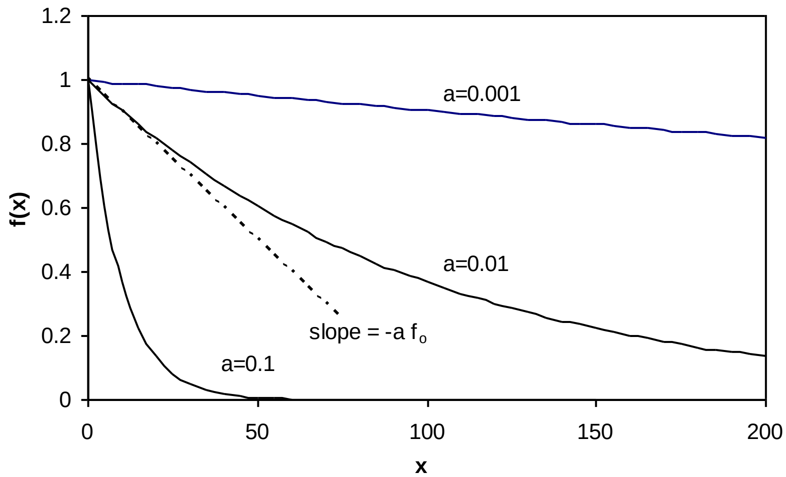
℘1 First-Order Homogeneous Differential Equations with a Constant Coefficient

$$df = -a\, f\, dx$$

$$\frac{df}{f} = -a\, dx$$

$$\int_{f_1}^{f_2} \frac{df}{f} = -\int_{x1}^{x2} a\, dx$$



$$\left(\ln f \right|_{f_1}^{f_2} = -a \left(x \right|_{x_1}^{x_2}$$

$$\ln f_2 - \ln f_1 = -a\,(x_2 - x_1)$$

$$\ln \frac{f_2}{f_1} = -a\,(x_2 - x_1)$$

$$f_2 = f_1\, e^{-a(x_2 - x_1)}$$

For $x_1 = 0$, rename $x_2 = x$, $f_1 = f_o$ and $f_2 = f$

$$\ln \frac{f}{f_o} = -a\,x \qquad\qquad f = f_o\, e^{-ax}$$

$x \to 0$ then $e^{-ax} \to e^0 = 1$ so that $f \to f_o$

$x \to \infty$ then $e^{-ax} = \frac{1}{e^{ax}} \to \frac{1}{e^{\infty}} = 0$ so that $f \to 0$

$$f \approx f(x_o) + \left(\frac{df}{dx}\right)_{x=x_o} (x - x_o) + \left(\frac{d^2f}{dx^2}\right)_{x=x_o} \frac{(x - x_o)^2}{2} + \left(\frac{d^3f}{dx^3}\right)_{x=x_o} \frac{(x - x_o)^3}{3!} + \ldots$$

$$e^{-ax} \approx 1 - a\,x + a^2 \frac{x^2}{2} - a^3 \frac{x^3}{6} + \ldots$$

$$f \approx f_o - f_o\, ax$$

$$f_2 \approx f_1 - f_1 a(x_2 - x_1)$$

$$\frac{df}{dx} = \frac{d\,(f_o\, e^{-ax})}{dx} = -f_o\, a\, e^{-ax}\Big|_{x=0} = -f_o\, a$$