Thermodynamic Definition of Temperature: \( \beta = 1/kT \)

\[
\left( \frac{\partial S}{\partial U} \right)_V = k\beta \quad \text{(cst. } V, \text{ equilibrium)}
\]

\( (\partial S/\partial U)_V > 0 \) \( \beta > 0 \)

\[
dS_1 = k\beta_1 dU_1 \quad \text{and} \quad dS_2 = k\beta_2 dU_2 \quad \text{(cst. } V)\]

\[
dS_{\text{tot}} = dS_1 + dS_2 = k\beta_1 dU_1 + k\beta_2 dU_2 \quad \text{(cst. } V)\]

\( dS_{\text{tot}} > 0 \) for a spontaneous process in an isolated system

\[
dU_1 = -dU_2
\]

\[
dS_{\text{tot}} = k(\beta_1 - \beta_2) dU_1 > 0 \quad \text{(cst. } V, \text{ spontaneous)}
\]

1. If \( \beta_1 < \beta_2 \) then \((\beta_1 - \beta_2) < 0 \) and we must have \( dU_1 < 0 \) to give \( dS_{\text{tot}} > 0 \). For \( dU_1 < 0 \) energy is transferred from system 1 to system 2. System 1 must be the hotter system.

2. If \( \beta_1 > \beta_2 \) then \((\beta_1 - \beta_2) > 0 \) and we must have \( dU_1 > 0 \) to give \( dS_{\text{tot}} > 0 \). For \( dU_1 > 0 \) energy is transferred from system 2 to system 1. System 2 must be the hotter system.

3. If \( \beta_1 = \beta_2 \) then \( dS_{\text{tot}} = 0 \), there will be no energy transfer and the two systems must be at equilibrium.

\[
\beta = \frac{1}{kT}
\]

1. If \( \beta_1 < \beta_2 \) then \((1/T_1 - 1/T_2) < 0 \) giving \( T_1 > T_2 \) making system 1 the hotter system,

2. If \( \beta_1 > \beta_2 \) then \((1/T_1 - 1/T_2) > 0 \) giving \( T_1 < T_2 \) making system 2 the hotter system,

3. If \( \beta_1 = \beta_2 \) then \( T_1 = T_2 \) and the systems are at the same temperature,

\[
\left( \frac{\partial S}{\partial U} \right)_V = \frac{1}{T} \quad \text{(cst. } V, \text{ equilibrium)}
\]

\[
p_i = \frac{e^{-E_i/kT}}{\sum_i e^{-E_i/kT}} \quad \text{(equilibrium)}
\]

\[
p_i = \frac{e^{-E_i/kT}}{Q} \quad \text{with } Q \equiv \sum_i e^{-E_i/kT} \quad \text{(equilibrium)}
\]

\[
dS = \frac{1}{T} \sum_i E_i \, dp_i \quad \text{(cst. } V, \text{ equilibrium)}
\]