Governance and Corruption in the Middle East and North Africa: 
A Neoclassical Analysis

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Abstract
With an abundant literature on corruption, few researchers focus on the Middle East and North Africa (MENA). The intersection of economics and politics in the Arab states of the MENA region lends itself to an analysis of not only the effects of corruption on the economic performance of the region, but also the political narrative that facilitates and reinforces corruption. I use a neoclassical stochastic growth model to quantitatively evaluate the effects of corruption on tax evasion and economic development. I find that as the level of corruption falls, so do the determinants of economic growth. I also observe increases in the provision and consumption of public goods. Lastly, I find that home production and tax evasion by households decline with improvements in the level of corruption in a sample of countries in the MENA region, supporting the idea of welfare gains accruing from increased government integrity.
1 Introduction

Corruption is a facet of the broader intersection between economic performance and governance. While one can speculate that corrupt practices have long existed in the public and private spheres, the study of corruption through the lens of economics first emerged in the 1970s, namely with Anne Kreuger’s article *The Political Economy of the Rent-Seeking Society*, published in 1974. At the time, the research mostly focused on the role of weak public institutions and economic policies in driving rent-seeking by and the corruptibility of public officials (Gupta and Abed, 2002). The economic study of corruption gained further traction in the 1990s following the disintegration of the Soviet Union and the consequent transformation of its former socialist economies, the growing awareness of the costs of corruption in both developed and developing countries, and the development of indicators of corruption used in empirical studies and in decision-making processes (Gupta and Abed, 2002).

The onset of globalization further increased the relevance of corruption in the realms of efficient market transfers, and international flows of capital, information and technology (Gupta and Abed, 2002). The associated demands for economic liberalization placed greater pressure on governments to be more transparent and accountable in managing their economies. To the extent that poor governing arrangements led to the spread of corruption, creating an environment not conducive to foreign direct investment nor trusted with flows of foreign aid, politics factored into economic decision-making. With the end of the Cold War, political, social and economic orders collapsed in developing countries that were only recently the focal point of geopolitical competition and the benefits it afforded. Therefore, weakened systems of governance and law encouraged rent-seeking and corrupt behavior at a time when the international community and the global economy placed a high value on accountable and transparent government institutions, the protection of property rights, and comprehensive regulations (Gupta and Abed, 2002). This forged a stronger link between corruption and the state of the economy in an increasingly integrated world.

The political order in a given state, thus, became more relevant to achieving both domestic and international economic goals. In a sense, the extent to which economic growth and development can be achieved became politicized. The type of political regime now had repercussions in the international arena. Regime structures that enabled rent-seeking behavior did so to derive legitimacy in the absence of a social contract that provided good governance. In the Middle East, rentier states that derive the bulk of their revenues primarily from the sale of oil fall within this category, as the political system is set up in order to develop an economic reliance of the populace on the state. The over-dependence of the population amounts to them sacrificing certain civil and political rights since the economic wellbeing of citizens is now in the hands of the state. While some countries possess resources that they exploit to generate rents, others in the region are recipients of those rents. Furthermore, the politics within a given country cannot be understood without understanding the broader regional political context. In either case, yet to varying degrees, the presence of patronage networks through which a regime distributes rents and maintains legitimacy not only enables, but also incentivizes citizens to engage in rent-seeking behavior.

In attempting to understand the consequences of corruption, one can motivate the need to counter it. While in political terms anti-corruption movements are driven by the need to
preserve and promote the wellbeing of citizens, in economic terms, the effects are not always as unambiguous. For instance, Vito Tanzi (1998) outlines arguments claiming that corruption is the efficient outcome in environments with high levels of bureaucratic red tape. Therefore, to better address issues of corruption in the Middle East and North Africa (MENA), it is important to understand the current political, social, and economic structures, as well as the motivations and inhibitions in place that determine the extent to which corruption is prevalent. Having understood the shape corruption takes in the MENA region, its consequences in terms of prospects for economic growth and development, as well as the drivers of corrupt activities, we can then formulate policies tailored to the domestic, regional, and international contexts within which corruption subsists.

Therefore, I seek to analyze the effect of corruption on the economic performance of and tax evasion behavior in a set of countries in the MENA region. To do so, I construct a stochastic neoclassical growth model that captures the decisions a representative household, public agent, and firm face in a corrupt environment. I then solve the model to discern how varying degrees of corruption affect economic indicators, including labor and capital supply, consumption, and investment. The remainder of the paper is organized as follows. In the upcoming subsections, I provide a more detailed overview of the MENA region and the forms of corruption referred to. Section 2 presents the model assuming a lump-sum tax. Section 3 further develops the model by introducing a tax rate, as well as home production and tax evasion by households. Section 4 outlines the calibration exercise and presents results. Lastly, Section 5 concludes through a discussion of the welfare implications of the results.

1.1 Why the Middle East and North Africa?

The focus of this paper is the MENA region, more specifically, its Arab states. Based on Transparency International’s 2017 Corruption Perception Index, 19 out of 21 MENA states scored below a 50, on a scale of 0 to 100, keeping in mind that the global average was 43 for the same year. Nonetheless, the MENA region performed better than Sub-Saharan Africa (average score 32), and Eastern Europe and Central Asia (average score 34). In addition to data limitations, perhaps it is the fact that the MENA region’s Arab states do not perform poorly enough that justifies the absence of literature specific to the region. Interestingly, in reporting their results, Transparency International stated:

\[ \text{In a region stricken by violent conflicts and dictatorships, corruption remains endemic in the Arab states while assaults on freedom of expression, press freedom and civil society continue to escalate.} \]

The state of the economy and governing arrangements in the MENA region are deeply intertwined, which provides grounds for a study that seeks to not only capture the effects of corruption on the economies of the MENA region, but also the extent to which corruption is embedded in and a byproduct of the political regimes in place. The figures below provide a general overview of the MENA region’s Arab states’ performance in terms of corruption and governance, based on the World Bank’s Worldwide Governance Indicators (WGI). The WGI reports indicators for 6 dimensions of governance, which are: ¹

²More information on the Worldwide Governance Indicators, including the definitions provided above, can be found here:http://info.worldbank.org/governance/WGI/faq

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1. **Voice and Accountability (VA):** Capturing perceptions of the extent to which a country’s citizens are able to participate in selecting their government, as well as freedom of expression, freedom of association, and a free media;

2. **Political Stability and Absence of Violence/Terrorism (PV):** Capturing perceptions of the likelihood of political instability and/or politically motivated violence, including terrorism;

3. **Government Effectiveness (GE):** Capturing perceptions of the quality of public services, the quality of the civil service and the degree of its independence from political pressures, the quality of policy formulation and implementation, and the credibility of the government’s commitment to such policies;

4. **Regulatory Quality (RQ):** Capturing perceptions of the ability of the government to formulate and implement sound policies and regulations that permit and promote private sector development;

5. **Rule of Law (RL):** Capturing perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence;

6. **Control of Corruption (CC):** Capturing perceptions of the extent to which public power is exercised for private gain, including both petty and grand forms of corruption, as well as “capture” of the state by elites and private interests.

The WGI indicators, by including various indicators of governance, are able to provide a broader understanding of trends in Arab states, particularly in terms of governing arrangements and corruption. All indicators are in units of a standard normal distribution, with mean zero, and standard deviation of one. They use a -2.5 to 2.5 metric, where higher values correspond to better governance. The figures below compare the MENA region’s 17 Arab states to the global average, as well as Arab states (17 in MENA in addition to Djibouti, Mauritania, Sudan, Somalia, and South Sudan) to the global average. This demonstrates that while the MENA region’s Arab states perform poorly, countries in other regions perform worse. However, the MENA’s consistent below-average performance reinforces, to an extent, Transparency International’s diagnosis.

The MENA region’s Arab states consistently score below the world average on all indicators. Arab states score lower than the regionally restricted sample of Arab states, further demonstrating the idea that while the MENA region’s Arab states perform poorly, in terms of governance, relative to the global average, they perform better relative to other countries and regions. Additionally, almost all indicators, with the exception of that on voice and accountability, experience slight improvements leading up to 2010 and 2011, before beginning to decline once again. Historically, this immediately precedes or coincides with the outbreak of the Arab Spring, a wave of sustained demonstrations calling for reform and regime change.

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2The 17 Arab states in the MENA region are: Algeria, Bahrain, Egypt, Iraq, Jordan, Kuwait, Lebanon, Libya, Morocco, the Occupied Palestinian Territories, Oman, Qatar, Saudi Arabia, Syria, Tunisia, the United Arab Emirates, and Yemen.
that swept the region starting in late-2010. The general decline in the different governance indicators is expected, since the glimmer of hope that first accompanied the Arab Spring was quick to die out. Ultimately, most countries either broke out in civil war, or experienced a more repressive reassertion of old regimes. Tunisia, where the Arab Spring first took off, is considered the only triumphant country throughout the entire ordeal. That triumph, however, is not without qualifications.

Having considered how the different governance indicators on average evolve over time, I turn to the economic trends in the region, as they relate to the relevant countries’ levels of corruption. Considering GDP per capita and levels of corruption, Figure 7 shows that higher-income countries in the sample of Arab states in the MENA region perform poorly on the Control of Corruption index relative to countries within the same income range. These countries include Qatar, the United Arab Emirates, Kuwait, Bahrain, Saudi Arabia, and Oman. Among lower-income countries, corruption levels are generally higher. However, in comparison to countries within the same income range, this set of Arab states exhibits more variation in Control of Corruption scores. The trend is that countries with lower incomes tend to have higher levels of corruption, as indicated by the lower Control of Corruption scores they attained. Therefore, there is cause to further explore the effects of corruption on economic performance, and to attempt to understand the mechanisms through which these effects manifest themselves.

![Figure 7: Country corruption levels by GDP per capita ($), 1996-2016](image)

**FIGURE 7:** Country corruption levels by GDP per capita ($), 1996-2016

### 1.2 Forms of Corruption in the MENA Region

In this section, I seek to provide some clarity to corruption, both conceptually and in practice, within the context of the Middle East and North Africa. Corruption can be categorized into petty and large-scale corruption. Petty corruption is the more visible of the two. It includes low-level government employees requesting payments to expedite services they should perform
as part of their jobs. For instance, you might need to bribe a government employee to get a birth certificate registered, renew a license, or to clear items through customs (Ottaway, 2016). Typically, this involves bureaucracies with high levels of red tape, relating back to the idea Tanzi (1998) presents in arguing that in some environments engaging in corrupt activities is more economically efficient. With money exchanging hands, whether directly or indirectly, economic activities are expedited. On a large scale, corruption in such a setting leads to greater efficiency. Petty corruption is primarily a consequence of inadequate salaries. Thus, policies directed toward civil service reform are often recommended to combat petty corruption. While it is not detrimental to the development of a state, it disproportionately harms those on the lower end of the socioeconomic spectrum.

The second broad category is that of large-scale corruption, which takes different forms. Generally, large sums of money designated for public spending are diverted into private hands. At the root of large-scale corruption are weak institutions and enforcement mechanisms, and personal greed. Anti-corruption policies recommended by economists and international organizations often focus on the strengthening of institutions. Civil societies in different countries play a role in trying to expose the face of corruption, demanding accountability, and calling for reforms. However, they are operating in increasingly restrictive environments, as Figure 1 further demonstrates. Restrictions include the prohibition on foreign funding for their activities (Ottoway, 2016). Large-scale corruption can take place during multi-million dollar contract negotiations between government officials and business leaders. One such example involves deals by BAE Systems PLC, Britain’s largest and most influential arms company, and Saudi Arabia (Alissa, 2008). A corruption probe revealed a $110 million slush fund claimed to have been kept by BAE to bribe Saudi officials in order to secure contracts as part of the Al Yamamah arms deal in the 1980s, a series of record arms sales (Alissa, 2008). At the time, Prime Minister Tony Blair intervened to stop the probe under the pretext of British national security. This episode highlights the international dynamics of corruption in the region and demonstrates the complicity of Western powers in the Arab world.

Government-sanctioned corruption is widespread. Egypt provides numerous examples. The military in Egypt controls a good share of the economy, though the figures are ambiguous given the lack of transparency. It owns increasingly valuable property that it sells in deals at below-market prices with bribes making their way into private pockets or back into the military’s obscure economy. Another example demonstrates the involvement of the state and the impunity, with which corruption often takes place (Ottaway, 2016). The Central Auditing Organization (CAO), established in 1942, is meant to be an independent organization. In December 2015, the head of the CAO, Hisham Geneina, publicly stated that corruption cost Egypt E£600 billion (over $33 billion) between 2012 and 2015 (Ottaway, 2016). A presidential commission was immediately set up to investigate Mr. Geneina. While large-scale corruption takes on different forms, the involvement of governments and their facilitation of corruption further emphasize the interconnectedness of governance and corruption in the MENA region.

In a final excursion, Alissa (2016) notes the elusive nature of corruption in some countries. This primarily refers to the exemption of individuals or groups from scrutiny. In Arab Gulf states, ruling families enjoy such immunity. The obscure line separating what belongs to the state and the monarchy provides them with some leeway in avoiding accusations of corruption. Furthermore, parliaments in many countries have no oversight over militaries’ budgets - a constitutional provision insulates the military from civilian oversight in Egypt’s case.
1.3 Existing Literature on Corruption

The literature abounds with studies on corruption and its effects on growth and development. Barreto (2000) adapts a neoclassical growth model to include endogenous corruption. In his model, the effects of corruption on income, consumption, and growth are identifiable. Bureaucratic red tape is later added to the model. The author defines corruption as the illegal profiteering by a public agent from her position as a representative of government. In corrupt transactions, both parties benefit, as in any other economic transaction. He considers three scenarios, one where there is no corruption, the second where there is corruption with a probability of detection, and the last where there is corruption and bureaucratic red tape. He concludes that relative to the competitive equilibrium, the corruption equilibrium is defined by lower growth rates and by sub-optimal levels of private sector income. Simultaneously, the corruption equilibrium is Pareto superior to the bureaucracy-plagued competitive equilibrium. In both cases, corruption leads to income redistribution.

Studies that further focus on the consequences of corruption on development include those by Shleifer and Vishny (1993) and Mauro (1998). In the former, the authors assume a principal-agent model of corruption as they proceed to consider two cases of corruption: with and without theft. They then consider three possible structures or corruption networks: joint monopoly, independent monopoly, and competition. In the case of joint corruption, all government agencies and individuals maximize their revenues from bribes collectively, whereas in the independent monopoly scenario, each agent providing a government-produced good maximizes their own revenues. Thus, while the latter charges higher bribes, it generates less revenue from bribes relative to the joint monopoly structure. Lastly, they consider the greater distortionary effect of corruption relative to taxes due to the element of secrecy inherent in corruption. Consequently, they conclude that corruption can shift a country’s investment away from high value projects, like health and education, and toward less useful projects. Mauro (1998) corroborates their claim that corruption correlates with lower spending on education. He also finds a potentially inverse relationship between corruption and spending on health. The theoretical explanation for this is that corruption shifts expenditure to sectors that have more lucrative corruption opportunities, and where corruption is less detectable.

Lastly, in an earlier study, Mauro (1995) set out to identify the channels through which corruption and other institutional factors affect economic growth. He found that corruption lowers private investment, thereby reducing economic growth, even in subsamples of countries where bureaucratic regulations are very cumbersome. Thus, he did not find evidence in support of previous findings where increased red tape increases the incidence of corruption, in turn enhancing economic growth. The author used data from Business International (BI) to construct two indices: bureaucratic efficiency and political stability. Furthermore, to address the possibility of endogeneity, he constructed an instrument of ethno-linguistic fractionalization and other variables that control for colonial history. He concluded that bureaucratic efficiency causes high investment and growth.

Furthermore, the literature considers the effect of tax evasion and tax corruption on growth and development. Pappa et al. (2015) and Orsi et al. (2014) examine the effects of fiscal policies on the economy, within the context of tax evasion, corruption, or the underground economy. Pappa et al. (2015) study the effects of fiscal consolidation in the presence of corruption and tax evasion on output, unemployment, and welfare. They use tax evasion as
a proxy for the shadow economy. Additionally, they use a New Keynesian dynamic stochastic general equilibrium (DSGE) model to determine the effects of labor tax hikes and government spending cuts in Greece, Italy, Portugal, and Spain. They find that the negative effects of labor tax hikes on output and unemployment are amplified by the presence of tax evasion and corruption. Tax hikes exacerbate output losses by inducing workers to reallocate resources to the informal sector. However, tax evasion and corruption mitigate the negative effects of government expenditure cuts. Furthermore, they find that labor tax hikes are costly in terms of welfare, while government spending cuts involve welfare gains as long as they do not enter the utility of households, and agents are not liquidity constrained. They observe the largest output losses in Portugal and Greece. The former due to the size of the tax hikes, and the latter given the severity of austerity measures. Portugal also seems to experience the largest welfare loss due to the significant tax hike in its consolidation package. These findings are within the context of consolidation packages that relied heavily on spending cuts as opposed to labor tax hikes.

While also using a DSGE model, Orsi et al. (2014) incorporate an underground economy in the model, with a focus on Italy. Their model assumes incomplete tax enforcement and a standard two-sector neoclassical stochastic growth model, in which the underground emerges as a consequence of incentives for agents to conceal transactions to avoid tax payments. Similar to Pappa et al. (2015), they find that the growing size of the underground economy in Italy is a result of a steady increase in taxation. Thus, moderate tax cuts, along with stronger monitoring efforts, can cause reductions in the size of the underground economy.

Lastly, Célimène et al. (2016) use a stochastic growth model to study the effects of tax evasion and corruption on the level and volatility of private investment and public spending, both of which are determinants of growth. They suggest alternative solutions to a government’s inability to reduce the levels of corruption and tax evasion. One is through allowing resources of evaded taxes to be invested in equities, and the other to raise the efficiency of public spending to dampen the negative externalities of tax evasion on productive public expenditure. Importantly, in their model, the decision of an agent to cheat and corrupt a bureaucrat is a rational decision. Thus, they conclude that tax evasion is not necessarily a burden, but an opportunity and an optimal response to governance failures and inadequate tax administration. Furthermore, they conclude that tax evasion and corruption influence the volatility of public spending and private investment, in turn affecting the volatility of the economy.

I will be following the example of Barreto (2000) by developing a neoclassical stochastic growth model with endogenous corruption. Furthermore, similar to studies by Pappa et al. (2015), Orsi et al. (2014), and Célimène et al. (2016) I examine the effects of corruption on tax evasion and the determinants of growth. In terms of fiscal policy, I focus only on how changes in the tax rate affect tax evasion by households. I restrict the calibration of the model to countries in the MENA region, unlike previous studies. For the purpose of this study, I will be turning my attention to the effect of corruption on the determinants of economic performance. The implications of corruption on development is a topic that merits further exploration, and which I hope to pursue in later stages.
2 Model: Lump-sum Tax

In addressing the research question of the effects of corruption on economic performance, I use a neoclassical growth model that I will later calibrate to provide further insight on the effects of corruption on the economic performance of Arab states in the MENA region. The base model is a neoclassical model with non-separable preferences. Having non-separable preferences is able to capture the effects of corruption on household utility-maximizing decision-making, since the level of corruption determines the supply of public goods, which a household observes before optimizing its utility. The model captures a closed economy in which firms, the government, and households are making decisions in a corrupt environment. Each problem will be considered separately below.

2.1 Representative Household’s Problem

Household agents in the model are assumed to live forever, and so they are maximizing utility over infinitely many periods. Furthermore, they have perfect foresight and have increasing, concave utility functions. The representative household’s lifetime utility-maximization problem is as follows:

\[
\max_{c_t, n_{g,t}, n_{m,t}, k_{g,t}, k_{m,t}, k_t} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left[ \ln \left( \gamma c_t^\rho + (1 - \gamma) g_{h,t}^\rho \right)^{1/\rho} + \nu \ln \left( 1 - n_{g,t} - n_{m,t} \right) \right] \right] \tag{1}
\]

subject to,

\[
c_t + k_{t+1} - (1 - \delta) k_t = w_{g,t} n_{g,t} + w_{m,t} n_{m,t} + R_{g,t} k_{g,t} + R_{m,t} k_{m,t} - \Gamma_t \tag{2}
\]

\[
k_t = k_{g,t} + k_{m,t} \tag{3}
\]

In their decisions, households are maximizing utility subject to a budget constraint. They solve for consumption \((c_t)\), labor supplied to the government \((n_{g,t})\), labor supplied to the private sector \((n_{m,t})\), capital supplied to the government \((k_{g,t})\), capital supplied to the private sector \((k_{m,t})\), and capital stock \((k_{t+1})\). Households derive utility from consumption, public goods \((g_{h,t})\), and leisure \((1 - n_{g,t} - n_{m,t})\). In Equation 1, \(\beta\) is a discounting factor, \(\rho\) is the elasticity of substitution, \(\gamma\) is a share parameter, and \(\nu\) the disutility of labor parameter. In the budget constraint, Equation 2, \(\delta\) is the depreciation rate of capital.

Furthermore, households generate income from supplying labor and capital to the government and private sector. They receive wages \(w_{g,t}\) and \(w_{m,t}\) for their labor in the public and private sectors, respectively. They also receive rents \(R_{g,t}\) and \(R_{m,t}\) for supplying capital to the public and private sectors, respectively. I assume that household agents divide their capital between the government and the market, as indicated in Equation 3. Lastly, they pay a lump sum tax of \(\Gamma_t\).

2.2 Government Optimization Problems

The representative government agent faces two sets of problems: cost-minimization and utility-maximization. The cost-minimization problem is set up below:
\[
\min_{n_{g,t}, k_{g,t}} w_{g,t} n_{g,t} + R_{g,t} k_{g,t} \tag{4}
\]
subject to,
\[
y_{g,t} = A_{g,t} n_{g,t} \left(1 - \alpha_{g,t}\right) k_{g,t}^{\alpha_{g,t}} \tag{5}
\]
The government agent makes decisions as to how much household-supplied labor \((n_{g,t})\) and capital \((k_{g,t})\) to use so as to minimize the costs of government production, given wage \(w_{g,t}\) and rent \(R_{g,t}\). Equation 2 is the government’s production function to produce government output \(y_{g,t}\). The government has an increasing, concave, constant returns to scale production function, in which \(A_{g,t}\) is total factor productivity (TFP), and \(\alpha\) is the capital share of income.

In addition to making government production decisions, the government agent must maximize utility. Once again, government agents have an increasing, concave utility function. The level of corruption in an economy is only directly observed by government agents. The public agent’s utility-maximization problem is:
\[
\max_{g_{h,t}, g_{c,t}} \psi \ln g_{h,t} + (1 - \psi) \ln g_{c,t} \tag{6}
\]
subject to,
\[
y_{g,t} = g_{h,t} + g_{c,t} \tag{7}
\]
The government agent derives utility from providing a public good to households \((g_{h,t})\), and from using government output for some private benefit \((g_{c,t})\). The parameter \(\psi\) inversely captures the level of corruption. It takes on values between 0 and 1. The higher \(\psi\) is, the less corrupt is the government. Lower \(\psi\) values correspond to a higher weight a public agent places on private consumption. Thus, the public agent, by deciding how much government output to designate for public goods and personal use, determines the level of corruption in the economy. I assume that government output is fully allocated between public good provision and private benefit, as Equation 7 illustrates.

Households do not directly observe the level of corruption at any given time. However, they can observe the level of public service provision relative to the tax they pay. This is the mechanism through which different \(\psi\) values are able to affect household utility-maximizing allocations of labor and capital, as well as levels of consumption, holding all else equal.

### 2.3 Firm’s profit-maximization

The representative firm faces a standard profit-maximization problem and an increasing, concave, constant returns to scale production function. I express it below:
\[
\Pi = \max_{n_{m,t}, k_{m,t}} A_{m,t} n_{m,t}^{1 - \alpha_{m,t}} k_{m,t}^{\alpha_{m,t}} - R_{m,t} k_{m,t} - w_{m,t} n_{m,t} \tag{8}
\]
Firms choose the amount of household-supplied labor \((n_{m,t})\) and capital \((k_{m,t})\) to use in their production process to maximizing profits. Their costs comprise of total rent \((R_{m,t} k_{m,t})\) and total wages \((w_{m,t} n_{m,t})\). Meanwhile, they face a TFP level of \(A_{m,t}\), and a capital share of income of \(\alpha\), similar to the government cost-minimization problem.
2.4 Further Model Constraints

Having set up the decisions facing households, the government, and firms, I introduce further restrictions to discipline the model, tying the closed economy together before being able to solve for optimal decisions within it.

2.4.1 Total Output, Investment, and Capital Accumulation

Total output in the economy is defined as:

\[ y_{m,t} = I_t + c_t + y_{g,t} \]  

(9)

Total output includes government output \((y_{g,t})\) since all output in the economy is either directly consumed, invested, or directed toward the public sector to produce government output, part of which is consumed for private benefit \((g_{c,t})\), and the other consumed by households in the form of public services \((g_{h,t})\). The share of market output that goes to the government for public sector production is captured by the parameter \(\theta\), such that:

\[ \theta = \frac{y_{g,t}}{y_{m,t}} \]  

(10)

Investment in Equation 9 is expressed as:

\[ I_t = k_{t+1} - (1 - \delta)k_t \]  

(11)

Thus, capital accumulation is:

\[ k_{t+1} = I_t + (1 - \delta)k_t \]  

(12)

2.4.2 Total Factor Productivity

Furthermore, I introduce shocks to TFP, captured by the following expressions:

\[ A_{m,t} = (1 - \phi)\overline{A}_m + \phi A_{m,t-1} + \epsilon_{m,t} \]  

(13)

\[ A_{g,t} = (1 - \phi)\overline{A}_g + \phi A_{g,t-1} + \epsilon_{g,t} \]  

(14)

Here, \(\phi\) is a parameter for shock persistence that takes a value between 0 and 1. In addition, \(\overline{A}_m\) and \(\overline{A}_g\) are steady-state values for TFP variables \(A_m\) and \(A_g\), respectively. Lastly, \(\epsilon_{m,t}\) and \(\epsilon_{g,t}\) are the stochastic elements of \(A_m\) and \(A_g\), respectively.

2.4.3 Market Clearing Condition

In order to solve for the competitive equilibrium in the model, I will need a market clearing condition, which is expressed as:

\[ c_t + k_{t+1} = (1 - \delta)k_t + y_{m,t} \]  

(15)
2.5 Parameterization

In order to run the model, I assign values to parameters. Since the parameter of interest in this model is the corruption parameter $\psi$, I will fix other parameters and only vary $\psi$ in order to draw conclusions as to how household decisions and determinants of economic performance behave in environments of different levels of corruption. I assign parameters the following values for the first iteration:

- $\beta = 0.99$
- $\gamma = 0.8$
- $\rho = 0.1$
- $\nu = 1.25$
- $\alpha = 0.4$
- $\delta = 0.025$
- $\theta = 0.21$
- $\phi = 0.8$
- $A_m = 1$
- $A_g = 1$

The value assigned to the disutility of labor, $\nu$, will be calibrated under Section 5. However, at this stage, I assign a conservative estimate of 1.25. In order to capture the effect of corruption on the determinants of economic performance, including labor and capital supply, consumption, and investment, I simulate the model per determinant across a range of $\psi$ values. In this iteration, $\psi$ takes on values between 0.01-0.98, where, theoretically, $0 < \psi < 1$, and where higher $\psi$ values correspond to higher government integrity and lower values to higher levels of corruption.

With the model set up, and parameters determined, I am able to simulate the model using Matlab. For the characterized solutions to the model, necessary to simulating it, see Appendix 7.1. Results are previewed and discussed in the next section.

2.6 Results

Using the characterized solutions and Matlab, I was able to calculate steady-state values for all endogenous variables in the model described above. By varying values of $\psi$, I obtain the results below, which can be explained using a negative income effect argument. The results are at first unexpected in that the determinants of economic performance fall as $\psi$ increases, or as the level of corruption falls in a given environment, holding all else equal.

Figure 8 captures the effect of lower levels of corruption on the primary pathway through which determinants of economic performance fall. As $\psi$ goes up, the public agent places more weight on the provision of public goods for households. For households, the more public goods and services they are able to enjoy, the more likely they will value leisure, as opposed to expending labor and capital to earn a higher income that will then enable them to consume goods that are otherwise publicly provided where low levels of government corruption exist.

The drop in labor supply is more pronounced in the private sector, with that in the public sector falling slightly before plateauing. The fall in the private sector might be driven by the fact that some public goods substitute private sector occupations now that they are publicly provided. With less labor supplied to both the government and the market, household agents generate lower income. Thus, their disposable income decreases. As Figure 9 demonstrates, as corruption levels fall, and so as $\psi$ values increase, consumption decreases at a diminishing rate. In turn, lower incomes correspond to lower saving and investment patterns, which is why we expect investment and capital factor supply to decrease as well. This is the case, as Figures 10 and 11 show.
FIGURE 8: The variation of labor supply with corruption levels ($\psi$)

FIGURE 9: The variation of consumption of market-produced goods and services with corruption levels

FIGURE 10: The variation of investment with corruption levels ($\psi$)

FIGURE 11: The variation of capital factor supply with corruption levels ($\psi$)
Even so, wages, and labor productivity remain relatively constant. We observe a drop in total output generated in the economy, as can be seen in Figure 12. Thus, the relative stability of output per worker suggests that total output \((y_{m,t})\) and labor supply \((n_{m,t})\) are falling at similar rates such that a decrease in one is offset by a decrease in the other in terms of productivity, or output per worker.

Other changes in endogenous variable behavior abound with changes in other parameters, namely the share of market output that goes to the government \((\theta)\), and the elasticity of substitution \((\rho)\) that captures how substitutable government spending is with consumption. However, in this section I display results that relate to changes in corruption levels, in order to provide the intuition of the base model with non-separable household preferences and lump sum taxes before considering tax rates and tax evasion behavior in following iterations.

At face value, the decreases observed in labor and capital supply, market-based consumption, investment, and total output seem counter-intuitive. Decreases in the consumption of market-produced goods and investment, and the simultaneous increase in public service provision creates ambiguous results for GDP, and GDP growth trends. This will be further explained after adapting the model to reflect a tax rate and tax evasion, and calibrating the model to generate country-specific results regarding the effects of corruption on economic performance.

However, the observed decreases have positive welfare implications. Households enjoy more leisure and consumption with lower corruption. With higher levels of corruption, households are being robbed of goods and services that the government should have otherwise provided, whether completely or on a wider scale. By reducing corruption levels, households no longer have to work and supply as much capital as they used to given the public goods now provided, as seen in the higher levels of leisure in less corrupt environments (see Figure 8). Thus, working more, in this model, reflects the cost of corruption. Furthermore, households are now better off, since more or better quality goods and services are provided and consumed (see Figure 13). In terms of taxes, they are being reallocated more efficiently in such a setting.
3 Model: Tax Rates and Home Production

In the second iteration of the model, I will further explore the pathways through which corruption changes household behavior. I do so by imposing a tax rate instead of a lump-sum tax system, and by introducing home production. The model will be developed to incorporate both changes in subsections 3.1 and 3.2.

3.1 Introducing a Tax Rate

The first model, by incorporating corruption under a lump-sum tax system, was able to capture the pure income effect of varying levels of corruption in an economy. However, the substitution effect is another channel through which different levels of corruption affect household behavior. Whereas the income effect operates by inducing individuals to work more (by supplying more labor and capital) the more corrupt an environment is, the substitution effect induces households to work less, given that the more corrupt a government is, the higher the share of tax revenue goes toward the private benefit of public officials. One such way to capture the latter effect is by introducing a tax rate, in place of a lump-sum tax system.

Imposing a tax rate only affects the budget constraint of households, such that, relative to Equation 2, it becomes:

\[ c_t + k_{t+1} - (1 - \delta)k_t = (w_{g,t}n_{g,t} + w_{m,t}n_{m,t} + R_{g,t}k_{g,t} + R_{m,t}k_{m,t})(1 - \tau) \]  

where \( \tau \) is the statutory tax rate. The remainder of the model remains unchanged by imposing a tax rate.

3.2 Home Production

The motive behind accounting for home production is the idea that corruption might have some feedback effects beyond labor and capital supply decisions between the public and private sectors. More specifically, home production represents a mechanism through which one might be able to capture tax evasion. Even so, a representative household might choose to produce at home because of their work environment preferences. Furthermore, home production is motivated by a household’s decision to consume home-produced goods as opposed to purchasing commodities from the market. Home production, broadly, can be a form of tax avoidance. In the model, home-produced goods and services are not taxed, implying that households do not report income generated from home production. Thus, I can use home production, conceptually, to capture, in part, how a household’s tax evasion behavior responds to changes in the level of government corruption.

Incorporating home production into the model only changes the household’s utility-maximization problem. I assume that household agents prefer to consume market goods and services, and home-produced goods and services equally. The representative household’s problem becomes:
\[
\max_{c_t, n_{g,t}, n_{m,t}, n_{h,t}, k_t, k_{g,t}, k_{m,t}, k_{h,t}, k_{t+1}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left[ \ln \left( \gamma \left( c_{m,t}^\sigma + c_{h,t}^\sigma \right) \right)^{\frac{\rho}{\sigma}} + (1 - \gamma) g_{h,t}^\sigma \right]^{1/\rho} + \nu \ln \left( 1 - n_{g,t} - n_{m,t} - n_{h,t} \right) \right] \tag{17}
\]

subject to,
\[
c_t + k_{t+1} - (1 - \delta) k_t = (w_{g,t} n_{g,t} + w_{m,t} n_{m,t} + R_{g,t} k_{g,t} + R_{m,t} k_{m,t}) (1 - \tau) + \pi_{f,t} \tag{18}
\]
\[
k_t = k_{g,t} + k_{m,t} + k_{h,t} \tag{19}
\]
\[
c_{h,t} = A_h n_{h,t}^{1-\alpha} k_{h,t}^\alpha \tag{20}
\]

First, consider the household’s utility function, seen in Equation 17. The variable \( c_t \) in Equation 1 is reintroduced as a combination of \( c_{m,t} \) and \( c_{h,t} \), where the former is a household’s consumption of market-produced goods and services, and the latter a household’s consumption of home-produced ones. The absence of a share parameter between the two implies that household’s prefer them equally. Furthermore, leisure is now a function of \( n_{g,t}, n_{m,t}, \) and \( n_{h,t} \), where \( n_{h,t} \) is labor dedicated to home-production processes. The utility function exhibits constant elasticity of substitution (CES) between the consumption of home-produced goods and market produced ones, with parameter \( \sigma \) representing the elasticity of substitution. It also exhibits CES preferences with regard to the consumption of home-produced and market-produced goods, on the one hand, and public goods on the other, with the parameter \( \rho \) representing the elasticity of substitution, similar to Equation 1. Thus, the household utility function exhibits CES preferences within CES preferences.

Second, consider Equation 18, the household’s budget constraint. I adapt Equation 16, the budget constraint with a tax rate, by adding profit from home production, \( \pi_{f,t} \), to net income on the right hand side. Note that profit accrued from home production is un-taxed income, thus capturing tax evasion, or a household’s under-reporting of income. Third, Equation 19 defines total capital as the sum of household capital supplied to the public sector (\( k_{g,t} \)), the private sector (\( k_{h,t} \)), and home production (\( k_{h,t} \)). Lastly, Equation 20, captures the household’s production function. I assume that the household consumes all that it produces at home. Thus, the household’s production function is equal to the household’s consumption of home-produced goods and services. The household’s TFP, \( A_t \), is assumed to be constant unlike those for government and market production functions (see Equations 13 and 14, respectively). Home production is also a function of labor and capital used in home production, captured by \( n_{h,t} \) and \( k_{h,t} \), respectively, with capital share of income \( \alpha \).

As a consequence, total output and the market clearing condition equations need to be updated. Relative to Equation 9, total output is now expressed as:
\[
y_{m,t} = I_t + c_{m,t} + y_{g,t} \tag{21}
\]
and compared to Equation 15, the market clearing condition is:
\[
c_{m,t} + c_{h,t} + k_{t+1} = (1 - \delta) k_t + y_{m,t} \tag{22}
\]


3.3 Parameterization

Similar to the parameterization procedure previously outlined, most parameters are assigned similar values. They retain the same values determined under Section 2.5, with the addition of two parameters. First, the elasticity of substitution parameter $\sigma$ is set at 0.1, similar to the elasticity of substitution parameter $\rho$. Second, $A_h$, the TFP parameter for home-production, is equal to 1, as is the case with steady-state TFP for both government and market production, $A_g$ and $A_m$, respectively.

However, the corruption parameter $\psi$ is not the only one of interest. Having introduced tax rates, and tax evasion by households, the effect of the tax rate $\tau$ on household behavior is also important to consider. Therefore, I simulate the model over $\psi$ values ranging 0.01-0.98, with higher values corresponding to higher levels of government integrity and lower corruption. I also simulate the model for tax rates $\tau$ ranging 0.00-0.87. When varying the level of corruption, I fix the tax rate at 0.25, whereas when varying the tax rate I fix the level of corruption at 0.35, a moderately high level of corruption.

3.4 Results

I am able to simulate the model using Matlab after characterizing the solutions, similar to the first model (see Section 2). Other than changes derived in the previous two subsections, the model retains the same structure. However, instead of solving for 19 endogenous variables and equations, I will now be solving for 22 endogenous variables and equations. For the characterized solutions, see Appendix 7.2. In this iteration, I will be concerned with how a household’s allocation of resources responds to changes in the tax rate and level of corruption.

I first turn to how varying the tax rate affects household behavior. As one would expect, an increase in the tax rate will divert a household’s supply of labor away from the public and private sectors, and toward home production, since only the latter produces untaxed income. Figure 14 demonstrates that behavior. In aggregate, labor supply falls as the tax rate increases, suggesting that the substitution effect of higher taxes dominates the income effect exhibited by households’ increase in the supply of labor to home production as tax rates increase. This fall in labor supply corresponds to a drop in net income, thus lower consumption and investment, which model results corroborate.

Furthermore, capital supply to all sectors drops at higher tax rates, see Figure 15. This implies that higher taxes result in less money to invest, and so to less capital, countering the effect of higher labor supply to home production that would otherwise lead to higher returns to capital. In turn, this justifies the drop in the consumption of home-produced goods despite the increase in labor supply to home production.

Lastly, with respect to tax rates, labor productivity and wages fall as tax rates rise. Labor productivity falls as the tax rate increases in both the public and private sectors. Since both labor supply to and output from the public and private sectors decline as the tax rate increases, falling labor productivity suggests that output falls at a faster rate than labor supply. This might be the result of lower labor motivation at higher tax rates, whereby a greater portion of their income will now go toward the government’s tax revenue, all else equal. As for wages, they drop at the same rate as the tax rate increases, which is the case...
Figure 14: The variation of labor supply with tax rates ($\tau$) by sector

Figure 15: The variation of capital supply with tax rates ($\tau$) by sector

since wage, whether government or market, is a function of labor productivity. Thus, labor productivity and wages move in the same direction.

Next, consider how household decisions change as the level of corruption changes. Labor and capital supply across all sectors drop as the level of government integrity increases. Lower labor supply corresponds to lower income levels, and, by extension, lower consumption, investment, and capital. Again, the income effect dominates the substitution effect. Alternatively, as corruption levels increase, households work more, providing more labor and capital to the government and market, to earn more, since public services are not as available as they otherwise would have been. Furthermore, labor productivity and wages in the government and market are unchanged by variations in the level of government corruption.
Of particular interest is the fact that the consumption of market goods and home-produced ones falls as the level of government integrity, $\psi$, rises (alternatively, as the level of corruption falls). The less corrupt a government is, the more value households place on leisure. Thus, they supply less labor and earn a lower income. The income effect at play justifies the results obtained. The fact that the consumption of home-produced goods falls as government integrity increases, see Figure 15, implies that in less corrupt environments, tax evasion by households will decline. Therefore, reductions in the level of corruption are likely to result not only in general welfare gains, but also in the reduction of tax evasion levels.

![Graph showing the variation of household consumption and production ($c_{h,t}$) with corruption ($\psi$) by sector](image)

**FIGURE 16:** The variation of household consumption and production ($c_{h,t}$) with corruption ($\psi$) by sector

## 4 The Quantitative Experiment

I calibrate the model developed in Section 3 in order to compare the performance of a set of countries in the MENA region to that of the USA, with differences resulting from their varying levels of corruption and tax rates. Comparing countries in the MENA region to the US is motivated by the size and health of the US economy globally, as well as the relatively low level of corruption observed. In 2014, with a Control of Corruption score of 1.38, the US was in the 89th percentile of countries for which data exists. Furthermore, a comparative analysis relative to the USA is a common thread in the literature. The variables I calibrate are the level of corruption ($\psi$), the tax rate ($\tau$), and the disutility of labor ($\nu$). Other parameters are determined through a procedure similar to that seen in Sections 2.5 and 3.3.

I calibrate the model for the year 2014 given data availability. I choose countries from the MENA region for which to calibrate the model based on their income levels and Control of Corruption scores. Therefore, in addition to calibrating the model for the USA as the baseline, I calibrate it for the UAE, Saudi Arabia, Jordan, Morocco, Egypt, and Lebanon. Relative to the sample, and with reference to Figure 7, the UAE (ARE) is a high-income
country, Saudi Arabia (SAU) a middle-income country, and Jordan (JOR), Morocco (MOR), Egypt (EGY), and Lebanon (LBN) are low-income countries. Considering the sample of low-income countries in the MENA region, Jordan has the highest Control of Corruption score, while Lebanon has the third lowest score, only higher than that of Iraq and Yemen. I do not consider Yemen or Iraq due to the high level of political instability that might be more directly related to their corruption score. Morocco and Egypt both have negative and rather similar scores, however, the difference in their levels of political stability and recent history merits individual attention. Egypt was the second country to witness mass protests calling for reform and regime change in late-2010 and early-2011. Without Egypt’s participation, protests demanding accountability and regime change would not have swept the region in what became known as the Arab Spring, given Egypt’s cultural and historical leadership in the region. At the same time, Morocco was able to contain the spread of the Arab Spring in its quarters. This difference motivates calibrating the model for both Egypt and Morocco despite their close corruption scores, which could inform further country-specific analysis.

To calibrate \( \psi \), I use data from the World Bank’s WGI Control of Corruption Index. I assume a uniform distribution between 0 and 1 from which I draw a corruption value for the US. Given the relatively low level of corruption observed in the US, I assign it a \( \psi \) value of 0.9. Based on the country of interest’s corruption score, relative to that of the US, I determine how far less that country’s relative score is in a uniform distribution. Since the corruption indicator is in units of standard normal distribution, with mean zero and standard deviation of one, the difference between the country of interest’s score \( CC_c \) and the US’s \( CC_{USA} \) captures how many standard deviations apart they are. Thus, the \( z \)-score I target is the difference between the scores of country of interest \( c \), and the US. Accordingly, I calculate the relative score, \( \psi_c \), of each country in the sample:

\[
z - score = \frac{\psi_c - 0.9}{1/\sqrt{12}} = CC_c - CC_{USA}
\]

Thus,

\[
\psi_c = (CC_c - CC_{USA})(1/\sqrt{12}) + 0.9
\]

where \( \psi_c \) is the level of corruption for country \( c \) based on the uniform distribution, \( CC_c \) is the Control of Corruption Index for country \( c \), and \( CC_{USA} \) is the Control of Corruption Index for the USA (here: 1.38). The standard deviation of the uniform distribution is \( 1/\sqrt{12} \). Since all countries considered have a corruption score lower than that of the US, all scores drawn from the uniform distribution will be less than 0.9. Therefore, the \( z \)-score I target is calculated as \( CC_c - CC_{USA} \) as opposed to \( CC_{USA} - CC_c \).

Furthermore, I calibrate \( \tau \) using data from the Penn World Table version 9.0. In the model, \( \tau \) represents the tax rate. The model assumes that the government’s tax revenue perfectly finances government production, and that the government uses market output to produce its own output. Therefore, I estimate the tax rate using government consumption as a share of GDP or the economy’s output \( y_{m,t} \) in the model. Empirically, I use 2014 data on government consumption as a share of output-side real GDP at current PPPs (in mil. 2011 US $). The calibrated values for \( \psi \) and \( \tau \) are summarized in Table 1.
Parameter values are assigned such that they follow the general convention in the literature, and enable the model to run for iterations with different $\psi$ and $\tau$ values. Thus, while they are similar to parameter values previously specified, they fall within a close range of each other. The parameter values used in this calibration exercise are as follows:

- $\beta = 0.98$
- $\rho = 0.1$
- $\alpha = 0.34$
- $\theta = 0.21$
- $\bar{A}_m = 1$
- $\bar{A}_h = 1$
- $\gamma = 0.8$
- $\sigma = 0.1$
- $\delta = 0.1$
- $\phi = 0.8$
- $\bar{A}_g = 1$

Lastly, I simulate a counter-factual by setting all $\psi$ values to the US value of 0.9, while maintaining the differentiated $\tau$ values. The observed difference in a household’s allocation of labor can then be attributed to differences in the levels of corruption. The results from the calibration and counter-factual can be seen in Table 2. I will only be observing the effect of the calibration exercise on the labor supply decisions of households since changes in labor are tied to and cause changes in the other determinants of economic performance.

### TABLE 1: Calibrated values for $\psi$ and $\tau$ per country

<table>
<thead>
<tr>
<th>Country</th>
<th>USA</th>
<th>UAE</th>
<th>Saudi Arabia</th>
<th>Jordan</th>
<th>Morocco</th>
<th>Egypt</th>
<th>Lebanon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control of Corruption score</td>
<td>1.38</td>
<td>1.2</td>
<td>0.09</td>
<td>0.14</td>
<td>-0.27</td>
<td>-0.62</td>
<td>-1.04</td>
</tr>
<tr>
<td>Calibrated $\psi$</td>
<td>0.9</td>
<td>0.848</td>
<td>0.528</td>
<td>0.542</td>
<td>0.424</td>
<td>0.323</td>
<td>0.201</td>
</tr>
<tr>
<td>Calibrated $\tau$</td>
<td>0.113</td>
<td>0.04</td>
<td>0.201</td>
<td>0.192</td>
<td>0.193</td>
<td>0.166</td>
<td>0.192</td>
</tr>
</tbody>
</table>

Calibration results indicate that all countries in the MENA region, with the exception of the UAE, experience lower labor supply to the market ($n_{m,t}$) and government ($n_{g,t}$) in comparison to the levels enjoyed by the US. Furthermore, they have higher labor supply to home-production relative to the USA, again, with the exception of the UAE. The behavior of the UAE is caused by differences in the countries’ share of government consumption ($\tau$),
since their levels of corruption are rather close. The results from the counter-factual reveal that the difference in the levels of corruption of the USA and the UAE has a negative effect on labor supplied to home-production. It must therefore be that differences in the value of $\tau$ drives the UAE to have a lower level of labor supply to home production and higher levels to both the market and the government, relative to the USA.

The percentage change captures the difference in the results of the calibration and counter-factual, moving from an economy with the calibrated level of corruption to an economy with a 0.9 level of government integrity. The income effect is the dominant mechanism through which labor supply to all sectors falls with an improvement in a government’s integrity. Of significance, however, is the fall in the labor supplied to home-production, which includes, or is a proxy for, tax evasion. Intuitively, the greater the difference between the the calibrated level of corruption and the counter-factual level of 0.9, the greater the reduction in labor supply to, and by extension production by, households. Lastly, as one would expect, labor supply levels to all sectors drops by a similar proportion per country.

The calibration exercise produced results that are able to capture how countries with different levels of corruption and government consumption fair relative to the USA. Meanwhile, the counter-factual’s results isolate the effect of corruption, rather the cost of having a higher level of corruption relative to the level enjoyed by the US, on labor supply to all sectors in an economy. While labor supply across all sectors falls, due to the income effect, the drop in labor supply that leads to lower levels of home-production activities is encouraging insofar as it curbs tax evasion behavior by households.

5 Conclusion

Similar to results found by Pappa et al. (2015) and Orsi et al. (2014), higher tax rates correspond to higher levels of tax evasion. Meanwhile, lower levels of corruption, or higher levels of government integrity, are associated with lower labor supply, and household consumption and production, as the negative wealth effect dominates the substitution effect in all iterations. However, the increase in the consumption of public goods with improvements in the level of government integrity supports the claim of welfare gains.

The results of the calibration exercise indicate that for MENA region countries lowering their levels of corruption to that of the US will experience a reduction in tax evasion by households. The greater the improvement in government integrity, the greater the reduction in tax evasion and home-production. Combined with the increased consumption of public goods at lower levels of corruption, there is greater support and evidence for the welfare gains of households in the MENA region. The main mechanism through which these dynamics play out is a dominant negative wealth effect. These findings support the positive correlation between corruption and tax evasion found in the literature. It is worth emphasizing that while the calibration results are in line with the findings of previous models, the magnitude of the effects is very small. Even so, the results obtained can be used for further study of corruption and tax evasion. Future studies can adapt the model to an open economy as opposed to the closed one assumed in this study. Furthermore, we can introduce more dynamics in the economy by incorporating a probability of detection and punishment of tax evasion, as a proxy for the robustness of monitoring processes and tax administration systems.
6 Acknowledgments

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7 Appendix

7.1 Characterized solutions: Lump-sum Tax

This section outlines the characterized solutions of the model covered in section 2. Each of the solutions for the household, government, and firm will be characterized individually. This will be followed by the remaining equations and conditions that further discipline the model. Lastly, this section will conclude with a list of 19 characterized solutions and endogenous variables that the model solves for.

7.1.1 Household’s Characterized Solutions

I set up the Lagrangian using Equations 1, 2, and 3 laid out in Section 2.1. Here, $\lambda_{h,t}$ and $\mu_t$ are Lagrangian multipliers. I then take the partial derivative of the Lagrangian with respect to each of the six choice variables for households ($c_t, n_{g,t}, n_{m,t}, k_{g,t}, k_{m,t}$, and $k_{t+1}$). To optimize household decisions, I equate each of the partial derivatives to zero.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[ \ln \left( \gamma c_t^\rho + (1 - \gamma) g_{h,t}^\rho \right)^{1/\rho} + \nu \ln \left( 1 - n_{g,t} - n_{m,t} \right) + \lambda_{h,t} \left( w_{g,t} n_{g,t} + w_{m,t} n_{m,t} \right) 
+ R_{g,t} k_{g,t} + R_{m,t} k_{m,t} - \Gamma_t - c_t - k_{t+1} + (1 - \delta) k_t \right] + \mu_t (k_t - k_{g,t} - k_{m,t})$$

1. Consumption:

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t \cdot \frac{1}{\rho} \cdot \frac{\rho c_t^{\rho-1}}{\gamma c_t^\rho + (1 - \gamma) g_{h,t}^\rho} - \beta^t \cdot \lambda_{h,t} = 0$$

$$\Leftrightarrow \frac{\gamma c_t^{\rho-1}}{\gamma c_t^\rho + (1 - \gamma) g_{h,t}^\rho} = \lambda_{h,t}$$

2. Labor supply to the government:

$$\frac{\partial \mathcal{L}}{\partial n_{g,t}} = -\beta^t \cdot \nu \cdot \frac{1}{1 - n_{g,t} - n_{m,t}} + \beta^t \cdot \lambda_{h,t} \cdot w_{g,t} = 0$$

$$\Leftrightarrow \nu \frac{1}{1 - n_{g,t} - n_{m,t}} = \lambda_{h,t} w_{g,t}$$

3. Labor supply to the market:

$$\frac{\partial \mathcal{L}}{\partial n_{m,t}} = -\beta^t \cdot \nu \cdot \frac{1}{1 - n_{g,t} - n_{m,t}} + \beta^t \cdot \lambda_{h,t} \cdot w_{m,t} = 0$$

$$\Leftrightarrow \nu \frac{1}{1 - n_{g,t} - n_{m,t}} = \lambda_{h,t} w_{m,t}$$

24
4. Capital supply to the government:

\[ \frac{\partial L}{\partial k_{g,t}} = \beta^t \cdot \lambda_{h,t} \cdot R_{g,t} - \beta^t \cdot \mu_t = 0 \]

\[ \Leftrightarrow \lambda_{h,t} R_{g,t} = \mu_t \]

5. Capital supply to the market:

\[ \frac{\partial L}{\partial k_{m,t}} = \beta^t \cdot \lambda_{h,t} \cdot R_{m,t} - \beta^t \cdot \mu_t = 0 \]

\[ \Leftrightarrow \lambda_{h,t} R_{m,t} = \mu_t \]

6. Capital stock:

\[ \frac{\partial L}{\partial k_{t+1}} = -\beta^t \cdot \lambda_{h,t} + \beta^{t+1} \cdot (1 - \delta) + \beta^{t+1} \cdot \mu_{t+1} = 0 \]

\[ \Leftrightarrow \beta[(1 - \delta) + \mu_{t+1}] = \lambda_{h,t} \]

7.1.2 Government’s Characterized Solutions

For the government, I characterize solutions for the cost-minimization and utility-maximization problems covered in Section 2.2. I first turn to the cost-minimization problem where I construct the Lagrangian using Equations 4 and 5. The Lagrangian multiplier is \( \lambda_{h,t} \), and I take the partial derivative of the Lagrangian with respect to \( n_{g,t} \) and \( k_{g,t} \).

\[ L = w_{g,t} n_{g,t} + R_{g,t} k_{g,t} - \lambda_{g,t}(A_{g,t} n_{g,t}^{1-\alpha} k_{g,t}^\alpha) \]

1. Labor supply to the government:

\[ \frac{\partial L}{\partial n_{g,t}} = w_{g,t} - \lambda_{g,t}(1 - \alpha) A_{g,t} k_{g,t}^{\alpha} n_{g,t}^{-\alpha} = 0 \]

\[ \Leftrightarrow w_{g,t} = \lambda_{g,t}(1 - \alpha) A_{g,t} k_{g,t}^{\alpha} n_{g,t}^{-\alpha} \]

\[ \Leftrightarrow w_{g,t} = \lambda_{g,t}(1 - \alpha) \frac{y_{g,t}}{n_{g,t}} \]

2. Capital supply to the government:

\[ \frac{\partial L}{\partial k_{g,t}} = R_{g,t} - \lambda_{g,t} \alpha A_{g,t} n_{g,t}^{1-\alpha} k_{g,t}^{\alpha-1} = 0 \]
\[ \Leftrightarrow R_{g,t} = \lambda_{g,t} A_{g,t} n_{g,t}^{1-\alpha} k_{g,t}^\alpha \]

\[ \Leftrightarrow R_{g,t} = \lambda_{g,t} y_{g,t} k_{g,t} \]

Next, I characterize the solution to the public agent’s utility-maximization problem. Instead of setting up a Lagrangian, I substitute the constraint, Equation 7, into the utility function, Equation 6. I then take the partial derivative of the utility function with respect to public good provision \((g_{h,t})\).

\[ U(g_{h,t}) = \psi \ln g_{h,t} + (1 - \psi) \ln (y_{g,t} - g_{h,t}) \]

1. Public good provision:

\[ \frac{\partial U}{\partial g_{h,t}} = \psi \cdot \frac{1}{g_{h,t}} - (1 - \psi) \cdot \frac{1}{y_{g,t} - g_{h,t}} = 0 \]

\[ \Leftrightarrow \psi = \frac{1 - \psi}{y_{g,t} - g_{h,t}} \]

\[ \Leftrightarrow \frac{\psi}{1 - \psi} = \frac{g_{h,t}}{y_{g,t} - g_{h,t}} \]

7.1.3 Firm’s Characterized Solution

For firms, I characterize solutions for their profit-maximization problem found under Section 2.3 by taking the partial derivative of firm profit, Equation 9, with respect to capital \((k_{m,t})\), and labor \((n_{m,t})\). Note that \(y_{m,t} = A_{m,t} n_{m,t}^{1-\alpha} k_{m,t}^\alpha\)

\[ \Pi = A_{m,t} n_{m,t}^{1-\alpha} k_{m,t}^\alpha - R_{m,t} k_{m,t} - w_{m,t} n_{m,t} \]

1. Labor supply to the market:

\[ \frac{\partial \Pi}{\partial n_{m,t}} = (1 - \alpha) A_{m,t} k_{m,t}^\alpha n_{m,t}^{-\alpha} - w_{m,t} = 0 \]

\[ \Leftrightarrow w_{m,t} = (1 - \alpha) A_{m,t} k_{m,t}^\alpha n_{m,t}^{-\alpha} \]

\[ \Leftrightarrow w_{m,t} = (1 - \alpha) \frac{y_{m,t}}{n_{m,t}} \]

2. Capital supply to the market:

\[ \frac{\partial \Pi}{\partial k_{m,t}} = \alpha A_{m,t} n_{m,t}^{1-\alpha} k_{m,t}^{\alpha-1} - R_{m,t} = 0 \]
\[ R_{m,t} = \alpha A_{m,t} n_{m,t}^{1-\alpha} k_{m,t}^{\alpha-1} \]

\[ R_{m,t} = \alpha \frac{y_{m,t}}{k_{m,t}} \]

### 7.1.4 Further Constraints

In Subsections 7.1.1-7.1.3 I derived characterized solutions from household, government, and firm problems. To further discipline the model, I need conditions specified under Section 2.4, along with implied ones that should be made explicit.

1. Market production function:
   \[ y_{m,t} = A_{m,t} n_{m,t}^{1-\alpha} k_{m,t}^\alpha \]

2. Government production function:
   \[ y_{g,t} = A_{g,t} n_{g,t}^{1-\alpha} k_{g,t}^\alpha \]

3. Government consumption as a share of GDP:
   \[ \theta = \frac{y_{g,t}}{y_{m,t}} \]

   \[ \iff \quad y_{g,t} = \theta g_{h,t} \]

4. Total output:
   \[ y_{m,t} = I_t + c_t + y_{g,t} \]

5. Investment:
   \[ I_t = k_{t+1} - (1 - \delta) k_t \]

6. Total capital:
   \[ k_t = k_{g,t} + k_{m,t} \]

7. Market TFP:
   \[ A_{m,t} = (1 - \phi) \bar{A}_m + \phi A_{m,t-1} + \epsilon_{m,t} \]

8. Government TFP:
   \[ A_{g,t} = (1 - \phi) \bar{A}_g + \phi A_{g,t-1} + \epsilon_{g,t} \]
7.1.5 Endogenous Variables

The characterized solutions derived in Sections 7.1.1-7.1.4 add up to 19 equations that correspond to the following endogenous variables:

1. $c_t$: household consumption
2. $n_{m,t}$: household supply of and market demand for labor
3. $n_{g,t}$: household supply of and government demand for labor
4. $k_{m,t}$: household supply of and market demand for capital
5. $k_{g,t}$: household supply of and government demand for capital
6. $k_{t+1}$: capital stock
7. $w_{m,t}$: market labor wage
8. $w_{g,t}$: government labor wage
9. $R_{m,t}$: market capital rent
10. $R_{g,t}$: government capital rent
11. $y_{m,t}$: market production
12. $y_{g,t}$: government production
13. $g_{h,t}$: government provision and household consumption of public goods
14. $I_t$: total investment
15. $A_{m,t}$: market TFP
16. $A_{g,t}$: government TFP
17. $\lambda_{h,t}$: household Lagrangian multiplier (for household budget constraint)
18. $\lambda_{g,t}$: government Lagrangian multiplier (for government production)
19. $\mu_t$: household Lagrangian multiplier (for total capital)

7.2 Characterized Solutions: Tax Rates and Home Production

When considering the model developed in Section 3, only the household utility-maximization problem changed. Thus, the characterized solutions for the representative household will change, along with a number of model constraints that are updated to match the new division of consumption between the consumption of market- and home-produced goods.
7.2.1 Household’s Characterized Solution

I set up the Lagrangian using Equations 17-19 laid out in Section 3.2. I then substitute in Equation 20, home-production and -consumption. As previously defined, \( \lambda_{h,t} \) and \( \mu_t \) are Lagrangian multipliers. I then take the partial derivative of the Lagrangian with respect to each of the eight choice variables for households (\( c_{m,t}, n_{g,t}, n_{m,t}, n_{h,t}, k_{g,t}, k_{m,t}, k_{h,t} \) and \( k_{t+1} \)). To optimize household decisions, I equate each of the partial derivatives to zero.

\[
L = \sum_{t=0}^{\infty} \beta^t [\ln(\gamma(c_{m,t} + c_{h,t})\frac{1}{\beta} + (1 - \gamma)g_{h,t})^{1/\beta} + \nu \ln(1 - n_{g,t} - n_{m,t} - n_{h,t}) + \lambda_{h,t}((w_{g,t}n_{g,t} + w_{m,t}n_{m,t} + R_{g,t}k_{g,t} + R_{m,t}k_{m,t})(1 - \tau) - c_t - k_{t+1} + (1 - \delta)k_t + \pi_{f,t}) + \mu_t(k_t - k_{g,t} - k_{m,t} - k_{h,t})]
\]

Substituting in Equation 20,

\[
L = \sum_{t=0}^{\infty} \beta^t [\ln(\gamma(c_{m,t} + (\bar{A}_h n_{h,t}^{-\alpha} k_{h,t}^{\alpha})\sigma)\frac{1}{\beta} + (1 - \gamma)g_{h,t})^{1/\beta} + \nu \ln(1 - n_{g,t} - n_{m,t} - n_{h,t}) + \lambda_{h,t}((w_{g,t}n_{g,t} + n_{m,t} + R_{g,t}k_{g,t} + R_{m,t}k_{m,t})(1 - \tau) - c_t - k_{t+1} + (1 - \delta)k_t + \pi_{f,t}) + \mu_t(k_t - k_{g,t} - k_{m,t} - k_{h,t})]
\]

1. Consumption of market-produced goods:

\[
\frac{\partial L}{\partial c_{m,t}} = \frac{\beta^t \cdot \frac{1}{\sigma} \cdot \gamma(c_{m,t} + (\bar{A}_h n_{h,t}^{-\alpha} k_{h,t}^{\alpha})\sigma)\frac{1}{\beta} + (1 - \gamma)g_{h,t})^{1/\beta} - \beta^t \lambda_{h,t} = 0
\]

\[
\Leftrightarrow \lambda_{h,t} = \frac{\gamma(c_{m,t} + (\bar{A}_h n_{h,t}^{-\alpha} k_{h,t}^{\alpha})\sigma)\frac{1}{\beta} + (1 - \gamma)g_{h,t})^{1/\beta} \cdot c_{m,t}^{\rho-1}}{\gamma(c_{m,t} + (\bar{A}_h n_{h,t}^{-\alpha} k_{h,t}^{\alpha})\sigma)\frac{1}{\beta} + (1 - \gamma)g_{h,t})^{1/\beta} \cdot c_{m,t}^{\rho-1}}
\]

2. Labor supply to the market:

\[
\frac{\partial L}{\partial n_{m,t}} = -\beta^t \cdot \nu \cdot \frac{1}{1 - n_{g,t} - n_{m,t} - n_{h,t}} + \beta^t \cdot \lambda_{h,t} \cdot (1 - \tau)w_{m,t} = 0
\]

\[
\Leftrightarrow \lambda_{h,t}(1 - \tau)w_{m,t} = \frac{\nu}{1 - n_{g,t} - n_{m,t} - n_{h,t}}
\]

3. Labor supply to the government:

\[
\frac{\partial L}{\partial n_{g,t}} = -\beta^t \cdot \nu \cdot \frac{1}{1 - n_{g,t} - n_{m,t} - n_{h,t}} + \beta^t \cdot \lambda_{h,t} \cdot (1 - \tau)w_{g,t} = 0
\]

\[
\Leftrightarrow \lambda_{h,t}(1 - \tau)w_{g,t} = \frac{\nu}{1 - n_{g,t} - n_{m,t} - n_{h,t}}
\]
4. Labor supply to home-production:

\[
\frac{\partial L}{\partial n_{h,t}} = \frac{\beta^t \cdot \frac{1}{\sigma} \cdot \frac{\sigma}{\rho} \cdot \gamma(c_{m,t}^\rho + (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\rho) ^{\frac{\sigma}{\rho} - 1} \cdot \rho (1 - \alpha) (\bar{A}_h k_{h,t}^{\alpha})^\rho n_{h,t}^{\rho(1-\alpha)-1}}{\gamma(c_{m,t}^\sigma + (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\sigma) ^{\frac{\sigma}{\rho} + (1 - \gamma) g_{h,t}^\rho}} - \frac{\beta^t \nu}{1 - n_{g,t} - n_{m,t} - n_{h,t}} = 0
\]

\[\iff \nu = \frac{\gamma(c_{m,t}^\rho + (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\rho) ^{\frac{\sigma}{\rho} - 1} \cdot \rho (1 - \alpha) (\bar{A}_h k_{h,t}^{\alpha})^\rho n_{h,t}^{\rho(1-\alpha)-1}}{\gamma(c_{m,t}^\sigma + (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\sigma) ^{\frac{\sigma}{\rho} + (1 - \gamma) g_{h,t}^\rho}} \] 

5. Capital supply to the market:

\[
\frac{\partial L}{\partial k_{m,t}} = \beta^t \lambda_{h,t}(1 - \tau) R_{m,t} - \beta^t \mu_t = 0
\]

\[\iff \mu_t = \lambda_{h,t}(1 - \tau) R_{m,t} \]

6. Capital supply to the government:

\[
\frac{\partial L}{\partial k_{g,t}} = \beta^t \lambda_{h,t}(1 - \tau) R_{g,t} - \beta^t \mu_t = 0
\]

\[\iff \mu_t = \lambda_{h,t}(1 - \tau) R_{g,t} \]

7. Capital supply to home-production:

\[
\frac{\partial L}{\partial k_{h,t}} = \frac{\beta^t \cdot \frac{1}{\sigma} \cdot \frac{\sigma}{\rho} \cdot \gamma(c_{m,t}^\rho + (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\rho) ^{\frac{\sigma}{\rho} - 1} \cdot \rho \alpha (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\rho n_{h,t}^{\rho(1-\alpha)-1}}{\gamma(c_{m,t}^\sigma + (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\sigma) ^{\frac{\sigma}{\rho} + (1 - \gamma) g_{h,t}^\rho}} - \beta^t \mu_t = 0
\]

\[\iff \mu_t = \frac{\gamma(c_{m,t}^\rho + (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\rho) ^{\frac{\sigma}{\rho} - 1} \cdot \alpha (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\rho n_{h,t}^{\rho(1-\alpha)-1}}{\gamma(c_{m,t}^\sigma + (\bar{A}_h n_{h,t}^{1-\alpha} n_{k,h,t}^{\alpha})^\sigma) ^{\frac{\sigma}{\rho} + (1 - \gamma) g_{h,t}^\rho}} \]

8. Capital stock:

\[
\frac{\partial L}{\partial k_{t+1}} = -\beta^t \lambda_{h,t} + \beta^{t+1} \lambda_{h,t+1}(1 - \delta) + \beta^{t+1} \mu_{t+1} = 0
\]

\[\iff \lambda_{h,t} = \beta[\lambda_{h,t+1}(1 - \delta) + \mu_{t+1}] \]
7.2.2 Further Constraints

Due to changes made to the household’s utility function, the following constraints are updated or added relative to the constraints defined under Appendix 7.1.4:

1. Home-production and -consumption function:

\[ c_{h,t} = A_h n_{h,t}^{1-\alpha} k_{h,t}^\alpha \]

2. Total (market) output:\(^3\)

\[ y_{m,t} = I_t + c_{m,t} + y_{g,t} \]

7.2.3 Characterized Solutions and Endogenous Variables

With the changes above, there are 22 equations that correspond to 22 endogenous variables for which the model is solving. The 22 equations include all 19 equations from Appendix 7.1 with the following changes:

- The 6 equations from the household’s characterized solution in Appendix 7.1.1 are replaced with the 8 equations from the household’s characterized solution in Appendix 7.2.1;
- The Total output equation from Appendix 7.1.4 is updated to the Total (market) output equation under Appendix 7.2.2;
- The function for home-production and consumption of home-produced goods \(c_{h,t}\) is added.

This leads to a net increase in the number of equations of 3. As for the endogenous variables the model is solving for, it also increases by 3. The 22 endogenous variables that correspond to the 22 equations are derived from the 19 endogenous variables listed under Appendix 7.1.5, with the following changes:

- Household consumption \(c_t\) is now differentiated into household consumption of market-produced goods \(c_{m,t}\) and home-produced goods \(c_{h,t}\);
- Households now provide both labor \(n_{h,t}\) and capital \(k_{h,t}\) to home-production.

These modifications result in the loss of \(c_t\) and the addition of \(c_{m,t}, c_{h,t}, n_{h,t},\) and \(k_{h,t}\), leading to a net increase of 3 endogenous variables.

\(^3\)Compare to equation 4 under Appendix 7.1.4 where unreported work and output did not exist.
References


