Some Handy Integrals

Gaussian Functions

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\left(\frac{\pi}{a}\right)^{1/2} \qquad \int_0^\infty x\,e^{-ax^2}\,dx = \frac{1}{2a}$$

$$\int_0^\infty x^2\,e^{-ax^2}\,dx = \frac{1}{4a}\left(\frac{\pi}{a}\right)^{1/2} \qquad \int_0^\infty x^3\,e^{-ax^2}\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^4\,e^{-ax^2}\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2} \qquad \int_0^\infty x^5\,e^{-ax^2}\,dx = \frac{1}{a^3}$$

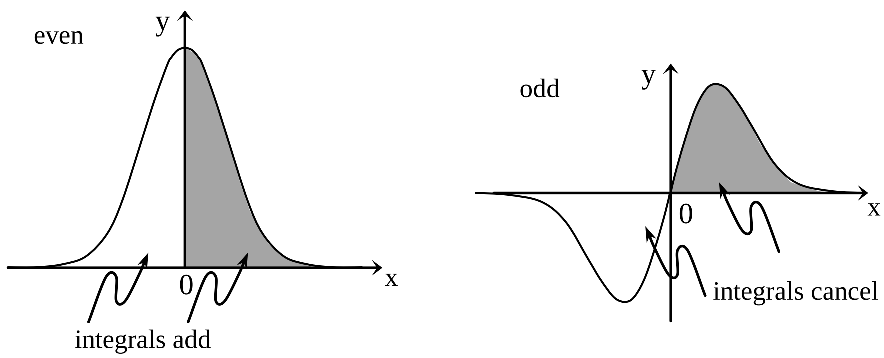
$$\int_0^\infty x^{2n}\,e^{-ax^2}\,dx = \frac{1\cdot3\cdot5\cdots(2n-1)}{2^{n+1}a^n}\left(\frac{\pi}{a}\right)^{1/2} \qquad \int_0^\infty x^{2n+1}\,e^{-ax^2}\,dx = \frac{n!}{2}\left(\frac{1}{a^{n+1}}\right)$$

Exponential Functions

$$\int_0^\infty x^n\,e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

Integrals from -∞ to ∞: Even and Odd Functions

The integral of any even function taken between the limits -∞ to ∞ is twice the integral from 0 to ∞. The integral of any odd function between -∞ and ∞ is equal to zero, see Figure 1.



(a). $f(x) = e^{-ax^2}$ *even*

(b). $[g(x)\,f(x)] = x\,e^{-ax^2}$ *odd*even*

Figure 1. Even and odd integrals.

To determine if a function is even, check to see if $f(x) = f(-x)$. For an odd function, $f(x) = -f(-x)$. Some functions are neither odd nor even. For example, $f(x) = x$ is odd, $f(x) = x^2$ is even, and $f(x) = x + x^2$ is neither odd nor even. The following multiplication rules hold:

even*even = even   odd*odd =even   odd*even = odd

Consider the integral of $f(x) = e^{-ax^2}$, Figure 1a. The function is even so that $\int_{-\infty}^{\infty} = 2\int_0^\infty$. Next consider $g(x) = x$, which is odd, giving $[g(x)\,f(x)] = x\,e^{-ax^2}$ as overall odd (Figure 1b). The integral is zero for the product function.