{\phi'(y)} = \frac{2i+\alpha}{\alpha}$. Given the functional form $u(c) = \frac{c^{1-a}}{1-a}$, we have that

$$c = (\frac{\alpha}{2i + \alpha})^{\frac{1}{\delta+a-1}} \tag{63}$$

and (61) becomes

$$\varepsilon_{\text{money demand}} = -\frac{2i}{a(2i + \alpha)}$$

Money demand ratio

We know from (37) that $L = \frac{m}{\frac{\alpha}{2}pc+A}$ and from (16) we must have $pc = m$. Since in the case of linear disutility of labor $p = 1$, (37) becomes $L = \frac{c}{\frac{\alpha}{2}c+A}$ with $c = (\frac{\alpha}{2i+\alpha})^{\frac{1}{a}}$ from (62).

In the case of linear disutility $p = \phi'(y)$. Since $\phi'(y) = y^{\delta-1}$ given the functional form chosen for the disutility of labor and $y = c$ from the market clearing condition, (37) becomes $L = \frac{c^{\delta}}{\frac{\alpha}{2}c^{\delta}+A}$ with $c = (\frac{\alpha}{2i+\alpha})^{\frac{1}{\delta+a-1}}$ from (63).

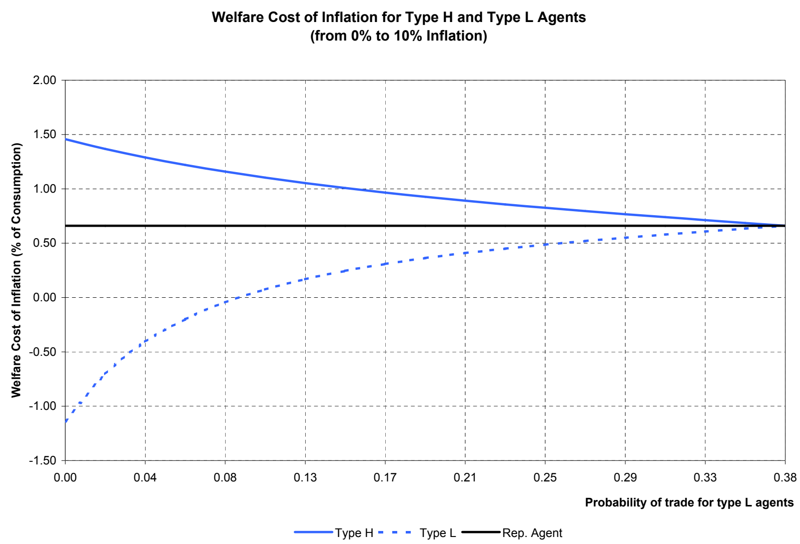

Figure 1: Welfare cost (zero inflation), heterogeneity in trading shocks

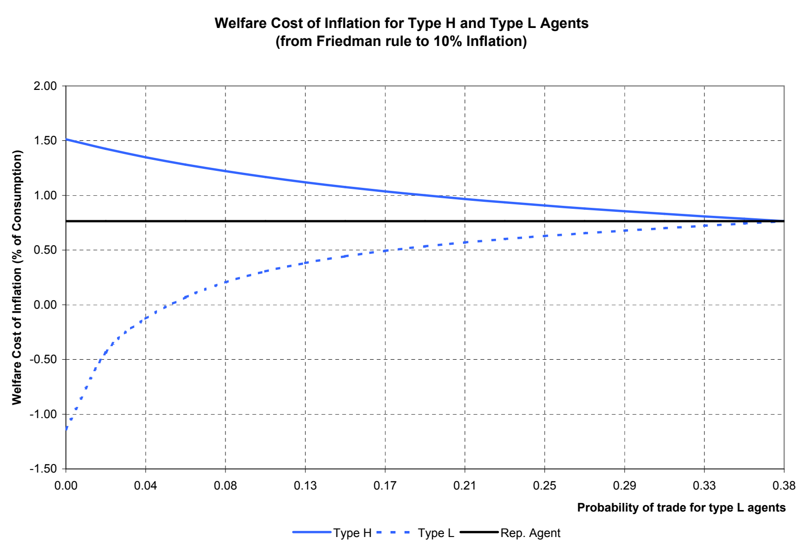

Figure 2: Welfare cost (Friedman rule), heterogeneity in trading shocks

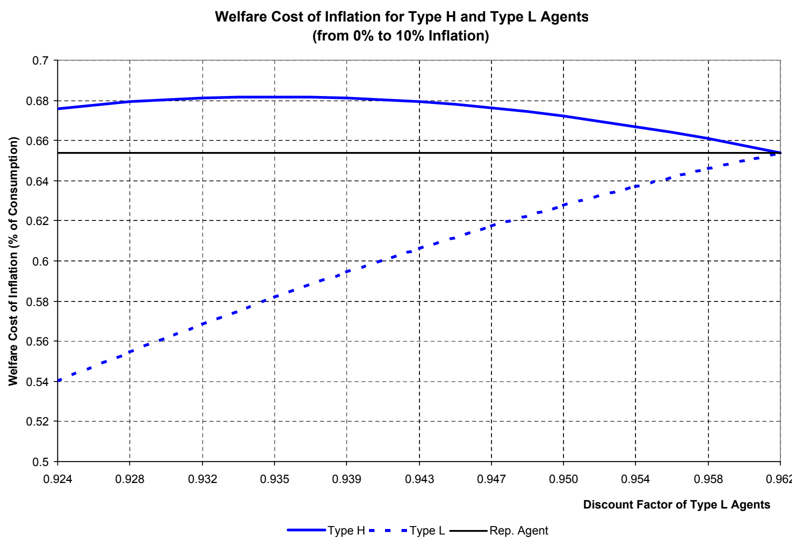

Figure 3: Welfare cost (zero inflation), heterogeneity in discounting

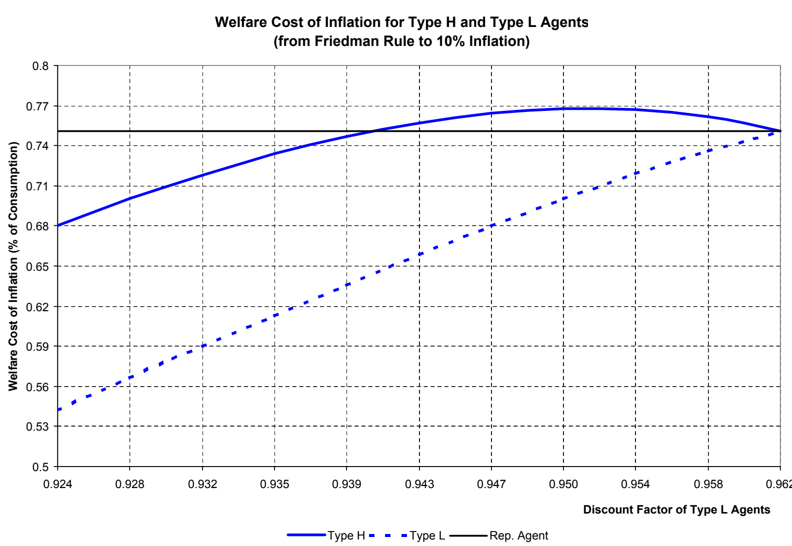

Figure 4: Welfare cost (Friedman rule), heterogeneity in discounting

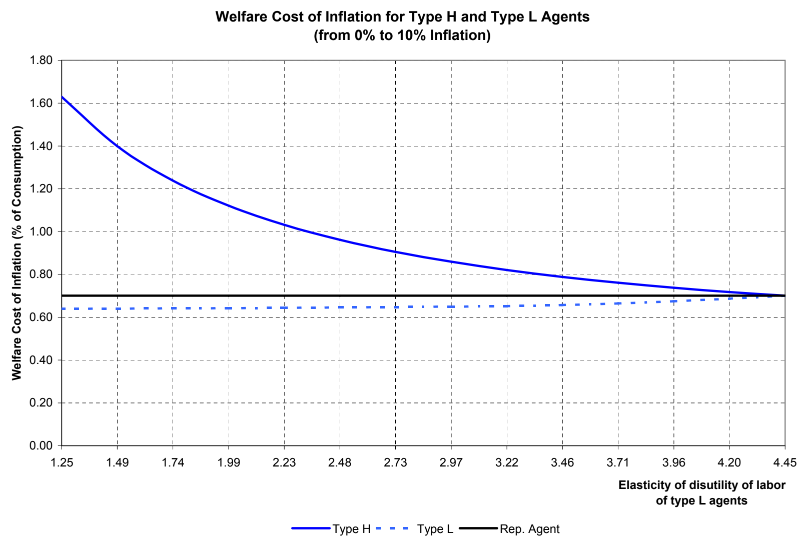

Figure 5: Welfare cost (zero inflation), heterogeneity in disutility from production

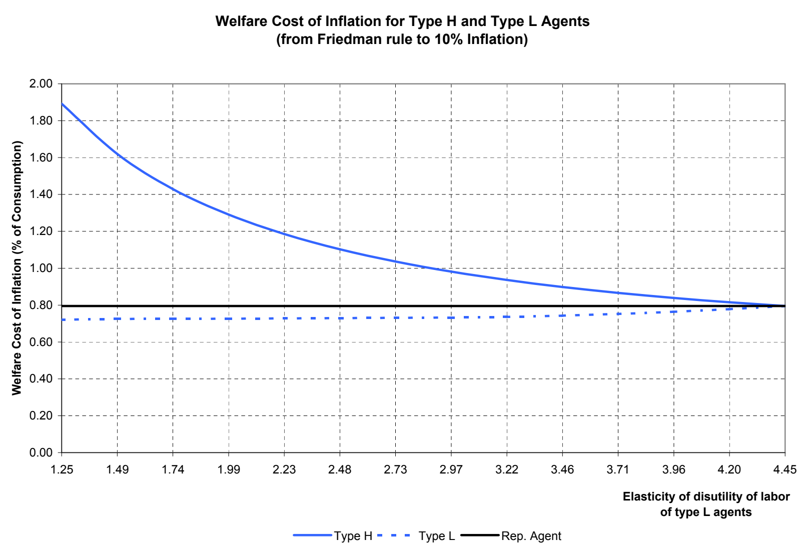

Figure 6: Welfare cost (Friedman rule), heterogeneity in disutility from production