**Entropy and the Partition Function**

\[
S = \frac{k}{\mathcal{N}} \ln W_{\text{max}} \quad \text{(Canonical ensemble)}
\]

\[
W = \frac{\mathcal{N}!}{n_0! n_1! n_2! \ldots}
\]

\[
\ln W = \ln \mathcal{N}! - \sum_{i=0}^{\infty} \ln n_i!
\]

(Sum over all energy states)

Sterling’s Formula: \(\ln x! = x \ln x - x\)

\[
\sum n_i = \mathcal{N} \quad \text{giving} \quad \ln W = \mathcal{N} \ln \mathcal{N} - \mathcal{N} \ln Q + \sum n_i \ln Q + \sum \frac{n_i E_i}{kT}
\]

\[
\ln W = \mathcal{N} \ln \mathcal{N} - \sum n_i \ln \mathcal{N} + \sum n_i \ln Q + \sum \frac{n_i E_i}{kT}
\]

\[
S = \frac{k}{\mathcal{N}} \ln W_{\text{max}} = k \ln Q + \frac{E}{\mathcal{N}T} = k \ln Q + \frac{U - U(0)}{T}
\]

(U(0) at 0 K)

**Ideal Gas**:

\(U - U(0) = \frac{3}{2} nRT = \frac{3}{2} NkT\)

\[S = k \ln Q + \frac{3}{2} nR\]

\(Q = \frac{q^N}{N!} \quad N! \equiv (N/e)^N \quad Q \equiv \left(\frac{q}{N}\right)^N\)

Ideal monatomic: \(q_t = \frac{(2\pi m kT)^{3/2}}{h^3}\)

\[
S = N_k \ln \left(\frac{N}{e}\right) + \frac{3}{2} nR = nR \ln \left[\frac{(2\pi m kT)^{3/2}}{N_A h^3} e^{5/2} V\right] + \frac{3}{2} nR
\]

\(m \sim \text{kg molecule}^{-1}, \quad V \sim \text{m}^3\)

Per mole: \(N = N_A, \quad n = 1 \text{ mol}\)

\(N_A k = R\)

\[
\frac{3}{2} = \ln e^{5/2}
\]

\[
S_m = R \ln \left[\frac{(2\pi m kT)^{3/2}}{N_A h^3} \frac{e^{5/2}}{V_m}\right]
\]

Sackur-Tetrode Equation

\[
S_m = R \ln V_m + \frac{3}{2} R \ln T + \frac{3}{2} R \ln M + R \ln \left[\frac{(2\pi k(1 \text{ kg/1000 g})/N_A)^{3/2} e^{5/2}(1 \text{ m}^3/1000 \text{ L})}{N_A h^3}\right]
\]

\[
S_m = R \ln (V_m/L) + \frac{3}{2} R \ln T + \frac{3}{2} R \ln (M/\text{g mol}^{-1}) + 11.037 \text{ J K}^{-1} \text{ mol}^{-1}
\]

\(C_v = \frac{3}{2} R \ln (T_2/T_1) = C_v \ln (T_2/T_1)\)

\(|P^* = 1 \text{ bar} \quad \text{V}_m^* = \text{RT}/P^* = 0.0247890 \text{ m}^3 = 24.7890 \text{ L} \quad \text{at 298.15 K} \)

\(|S_m^* = 298.15 \text{ K} = 26.6929 + 71.0587 + \frac{3}{2} R \ln (M/\text{g mol}^{-1}) + 11.037 \text{ J K}^{-1} \text{ mol}^{-1} \quad \text{(translation)}\)