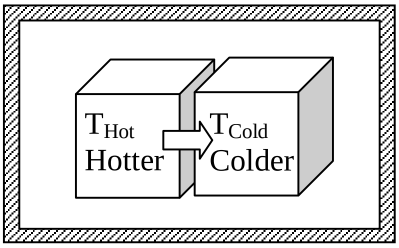
Thermodynamic Definition of Temperature: $\beta = 1/kT$



$$\left(\frac{\partial S}{\partial U}\right)_V = k\beta \qquad \text{(cst. V, equilibrium)}$$

$(\partial S/\partial U)_V > 0$ $\qquad$ $\beta > 0$

$dS_1 = k\beta_1 dU_1$ and $dS_2 = k\beta_2 dU_2$ $\qquad$ (cst. V)

$dS_{tot} = dS_1 + dS_2 = k\beta_1 dU_1 + k\beta_2 dU_2$ $\qquad$ (cst. V)

$dS_{tot} > 0$ for a spontaneous process in an isolated system

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$dU_1 = -\, dU_2$

$dS_{tot} = k(\beta_1 - \beta_2)\, dU_1 > 0$ $\qquad$ (cst. V, spontaneous)

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1. If $\beta_1 < \beta_2$ then $(\beta_1 - \beta_2) < 0$ and we must have $dU_1 < 0$ to give $dS_{tot} > 0$. For $dU_1 < 0$ energy is transferred from system 1 to system 2. System 1 must be the hotter system.
2. If $\beta_1 > \beta_2$ then $(\beta_1 - \beta_2) > 0$ and we must have $dU_1 > 0$ to give $dS_{tot} > 0$. For $dU_1 > 0$ energy is transferred from system 2 to system 1. System 2 must be the hotter system.
3. If $\beta_1 = \beta_2$ then $dS_{tot} = 0$, there will be no energy transfer and the two systems must be at equilibrium.

$$\beta = \frac{1}{kT}$$

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1. If $\beta_1 < \beta_2$ then $(1/T_1 - 1/T_2) < 0$ giving $T_1 > T_2$ making system 1 the hotter system,
2. If $\beta_1 > \beta_2$ then $(1/T_1 - 1/T_2) > 0$ giving $T_1 < T_2$ making system 2 the hotter system,
3. If $\beta_1 = \beta_2$ then $T_1 = T_2$ and the systems are at the same temperature,

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$$\left(\frac{\partial S}{\partial U}\right)_V = \frac{1}{T} \qquad \text{(cst. V, equilibrium)}$$

$$p_i = \frac{e^{-E_i/kT}}{\sum_i e^{-E_i/kT}} \qquad \text{(equilibrium)}$$

$$p_i = \frac{e^{-E_i/kT}}{Q} \quad \text{with } Q \equiv \sum_i e^{-E_i/kT} \qquad \text{(equilibrium)}$$

$$dS = \frac{1}{T}\sum_i E_i\, dp_i \qquad \text{(cst. V, equilibrium)}$$