Some Handy Integrals

Gaussian Functions

\[ \int_0^\infty e^{-ax^2} \, dx = \frac{\sqrt{\pi}}{\sqrt{a}} \]
\[ \int_0^\infty x \, e^{-ax^2} \, dx = \frac{1}{2a} \]
\[ \int_0^\infty x^2 \, e^{-ax^2} \, dx = \frac{1}{2a^2} \]
\[ \int_0^\infty x^3 \, e^{-ax^2} \, dx = \frac{1}{a^3} \]
\[ \int_0^\infty x^{2n} \, e^{-ax^2} \, dx = \frac{n!}{(2n+1)!} \left( \frac{1}{a^2} \right)^{n+1} \]

Exponential Functions

\[ \int_0^\infty x^n \, e^{-ax} \, dx = \frac{n!}{a^{n+1}} \]

Integrals from \(-\infty\) to \(\infty\): Even and Odd Functions

The integral of any even function taken between the limits \(-\infty\) to \(\infty\) is twice the integral from 0 to \(\infty\). The integral of any odd function between \(-\infty\) and \(\infty\) is equal to zero, see Figure 1.

![Even and odd integrals](image.png)

(a). \(f(x) = e^{-ax^2}\) \(\text{even}\)
(b). \([g(x) \cdot f(x)] = x \, e^{-ax^2}\) \(\text{odd*even}\)

Figure 1. Even and odd integrals.

To determine if a function is even, check to see if \(f(x) = f(-x)\). For an odd function, \(f(x) = -f(-x)\). Some functions are neither odd nor even. For example, \(f(x) = x\) is odd, \(f(x) = x^2\) is even, and \(f(x) = x + x^2\) is neither odd nor even. The following multiplication rules hold:
- \(\text{even*even} = \text{even}\)
- \(\text{odd*odd} = \text{even}\)
- \(\text{odd*even} = \text{odd}\)

Consider the integral of \(f(x) = e^{-ax^2}\), Figure 1a. The function is even so that \(\int_{-\infty}^{\infty} f(x) \, dx = 2\int_0^{\infty} f(x) \, dx\). Next consider \(g(x) = x\), which is odd, giving \([g(x) \cdot f(x)] = x \, e^{-ax^2}\) as overall odd (Figure 1b). The integral is zero for the product function.