# Problem Set 1
## Solutions

## Chapter 3

3.1 Any game of the following form, where $a > b > c > d$ and $w > x > y > z$.

| | | WIFE: Football | WIFE: Opera |
|---|---|---|---|
| HUSBAND | Football | *a,x* | *c,y* |
| | Opera | *d,z* | *b,w* |

3.2 Any game of the following form, where $a > b > c > d$ and $w > x > y > z$.

| | | WIFE: Football | WIFE: Opera |
|---|---|---|---|
| HUSBAND | Football | *a,y* | *b,x* |
| | Opera | *d,z* | *c,w* |

3.8 Strategies for both players are 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, and so on. In other words, any number greater than 2000 that is divisible by 1000.

3.9 We restrict attention to strategies up to 7000 as beyond that both players will have negative utility:

| | | PLAYER 2: 2000 | 3000 | 4000 | 5000 | 6000 | 7000 |
|---|---|---|---|---|---|---|---|
| PLAYER 1 | 2000 | 4000,0 | 0,5000 | 0,5000 | 0,5000 | 0,5000 | 0,5000 |
| | 3000 | 4000,0 | 3000,0 | 0,4000 | 0,4000 | 0,4000 | 0,4000 |
| | 4000 | 4000,0 | 3000,0 | 2000,0 | 0,3000 | 0,3000 | 0,3000 |
| | 5000 | 4000,0 | 3000,0 | 2000,0 | 1000,0 | 0,2000 | 0,2000 |
| | 6000 | 4000,0 | 3000,0 | 2000,0 | 1000,0 | 0,0 | 0,1000 |
| | 7000 | 4000,0 | 3000,0 | 2000,0 | 1000,0 | 0,0 | -1000,0 |

3.10

| PLAYER 1 \ PLAYER 2 | 2000 | 3000 | 4000 | 5000 | 6000 | 7000 |
|---|---|---|---|---|---|---|
| 2000 | 2000,2500 | 0,5000 | 0,5000 | 0,5000 | 0,5000 | 0,5000 |
| 3000 | 4000,0 | 1500,2000 | 0,4000 | 0,4000 | 0,4000 | 0,4000 |
| 4000 | 4000,0 | 3000,0 | 1000,1500 | 0,3000 | 0,3000 | 0,3000 |
| 5000 | 4000,0 | 3000,0 | 2000,0 | 500,1000 | 0,2000 | 0,2000 |
| 6000 | 4000,0 | 3000,0 | 2000,0 | 1000,0 | 0,500 | 0,1000 |
| 7000 | 4000,0 | 3000,0 | 2000,0 | 1000,0 | 0,0 | -500,0 |

3.11

| FIRM 1 \ FIRM 2 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 5/2,5/2 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| 2 | 0,5 | 4,4 | 8,0 | 8,0 | 8,0 | 8,0 |
| 3 | 0,5 | 0,8 | 9/2,9/2 | 9,0 | 9,0 | 9,0 |
| 4 | 0,5 | 0,8 | 0,9 | 4,4 | 8,0 | 8,0 |
| 5 | 0,5 | 0,8 | 0,9 | 0,8 | 5/2,5/2 | 5,0 |
| 6 | 0,5 | 0,8 | 0,9 | 0,8 | 0,5 | 0,0 |

3.12 If the rival's price is \$4 or less, each of the two strategies yields a profit of 0. If the rival's price is \$5, then the latter has a zero profit but the former has a strictly positive profit of 5/2. Finally, if the rival's price is \$6, then the former strategy yields a profit of 5 while the latter yields a profit of 0.

3.13 If the rival's price is \$3 or less, then a price of \$4 and a price of \$5 yields a profit of 0. If the rivals' price is \$4, then the latter has a zero profit but the former has a strictly positive profit of 4. If the rival's price is \$5, then the latter has a profit of 5/2 but the former has a strictly higher profit of 8. Finally, if the rival's price is \$6, then the former strategy yields a profit of 8 while the latter yields a profit of 5. Hence the price of \$4 (weakly) dominates a price of \$5. Similarly, a price of \$3 (weakly) dominates a price of \$4.

3.14 No, there is no dominant strategy. Consider any price between $p = 2$ and $p = 6$, and suppose that firm 2 happens to match that price. In that case, firm 1 would have been better off pricing at $p - 1$ (work the profit figures out for yourself). On the other hand $p = 1$ is not a dominant strategy because if the rival firm prices at 6, then firm 1 would have been better off pricing at 3 instead. Clearly a price of 0 is not a dominant strategy either.

## Chapter 5

5.7 Any price below \$6 captures the market. By the definition of a monopoly price, \$3 yields a higher profit than any of the alternatives. Similarly, in the next two cases, any price below either \$5 or \$4 captures the market. By the definition of a monopoly price, \$3 yields a higher profit than any of the alternatives. So the (identical) best response to 4, 5, or 6 dollars is the (monopoly) price of \$3.

5.8 From the normal form, we can see that the best response to \$3 is to price at \$2 since the payoff to \$2 is 8 which is higher than any other price:

| | | FIRM 2 | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| FIRM 1 | 1 | 5/2,5/2 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| | 2 | 0,5 | 4,4 | 8,0 | 8,0 | 8,0 | 8,0 |
| | 3 | 0,5 | 0,8 | 9/2,9/2 | 9,0 | 9,0 | 9,0 |
| | 4 | 0,5 | 0,8 | 0,9 | 4,4 | 8,0 | 8,0 |
| | 5 | 0,5 | 0,8 | 0,9 | 0,8 | 5/2,5/2 | 5,0 |
| | 6 | 0,5 | 0,8 | 0,9 | 0,8 | 0,5 | 0,0 |

5.9 The best response function, $b(p)$, (identical for the two firms) is:

$$b(1) = 1,\ b(2) = 1,\ b(3) = 2,\ b(4) = b(5) = b(6) = 3,$$

Since Nash equilibrium is given by a price $p^*$ such that $p^* = b(p^*)$, it follows that the only NE of the game is $p^* = 1$.

5.10 Call the price that the rival firm charges $p$ (where $p \geq p_m$). Any price below $p$ dollars captures the market. By the definition of a monopoly price, $p_m$ dollars yields a higher profit than any of the alternatives.

5.11 The profits at $p$ are $\frac{D(p)}{2}p$ whereas the profits at $p-1$ are $D(p-1)(p-1)$. $D(p-1) > D(p)$ (since demand is downward sloping) while $p - 1 \geq p/2$ since $p \geq 2$.

5.12 So long as the rival charges a price $p \geq 2$, it will be a best response to undercut, and the rival's best response will be to further undercut. However, when $p = 1$, then the best response must be to price at \$1 (since at \$0, profits are zero). Thus, a price of \$1 for each firm is a Nash equilibrium.

5.13 There is no change since the benefit to undercutting the rivals remains the same, since the entire market is captured by the lowest price. But the profits from not undercutting are now $\frac{D(p)}{N}p$. Hence, each firm would undercut if $\frac{D(p)}{N}p < D(p-1)(p-1)$. As before, $D(p-1) > D(p)$, and $p - 1 \geq p/N$ so long as $p \geq \frac{N}{N-1}$. Thus, the Nash equilibrium will still be for every firm to price at \$1.

## Additional Exercises

1. (a) {D} x {R} (b) {U,D} x {L,R} (c) {X} x {C}

   (d) {U,M,D} x {L,R} (e) {U,M} x {L,R} (f) {U,M} x {L,C,R}

2. For "give in" to be a rational strategy choice, it must be that $x \leq 0$. ($x < 0$ is also fine). Otherwise, Employee will definitely play "Be tough," as it would be their dominant strategy. Against "Be tough," You are better off playing "Hire attorney."

3. The IEDS solution to the game is {(7:00, 6:00, 6:00)}.

4. (a) The simplest model uses dollar gain/loss as the payoff for both players:

| | | POLICEMAN: Mayflower | POLICEMAN: North St |
|---|---|---|---|
| YOU | Mayflower | -50,<u>50</u> | <u>0</u>,0 |
| YOU | North St | <u>0</u>,0 | -50,<u>50</u> |

(b) Neither player has a dominant strategy, which implies that neither has a dominated strategy. Furthermore, there is no cell in the matrix where both players are playing a best response to the other player's strategy. Therefore, no Nash equilibrium exists in the game. (More precisely, no Nash equilibrium *in pure strategies* exists; we will see shortly that a Nash equilibrium of another sort does exist.)

(c) With this additional strategy available to You, the game becomes

| | | POLICEMAN: Mayflower | POLICEMAN: North St |
|---|---|---|---|
| YOU | Mayflower (fast) | -50,<u>50</u> | <u>0</u>,0 |
| YOU | North St (fast) | <u>0</u>,0 | -50,<u>50</u> |
| YOU | Drive Slow | $x$, <u>0</u> | $x$, <u>0</u> |

where $x < 0$. In fact, this does not change the answer to part (b) at all. Because $x < 0$, your best response to Policeman playing Mayflower is still to drive fast on North St, and your best response to Policeman playing North St. is to drive fast on Mayflower. Drive Slow is never a best response for You. There is still no Dominant Strategy solution, IEDS solution, or (pure strategy) Nash equilibrium.

[Note: If $x \leq -50$, then "Drive Slow" is a dominated strategy. There is still no IEDS solution to the game, but "Drive Slow" can be ruled out. This point need not be made for full credit on the question.]