Part 1: Answer 4 of the following 5 questions. If you answer more than 2, cross out the question that you don’t wish to be graded. (8 points each)

1. (a). Give the angular momentum quantum number that corresponds to the following angular momentum vector diagram:

(b). Label the diagram with the corresponding values for the z-axis projection of the angular momentum.

2. Draw the energy level diagram for a nucleus with spin quantum number $I = \frac{3}{2}$ in the presence of an external magnetic field (as appropriate for an NMR experiment).

3. For the rigid rotor wave functions, why can’t $m_l$ be greater than $l$?

4. Show that the wave function $\Psi = a e^{im\phi}$ is an eigenfunction of the z-component of the angular momentum, for rotation in the x-y plane (particle in a ring).
5. Find the average radius for the ground state of the hydrogen atom. Do the angular part of the integral, and then just set up the radial part of the integral. Remember to give all necessary information to complete the integral. You don’t need to do the integral over r.

**Part 2:** Answer 3 of the following 4 questions. If you answer more than 3, cross out the question that you don’t wish to be graded. (12 points each)

6. Give the wave function and energy for a 3D-particle in a box with quantum numbers (1,1,2) and side lengths a, b, and c for a particle of mass m.

7. Discuss the relationship between the commutation relationships:

\[ [\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z \quad [\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y \quad [\hat{L}_y, \hat{L}_x] = i\hbar \hat{L}_z \quad [\hat{L}^2, \hat{L}_z] = 0 \]

and the angular momentum vector diagram shown in problem 1.
8. Show that $<p^2>$ for the ground state of the particle in a box of length $a$ is:  
\[<p^2> = \hbar^2 \left( \frac{\pi}{a} \right)^2\]

9. Use the recursion relationship for Hermite polynomials to generate the third excited state wave function for the harmonic oscillator given:

\[H_0 = 1 \quad H_1 = 2y \quad H_2 = 4y^2 - 2\]

with $y = \alpha x$. You don’t need to normalize the wave function. You can leave the wave function expressed as a function of $\alpha x$. [Make sure to show your work.]
Part 3: Answer 2 of the following 3 questions. If you answer more than 2, cross out the question that you don’t wish to be graded. (16 points each)

10. Determine the value for the commutation relationship between the observation time and total energy of a particle, \([\hat{t}, \hat{E}]\). The operator for total energy as a function of time is \(\hat{E} = i\hbar (d/dt)\) and \(\hat{t} = t\) (just multiply by \(t\)). [Hint: apply the commutator as an operator for an arbitrary function, \(f(t)\)]

11. Consider a particle in a box of length “\(a\)” with a potential that goes to infinity at \(-\frac{a}{2}\) and \(+\frac{a}{2}\). The general form of the wave function is given by \(\Psi(x) = N \cos(kx)\). Apply the boundary conditions to determine the wave functions for the particle, including the possible values for \(k\). You don’t need to normalize the final wave function.
12. Show that the operator \( \hat{\partial} = (i \, d/dx) \) is Hermitian. Assume \(-\infty < x < \infty\) and that the wave functions approach zero for \( x \to \pm \infty \). [Hint: use integration by parts: \( \int u \, dv = uv - \int v \, du \)]

Extra credit (8 points): Do the integral in problem 5
Definite Integrals

\[ \int_{0}^{\pi/2} \sin^2(x) \, dx = \int_{0}^{\pi/2} \cos^2(x) \, dx = \frac{\pi}{4} \]

\[ \int_{0}^{\pi/2} \sin^3(x) \, dx = \int_{0}^{\pi/2} \cos^3(x) \, dx = \frac{2}{3} \]

\[ \int_{0}^{\pi} \sin^2(ax) \, dx = \int_{0}^{\pi} \cos^2(ax) \, dx = \frac{\pi}{2} \]

\[ \int_{0}^{\pi/2} \cos(ax) \sin(ax) \, dx = \int_{0}^{\pi/2} \cos(ax) \sin(ax) \, dx = 0 \]

\[ \int_{0}^{\pi} \sin(ax) \sin(bx) \, dx = \int_{0}^{\pi} \cos(ax) \cos(bx) \, dx = 0 \quad (a \neq b; \, a, b \text{ integers}) \]

\[ \int_{0}^{\pi} \sin(ax) \cos(bx) \, dx = \frac{2a}{a^2 - b^2} \quad \text{if} \ (a - b) \text{ is odd, or zero if} \ (a - b) \text{ is even} \]

\[ \int_{0}^{\pi} \cos(ax) \sin(ax) \, dx = 0 \]

Gaussian Functions

\[ \int_{0}^{\infty} e^{-ax^2} \, dx = \frac{1}{\sqrt{a}} \left( \frac{\pi}{2} \right)^{\frac{1}{2}} \]

\[ \int_{0}^{\infty} x^2 e^{-ax^2} \, dx = \frac{1}{4a} \left( \frac{\pi}{a} \right)^{\frac{1}{2}} \]

\[ \int_{0}^{\infty} x^4 e^{-ax^2} \, dx = \frac{3}{8a^2} \left( \frac{\pi}{a} \right)^{\frac{1}{2}} \]

\[ \int_{0}^{\infty} x^{2n} e^{-ax^2} \, dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1}a^n} \left( \frac{\pi}{a} \right)^{\frac{1}{2}} \]

\[ \int_{0}^{\infty} x^{2n+1} e^{-ax^2} \, dx = \frac{n!}{a^{n+1}} \left( \frac{1}{a} \right)^n \left( \frac{\pi}{a} \right)^{\frac{1}{2}} \]

Exponential Functions

\[ \int_{0}^{\infty} x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}} \]