Economics 379
Game Theory

Department of Economics
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Problem Set 3

Due Wednesday, March 9

From Dutta

CH 8: 16 – 20. [Important note on 8.20: Change the payoffs to the (*s*, *n*, *n*) outcome (i.e. Player 1 plays *s*, Player 2 plays *n*, and Player 3 plays *n*) to (-1, 1, 1). (This is the top right corner of the game matrix as drawn in the textbook.) Note – and use! – the fact that this makes the game symmetric.]

Additional Exercises

1. Find all the Nash equilibria (including both pure and mixed strategies) of the following games.

		PLAYER 2	
		L	R
PLAYER 1	U	2,4	0,0
	D	1,6	3,7

(a)

		PLAYER 2	
		L	R
PLAYER 1	U	2,2	0,3
	D	3,0	1,1

(b)

		PLAYER 2	
		L	R
PLAYER 1	U	2,2	3,1
	D	3,1	2,2

(c)

		PLAYER 2		
		L	C	R
PLAYER 1	U	8,3	3,5	6,3
	M	3,3	5,5	4,8
	D	5,2	3,7	4,9

(d)

		PLAYER 2		
		L	C	R
PLAYER 1	U	8,1	0,2	4,3
	M	3,1	4,4	0,0
	D	5,0	3,3	1,4

(e)

2. For (a), (b), and (c) of question 1 above, provide a graphical solution that shows every Nash equilibrium. In other words, graph each player's best response function to the other players mixed strategy, and indicate on your graph the location of every Nash equilibrium.

3. Consider the following social problem. A pedestrian is hit by a car and lies injured on the road. There are n people near the accident. The injured pedestrian requires immediate medical attention, which will be forthcoming if at least one of the n people calls for help. Simultaneously and independently, each of the n bystanders decides whether to call for help. Each bystander obtains v units of utility if someone (anyone) calls for help. Those who call for help pay a personal cost of c. That is, if person i calls for help, then he obtains the payoff $v - c$; if person i does not call but at least one other person calls, then person i gets v; finally, if non of the n people calls for help, then person i obtains zero. Assume $v > c$.

   (a) Find the symmetric Nash equilibrium of this N-player normal-form game. (Hint: The equilibrium is in mixed strategies.)

   (b) Compute the probability that at least one person calls for help in equilibrium. (This is the probability that the injured pedestrian gets medical attention.) Note how this depends on N. Is this a perverse or intuitive result? (Hint: If the probability of event A happening is p_A, then the probability of A not happening is $1–p_A$.)

4. Imagine that a prosecutor (P) has accused a defendant (D) of committing a crime. Suppose that the trial involves evidence production by both parties and that, by producing evidence, a litigant increases the probability of winning the trial. Specifically, suppose that the probability that the defendant wins is given by $e_D/(e_D + e_P)$, where e_D is the expenditure on evidence production by the defendant and e_P is the expenditure on evidence production by the prosecutor. Assume that $e_D, e_P \geq 0$. The defendant must pay 8 if he is found guilty, whereas he pays 0 if he is found innocent. The prosecutor receives 8 if she wins and 0 if she loses the case.

   (a) Represent this game in normal form (i.e. list the players and the strategies and payoffs for each player).

   (b) Write the first-order condition and derive the best-response function for each player. (You may find it helpful to remember the Quotient Rule: the derivative of (u/v) where u and v are differentiable functions is $\left[(v \times u') - (u \times v')\right] / v^2$. Or just use the Chain Rule; it always works in situations like this.)

   (c) Find the (pure strategy) Nash equilibrium of this game. What is the probability that the defendant wins in equilibrium?

   (d) A strategy profile $s = (s_1, \ldots, s_N)$ is said to be **Pareto efficient** if there is no other strategy profile s' such that $u_i(s') \geq u_i(s)$ for every player i and $u_i(s') > u_i(s)$ for some player j (that is, no other set of strategies that every player would find at least as good as s and at least one player would find better than s). Is the outcome of this game Pareto efficient? Explain.