1. Identify the four main components (as discussed in class) of the following instruments. What is the domain of the analytical and the transduced signal?

a) mechanical balance

Signal Generation: mass differential causes one lever arm to drop
Analytical Signal: movement of lever/needle
Transducer: calibrated scale relating displacement to mass
Transduced Signal: needle position on scale
Processor: your brain
Output: visual (optical)

b) digital analytical balance

Signal Generation: force on object due to gravity
Analytical Signal: displacement of object relative to transducer
Transducer: electromagnet or piezoelectric device
Transduced Signal: electrical current or voltage
Processor: analog to digital conversion
Output: digital numeric LED/LCD display

c) quartz crystal microbalance (QCM)

Signal Generation: analyte adsorbed on oscillating crystal
Analytical Signal: mass change of crystal
Transducer: quartz crystal oscillator
Transduced Signal: AC voltage applied to achieve resonance of crystal+analyte
Processor: frequency counting of applied voltage
Output: chart recorder

d) 0.1° mercury thermometer

Signal Generation: Hg column in glass placed in thermal contact with analyte
Analytical Signal: expansion/contraction of Hg
Transducer: calibrated scale relating Hg volume to temperature
Transduced Signal: position of Hg level on scale
Processor: your brain
Output: visual
e) thermocouple

**Signal Generation:** temperature-dependent voltage produced at bimetal junction
**Analytical Signal:** voltage difference between junction at sample and junction in reference bath
**Transducer:** N/A
**Transduced Signal:** N/A
**Processor:** reference T vs. V table; analog to digital conversion
**Output:** digital numeric LED/LCD display

f) infrared (IR) digital thermometer

**Signal Generation:** sample radiates as blackbody
**Analytical Signal:** infrared radiation emitted from sample
**Transducer:** thermoelectric sensor
**Transduced Signal:** voltage or current proportional to radiant power from emissive surface
**Processor:** calculate source temperature (Planck’s equation)
**Output:** digital numeric LED/LCD display

What are the relative merits of these instruments which are used to measure the same physical variables? What factors might be considered in selecting one over another for a given application?

The mechanical balance is quick and easy, but not very accurate as it depends on our perception to evaluate the equivalence point. It requires no electronics, but is limited to relatively large masses. The digital analytical balance is also quick, easy and remarkably precise, but its use is limited to a range of about $10^{1} - 10^{-4}$ g. The QCM can measure incredibly small amounts of analyte (even submonolayer), but requires sophisticated electronics and the data is typically noisy. Cost, required accuracy and the order of magnitude of the analytical mass will dictate which method is chosen.

The Hg thermometer is reliable and easy to use for temperatures near room temp. Unfortunately, the transduced signal is not electrical, so it cannot be integrated into an automated data acquisition routine. A thermocouple can measure temperatures over a very broad range and is relatively simple to build yourself and integrate into PC-based data acquisition. However, it requires thermal contact with the analyte, which can sometimes be problematic. The IR thermometer does not require physical contact and can measure a broad temperature range (but typically with low accuracy). Cost, required accuracy, sample access and data acquisition needs will dictate which instrument is chosen.
2. Why is it important that a good voltage measuring device (like the DMMs you use in lab) has a high internal resistance? What is the minimum internal resistance of a voltmeter necessary to accurately measure the output from a voltage divider with $R_1 = R_2 = 100 \, \Omega$ to within 0.5% error? From a divider with $R_1 = R_2 = 10 \, \text{M}\Omega$? Which of the above dividers would you choose for a generic load of unknown resistance?

A good voltage measuring device should have a high internal resistance so that it draws little current itself, reducing the likelihood of drawing down the voltage being measured by “loading” the circuit. We can examine the effect of loading by the DMM by calculating the minimum meter resistance necessary to measure a voltage to within a certain tolerance of the “true” voltage present without the meter. Consider the following voltage divider circuit:

![Voltage Divider Circuit Diagram]

In the absence of the meter, the voltage at $V_{\text{out}}$ is

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{2}$$

With the meter in the circuit, the current $I$ is equal to

$$I = \frac{V_{\text{out}} (R + R_M)}{RR_M} = \frac{V_{\text{in}} - V_{\text{out}}}{R}$$

Then the voltage at $V_{\text{out}}$ is

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_M}{(R + 2R_M)}$$

For this to be within 0.5% of the voltage output without the meter
\[
0.4975 \leq \frac{R_M}{(R + 2R_M)}
\]

For \( R = 100\Omega \) then \( R_M \geq 9.95k\Omega \) and for \( R = 100M\Omega \), \( R_M \geq 9.95G\Omega \). This demonstrates why DMMs have large internal resistances, so that the effect of loading by the meter is minimized, whether the circuit being measured has a small or large circuit resistance. Conversely, if you were to make a voltage divider to use with a generic load of unknown resistance, you would do better to choose \( R = 100\Omega \) so that the output voltage won’t be drawn down if the load happens to be of low resistance.

I changed this problem because of some awkward wording originally. Most of you assumed that I meant that we were measuring a voltage source with an internal source resistance of \( R_s = 100\Omega \) or \( R_s = 100M\Omega \) and used the equation in the text

\[
E_r = -\frac{R_s}{R_M + R_s} \times 100\%
\]

to determine the percent error in measuring the voltage output. This was not exactly what I intended. Since we have always assumed “perfect” voltage sources, I wanted to make an “imperfect” voltage source—a divider. Since there was confusion over this, and the problem as originally worded was unclear, I will accept answers using the textbook approach. In that case the meter resistance should be at least \( R_M \geq 19.9k\Omega \) for \( R = 100\Omega \) and \( R_M \geq 19.9G\Omega \) for \( R = 100M\Omega \).

3. Answer Skoog problem 2-5 for the following resistor values: \( R_1 = 10\Omega \), \( R_2 = 25\Omega \), \( R_3 = 15\Omega \), \( R_4 = 50\Omega \).

In a) and b) we want to determine basically all the voltages and currents in this circuit. This can be done using Kirchoff’s current and voltage laws (pick any circuit loops you like), or it’s probably easier to do it this way by determining equivalent circuits:
The series and parallel resistors can be represented by a single equivalent resistance as in the following circuit:

\[ R = \frac{R_1 R_3}{R_1 + R_3} + R_4 \]

The equivalent resistance \( R = 69.375 \Omega \), which allows calculation of \( I_1 = I_5 = 220 mA \) by \( V = IR \). With this current known, go back to the original circuit and calculate the voltage drops across the resistors.

\[
\begin{align*}
V_1 &= (220mA)(10\Omega) = 2.2V \\
V_2 &= V_3 = (220mA)(9.375\Omega) = 2.0V \\
V_4 &= (220mA)(50\Omega) = 11V
\end{align*}
\]

Now the voltage drop across the parallel resistors allows calculation of their currents

\[
\begin{align*}
I_2 &= \frac{2.0V}{25\Omega} = 81mA \\
I_3 &= \frac{2.0V}{15\Omega} = 140mA
\end{align*}
\]
Thus all the currents and voltages are determined.

c) The power dissipated by resistor $R_3$ is $P_3 = I_3V_3 = (140mA)(2.0V) = 270mW$

d) The potential difference between points 3 and 4 is the sum of the potential drops across resistors $R_3$ (or $R_2$) and $R_4$. $V_{34} = V_3 + V_4 = 2.0V + 11V = 13V$. Note that the answer in the text for the original resistor values for the problem is incorrect.

4. A current of 1 mA charges a 1\(\mu\)F capacitor. How long does it take the ramp to reach 10 V? Disconnect the current source. How long would it take to discharge the capacitor to 1% of its initial charge through a) a 10 k\(\Omega\) resistor, and b) a 10 M\(\Omega\) resistor?

For the first part, use

\[
Q = CV
\]

\[
\frac{dQ}{dt} = C \frac{dV}{dt} = I
\]

\[
C \int_0^{10V} dV = I \int_0^t dt
\]

\[
\frac{(10^{-6} F)(10V)}{1mA} = t = 10^{-2} \text{s}
\]

For the second part, use

\[
V_t = V_0e^{-t/RC} \text{ and } Q = CV
\]

\[
Q_t = Q_0e^{-t/RC}
\]

\[
\frac{Q_t}{Q_0} = 0.01 = e^{-t/RC}
\]

\[
t = -\ln(0.01)(10^4 \Omega)(10^{-6} F) = 0.5s \text{ for } R = 10k\Omega
\]

\[
t = -\ln(0.01)(10^4 \Omega)(10^{-6} F) = 50s \text{ for } R = 10M\Omega
\]
5. The following Op-Amp circuit can act as a current source for a load resistance. What values of R1, R2, R3 and V+ could you use to source 1mA of current through the load?

![Op-Amp Circuit Diagram]

We have wide latitude in choosing the various voltages and resistors to build this current source (as we would in a real situation). Let’s make a convenient choice of 15V for V+ (the same as a typical positive power source voltage for the op-amp). We can choose what voltage to set for the op-amp non-inverting input (V\text{in}+) by defining the voltage divider resistors R1 and R2. Let’s make V\text{in}+ relatively low, say 1V (there’s a good reason for this, which we’ll see later). Then R1 and R2 are related by:

\[
\frac{V_{\text{in}+}}{V} = \frac{R_2}{R_1 + R_2} = \frac{1V}{15V}
\]

\[R_1 = 14R_2\]

If we choose R2=1kΩ, then R2=14kΩ. Now let’s apply the op-amp “golden rules”:

1) For an op-amp with negative feedback (like this one), the output will do whatever is necessary to maintain both inputs at the same voltage. Thus, since we set V\text{in}+ at 1V with the divider, V\text{in}− is also at 1V.

2) The op-amp inputs draw negligible current. Thus, the current through R3 is the same as the current that the op-amp sources through the load. We want this current to be 1mA, so we choose R3 according to
\[ R_3 = \frac{V_{in}^-}{I_{load}} = \frac{1V}{1mA} = 1k\Omega \]

Under what loads will this current source work as required? The voltage divider sets \( V_{in}^- \) at 1V, so the op-amp output voltage can go anywhere between 1V and 15V (the power supply voltage). The op-amp output will be pulled low (minimum 1V) for small load resistances (down to 0\( \Omega \)), and will be pulled high (maximum 15V) for large load resistances (up to 14k\( \Omega \)). Use \( V=IR \) to find the maximum load. Now we see that choosing a low \( V_{in}^- = V_{in}^+ = 1V \) allows us to source a larger range of possible load resistances than if we had set the input voltage higher.

6. Design a circuit to perform each of the following functions. Use any number or combination of OP-AMPs, capacitors and resistors necessary.

a) Photomultiplier tubes are transducers that convert incident light into a current signal. They characteristically have a small “dark current” produced even in the absence of light. Design a circuit to convert the output of a photomultiplier tube to a voltage and correct for the dark current baseline.

There are several ways to do this. Keep in mind that there is only one PMT—it is not economical or necessary to employ two in a signal/reference scheme. What we want is to be able to zero the output signal by some variable amount in order to account for the dark current. The most transparent way to do this (but probably not the simplest) is to convert the PMT signal to a voltage and feed this into a difference amplifier with a variable offset voltage. The offset voltage is generated below with a variable resistor (potentiometer) with a voltage follower.
b) Correct for a baseline and integrate the area under a spectroscopic peak

This is accomplished with a difference amplifier followed by an integrator (see Skoog for examples of these). Again, keep in mind that the baseline voltage offset isn’t just sitting around somewhere to use as an input—you need to “make” it—perhaps with a pot and follower as in the circuit above.

c) Take the difference of two voltages (from a sample and reference signal) and amplify the difference by a factor of 50.

See Skoog p. 63 for an example. The key is that the relative ratio of the input and feedback resistors sets the gain.

7. For the following circuit with a time-dependent input signal, sketch the frequency dependence of $V_{out}/V_{in}$ for $C = 0.1 \ \mu F$, $R_1 = 1 \ \text{k}\Omega$ and $R_2 = 1 \ \text{k}\Omega$ (indicate $f_{3\text{dB}}$). Now replace with $R_2 = 10 \ \text{k}\Omega$. What is the advantage of using such a circuit for filtering?
This circuit acts as a low-pass filter and an amplifier. For low frequencies, the capacitor doesn’t have much effect ($Z_c = \frac{1}{\omega C}$ is large) and the gain approaches that of the amplifier without the capacitor:

$$Gain = -\frac{R_2}{R_1}$$

At high frequencies, gets smaller, so the lower impedance $Z_c = \frac{1}{\omega C}$ of the capacitor parallel to $R_2$ starts affecting performance (brings down gain).

When $f = f_{3dB} = \frac{1}{2\pi R_2 C}$, the total impedance of $R_2$ parallel to $Z_C$ is $\frac{R_2}{\sqrt{2}}$. So for the various frequency regimes, we have:

For $f << f_{3dB}$, $Gain = -\frac{R_2}{R_1}$

For $f = f_{3dB}$, $Gain = -\frac{1}{\sqrt{2}} \frac{R_2}{R_1}$

For $f >> f_{3dB}$, $Gain \rightarrow 0$

So, this amplifier has the advantage of being insensitive to high frequencies that you don’t want. When sketching the output of this circuit, you should indicate that $f_{3dB} = \frac{1}{2\pi R_2 C}$ is 1600 Hz for $R_2=1k\Omega$ and 160 Hz for $R_2=10k\Omega$. 
Also you should indicate that the zero frequency output voltage is:

\[ V_{out} = -V_{in} \cdot \frac{R_2}{R_1} \]. Of course this only shows amplification for \( R_2 = 10k\Omega \).