Problem Set 3
Solutions

From Dutta

8.16

a) Expected Payoff = $\frac{2}{3}(.2) + \frac{1}{3}(.7) = 0.36667$

b) Expected Payoff = $\frac{2}{3}(.8) + \frac{1}{3}(.3) = 0.63333$

c) 0.36667

8.17

a) Expected Payoff = $\frac{4}{10}(.2) + \frac{6}{10}(.7) = 0.5$

b) Expected Payoff = $\frac{4}{10}(.8) + \frac{6}{10}(.3) = 0.5$

c) 0.5

8.18

1\2	*tough*	*concede*
tough	−1, −1	10, 0
concede	0, 10	5, 5

Suppose player 2 plays t with probability q. Then, player 1's payoffs to playing t are $-q + 10(1-q)$ while from playing c are $5(1-q)$. Player 1 is indifferent - and will therefore be willing to play a mixed strategy himself - if $-q + 10(1-q) = 5(1-q)$, i.e., $q = \frac{5}{6}$. By the same logic, player 2 is indifferent between t and c only if player 1 plays t with probability $p = \frac{5}{6}$. Hence, $p = \frac{5}{6}$, $q = \frac{5}{6}$ is the only mixed strategy Nash equilibrium of Chicken.

8.19

No. The IEDS procedure leads to one outcome. From section 8.3.1, we know that the IEDS outcome with mixed strategies is the same as the outcome with pure strategies, if there is one IEDS outcome with pure strategies.

8.20

Note first that the game is symmetric. (Take a minute to convince yourself of this in this 3-player game. If all three players play *x*, where *x* = *s* or *n*, then they all get the same payoff. Being the only player to play *s* while the other players play *n* results in a payoff of -2, versus -1 for the other two players, regardless of which player plays *n*. Being the only player to play *n* while the other players play *s* results in a payoff of -1, while the other two receive a payoff of 1.) This implies that the game can be analyzed in full generality by considering only one scenario of players.

(a) No. For one player to mix while the other two play pure strategies, the mixing player must be indifferent between her two choices given the pure strategies played by the other two players. This is never the case for any player. For example, note that Player 1 is never indifferent between *s* and *n* for any two <u>pure</u> strategies played by players 2 and 3.

(b) No. Let Player 3 be the player playing a pure strategy. First suppose that she plays *s*. Let *q* be the probability that Player 2 plays *s*. Then Player 1 will mix when
$EU_1(s) = EU_1(n) \Rightarrow q - (1 - q) = -2q + 1 - q \Rightarrow q = 2/5$. By symmetry, Player 2 will also only mix when Player 1 plays *s* with probability 2/5. Now we ask, Is it in fact a BR for Player 3 to be playing *s* in the first place? The answer is no. $EU_3(s) = 4/25 - 6/25 - 6/25 - 9/25 = -17/25$, while
$EU_3(n) = -8/25 + 6/25 + 6/25 + 0 = 4/25$. So this cannot be a Nash equilibrium. Now suppose that Player 3 plays *n*. In this case, Players 1 and 2 will not mix, as *n* is a dominant strategy for each player. Therefore, it cannot be the case that Players 1 and 2 mix while Player 3 plays a pure strategy. By symmetry, it cannot be the case that any two players mix while the other player plays a pure strategy.

Additional Problems

1. (a) [U,L], [D,R], [(1/5,4/5),(3/4, 1/4)]

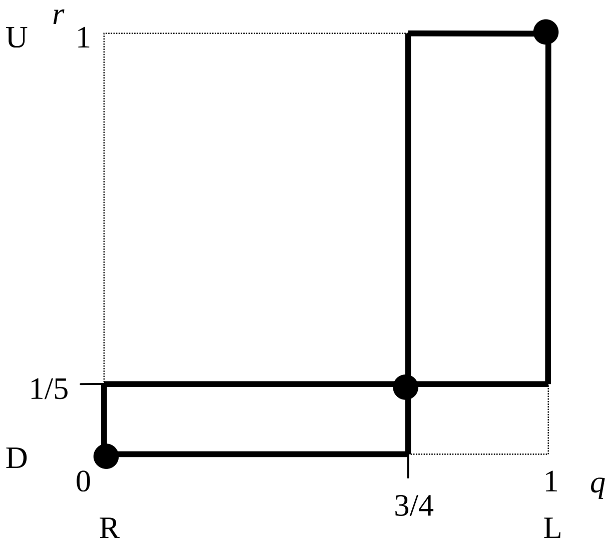
   (b) (D,R)

   (c) There are no pure strategy Nash equilibria. s1 = (1/2, 1/2) and s2 = (1/2, 1/2).

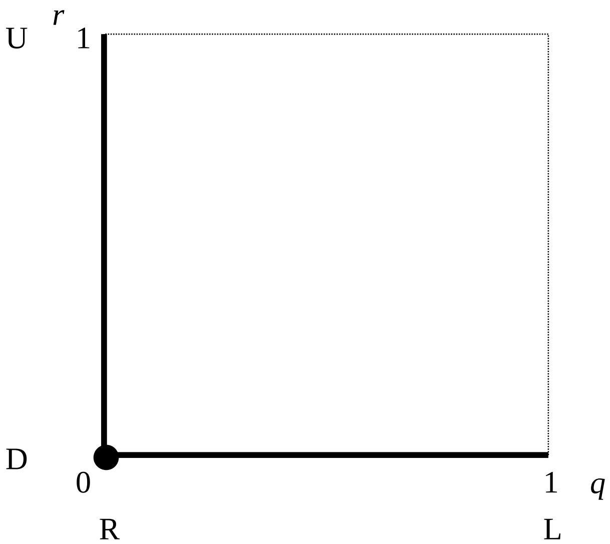
   (d) It is easy to see that C dominates L, and that (2/3, 1/3, 0) dominates D. Thus, player 1 will never play D, and player 2 will never play L. We need to find probabilities over U and M such that player 2 is indifferent between C and R. This requires $5p + 5 - 5p = 3p + 8 - 8p$ or $p = 3/5$. Thus, s1 = (3/5, 2/5, 0). We must also find probabilities over C and R such that player 1 is indifferent between U and M. This requires $3q + 6 - 6q = 5q + 4 - q$ or $q = 1/2$. Thus, s2 = (0, 1/2, 1/2).

   (e) Note that C dominates L. So player 2 chooses probabilities over C and R such that player 1 is indifferent between at least two strategies. Let q denote the probability with which C is played. Notice that the q which makes player 1 indifferent between any two strategies makes him indifferent between all three strategies. To see this note that $q = 1/2$ solves $4 - 4q = 4q = 3q + 1 - q$. Thus, s2 = (0, 1/2, 1/2). It remains to find probabilities such that player 2 is indifferent between playing C and R. Here p denotes the probability with which U is played and r denotes the probability with which M is played. Indifference between C and R requires $2p+4r +3(1 - p - r) = 3p+4(1 - p - r)$. This implies $r = 1/5$. Thus, s1 = (x, 1/5, y), where $x, y \geq 0$ and $x + y = 4/5$.

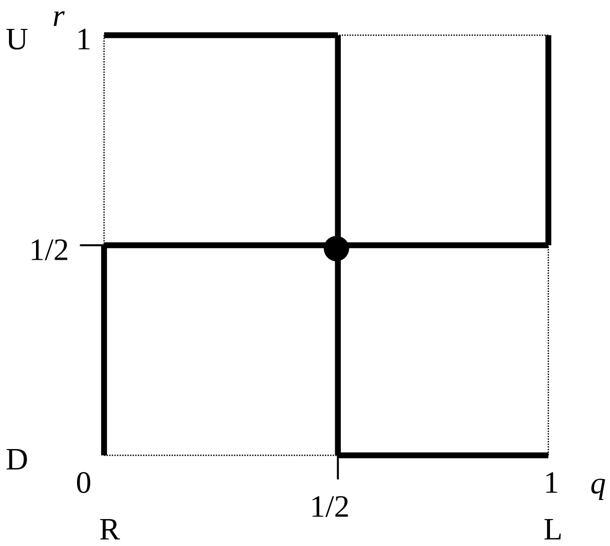
2. (a) For this and (b) and (c), let r be the probabililty that P1 plays U and q be the probability that P2 plays L.



(b)



(c)



3. (a) The symmetric mixed strategy Nash equilibrium requires that each player call with the same probability, and that each player be indifferent between calling and not calling. This implies that $(1 - p^{n-1})v = v - c$ or $p = (c/v)^{1/(n-1)}$.

(b) The probability that at least one player calls in equilibrium is $1 - p^n = 1 - (c/v)^{n/(n-1)}$. Note that this decreases as the number of bystanders n goes up.

4.

(a) The normal form is given by $N = \{P, D\}$, $e_i \in [0, \infty)$, $u_P(e_P, e_D) = 8e_P/(e_P + e_D) - e_P$, and $u_D(e_P, e_D) = 8e_D/(e_P + e_D) - e_D$.

(b) The prosecutor solves $\max_{e_P} 8e_P/(e_P + e_D) - e_P$. The first order condition is $8/(e_P + e_D) - 8e_P/(e_P + e_D)^2 = 1$. This implies $8(e_P + e_D) - 8e_P = (e_P + e_D)^2$, or $8e_D = (e_P + e_D)^2$. Taking the square root of both sides yields $2\sqrt{2e_D} = e_P + e_D$. Rearranging, we find $e_P^*(e_D) = 2\sqrt{2e_D} - e_D$.

You may conclude from symmetry that $e_D^*(e_P) = 2\sqrt{2e_P} - e_P$. Alternately, the defendant solves $\max_{e_D} (-8)\left(e_P/(e_P + e_D)\right) - e_D$. The first order condition is $(-8)\left(-e_P/(e_P + e_D)^2\right) = 1$. (Remember the old Quotient Rule: the derivative of (u/v) where u and v are differentiable functions is $\left[(v \times u') - (u \times v')\right]/v^2$). This implies that $8e_P = (e_D + e_P)^2$. Taking the square root of both sides and rearranging yields $e_D^*(e_P) = 2\sqrt{2e_P} - e_P$.

(c) Observe that $e_D^*(e_P)$ and $e_P^*(e_D)$ are symmetrical. This implies that $e_D^* = e_P^* = e^*$. Rewrite either best-response function as $e^* = 2\sqrt{2e^*} - e^*$. This implies that $e^* = \sqrt{2e^*}$. Taking the square root of both sides yields $\left(e^*\right)^2 = 2e^* \Rightarrow e^* = 2$. So $e_D^* = e_P^* = 2$. Alternately, substitute $e_P^*(e_D)$

into $e_D^*(e_P)$, yielding $e_D^* = 2\sqrt{2\left(2\sqrt{2e_D^*} - e_D^*\right)} - \left(2\sqrt{2e_D^*} - e_D^*\right)$. Solving this for e_D^* yields $e_D^* = 2$. Solving $e_P^*(e_D)$ for $e_D = 2$ yields $e_P^* = 2$.

(d) The outcome of the game is not Pareto efficient. If both spend 1 instead of 2, the probability of victory is unchanged, while the cost decreases by one. Both players will have an increase in expected utility of 1.