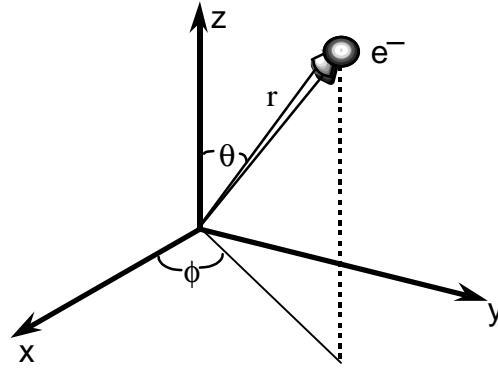
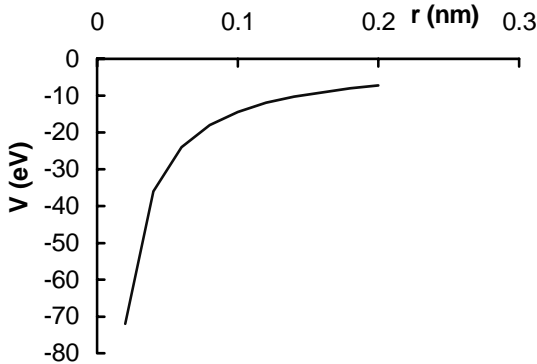


Ground State of the Hydrogen Atom



$$V(r) = \frac{-Z e^2}{4\pi\epsilon_0 r}$$

$$e = 1.60 \times 10^{-19} \text{C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ J}^{-1} \text{C}^2 \text{m}^{-1}$$

$$\frac{-\hbar^2}{2m} \nabla^2 \Psi + V \Psi = E \Psi$$

$$\frac{-\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Psi + \frac{-Z e^2}{4\pi\epsilon_0 r} \Psi = E \Psi$$

$$\nabla^2 = \frac{1}{r} \left(\frac{\partial^2}{\partial r^2} \right) r + \left(\frac{1}{r^2} \right) \Lambda^2 \quad \Lambda^2 = \frac{1}{\sin^2 \theta} \left(\frac{\partial^2}{\partial \phi^2} \right) + \left(\frac{1}{\sin \theta} \right) \left(\frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} \right)$$

$$\Psi = R(r) \Theta(\theta) \Phi(\phi) = R(r) Y_{\ell, m_\ell}$$

$$\frac{-\hbar^2}{2mr^2} \frac{\partial^2 \Phi}{\partial \phi^2} = \frac{-\hbar^2 m_\ell^2}{2mr^2} \Phi \quad l_z = \hbar m_\ell \quad \Phi(\phi) = \left(\frac{1}{2\pi} \right)^{1/2} e^{im_\ell \phi}$$

$$\Lambda^2 Y_{\ell, m_\ell} = -\ell(\ell+1) Y_{\ell, m_\ell}$$

$$\frac{-\hbar^2}{2m} \left(\frac{1}{r} \frac{\partial^2}{\partial r^2} r R(r) Y_{\ell, m_\ell} + \frac{\Lambda^2}{r^2} R(r) Y_{\ell, m_\ell} \right) - \frac{Z e^2}{4\pi\epsilon_0 r} R(r) Y_{\ell, m_\ell} = E R(r) Y_{\ell, m_\ell}$$

$$\frac{-\hbar^2}{2m} \left(Y_{\ell, m_\ell} \frac{1}{r} \frac{\partial^2}{\partial r^2} r R(r) - R(r) \frac{\ell(\ell+1)}{r^2} Y_{\ell, m_\ell} \right) - \frac{Z e^2}{4\pi\epsilon_0 r} R(r) Y_{\ell, m_\ell} = E R(r) Y_{\ell, m_\ell}$$

$$\frac{-\hbar^2}{2m} \left(\frac{1}{r} \frac{\partial^2}{\partial r^2} r R - \frac{\ell(\ell+1)}{r^2} R \right) - \frac{Z e^2}{4\pi\epsilon_0 r} R = E R$$

$$\frac{-\hbar^2}{2m} \left(\frac{1}{r} \frac{d^2}{dr^2} r R \right) - \frac{Z e^2}{4\pi\epsilon_0 r} R = E R \quad \text{for ground state } \ell = 0$$

Guess: $R(r) = A e^{-\alpha r}$

$$-\frac{\hbar^2}{2m} \left(\alpha^2 R - \frac{2\alpha}{r} R \right) - \frac{Z e^2}{4\pi\epsilon_0 r} R = E R$$

$$-\frac{\hbar^2\alpha^2}{2m} + \frac{\hbar^2 2\alpha}{2m r} - \frac{Z e^2}{4\pi\epsilon_0 r} = E$$

$$-\frac{\hbar^2\alpha^2}{2m} = E \qquad \frac{\hbar^2\alpha}{m r} - \frac{Z e^2}{4\pi\epsilon_0 r} = 0$$

$$\alpha = \frac{\sqrt{-2mE}}{\hbar} \qquad \alpha = \frac{Z e^2}{4\pi\epsilon_0 \hbar^2} \frac{m}{\hbar^2}$$

$$E = - \left(\frac{Z^2 e^4 m}{32\pi^2\epsilon_0^2 \hbar^2} \right) \qquad E_n = - \left(\frac{Z^2 e^4 m}{32\pi^2\epsilon_0^2 \hbar^2} \right) \frac{1}{n^2} \quad \text{for excited states}$$

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} \qquad E_n = - \frac{Z^2 \hbar^2}{2ma_0^2} \frac{1}{n^2} \qquad \alpha = \frac{Z}{a_0}$$

$$a_0 = 0.0529 \text{ nm} = 0.529 \text{ \AA}$$

$$E_n = -13.6 \text{ eV} \frac{Z^2}{n^2} = -109,678 \text{ cm}^{-1} \frac{Z^2}{n^2}$$

$$E_n = -1312 \text{ kJ mol}^{-1} \frac{Z^2}{n^2} = -\frac{H}{2} \frac{Z^2}{n^2} \qquad \text{where } 1H = 2625.5 \text{ kJ mol}^{-1}$$

Normalize

$$\int_0^{2\pi} \int_0^\pi \int_0^\infty \Psi^2 r^2 \sin\theta \, dr \, d\theta \, d\phi = A^2 \int_0^{2\pi} \int_0^\pi \int_0^\infty e^{-2Zr/a_0} r^2 \sin\theta \, dr \, d\theta \, d\phi =$$

$$A^2 \int_0^{2\pi} d\phi \int_0^\pi \sin\theta \, d\theta \int_0^\infty e^{-2Zr/a_0} r^2 \, dr = A^2 4\pi \int_0^\infty e^{-2Zr/a_0} r^2 \, dr =$$

$$A^2 4\pi \left(\frac{2}{(2Z/a_0)^3} \right) = ?$$

$$A = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2}$$

$$\Psi(r) = R(r) = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0}$$