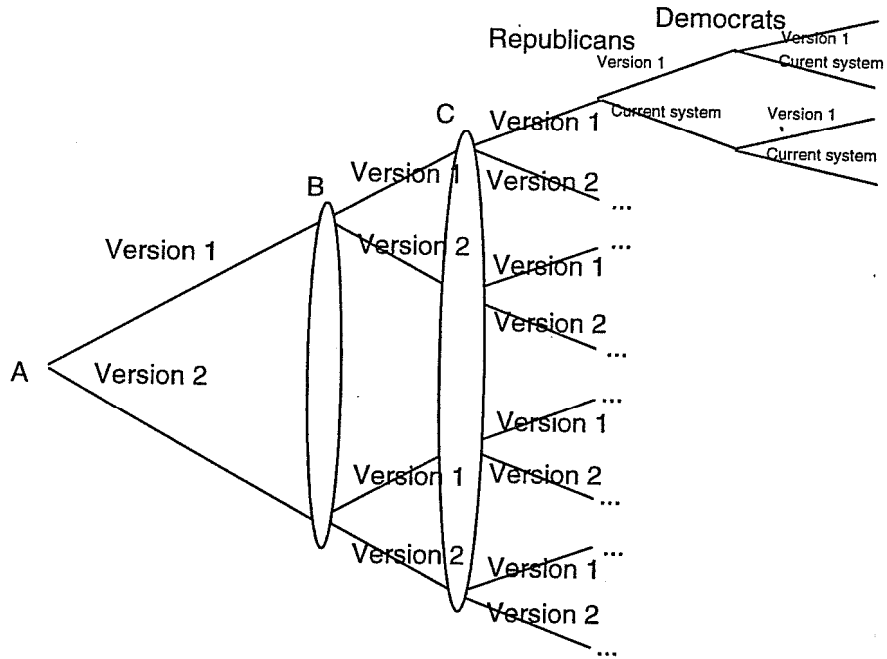


Problem Set 4 Solutions

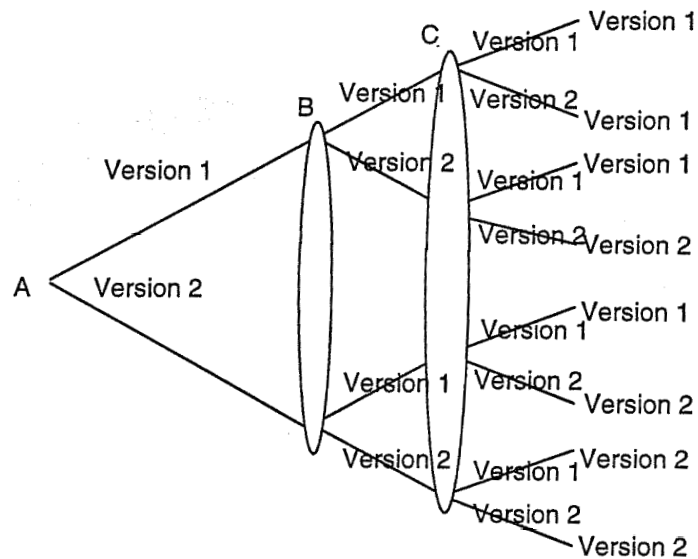
From Dutta

13.10



13.14

The reduced form game is



The strategic form of the reduced game is

A\B	Version 1	Version 2	A\B	Version 1	Version 2
Version 1	Version 1	Version 1	Version 1	Version 1	Version 2
Version 2	Version 1	Version 2	Version 2	Version 2	Version 2

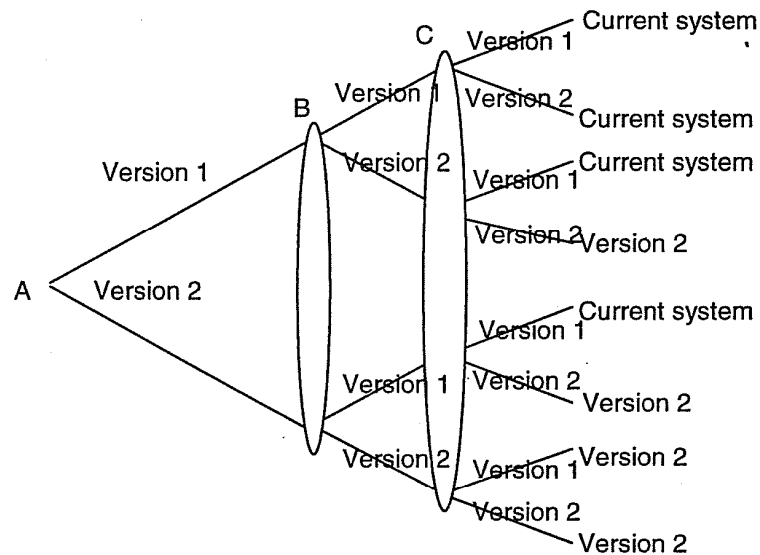
C votes in favor of Version 1

C votes in favor of Version 2

Note that Version 1 is dominated by Version 2 for A and B; Version 2 is dominated by Version 1 for C. So the subgame perfect equilibrium of this reduced game is (1,1,2) when no dominated strategies are allowed.

13.15

When there is a Democratic majority in the Senate, current system will pass if Version 1 is recommended; Version 2 if Version 2 is recommended. So the reduced voting game is



The strategic form of the reduced game is

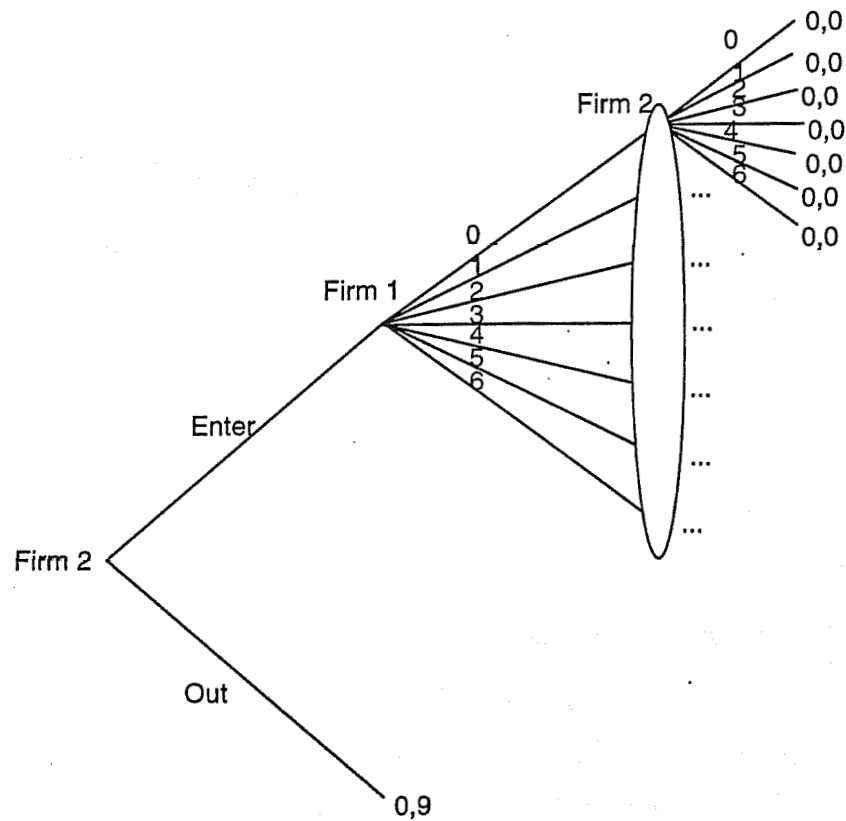
A\B	Version 1	Version 2	A\B	Version 1	Version 2
Version 1	Current system	Current system	Version 1	Current system	Version 2
Version 2	Current system	Version 2	Version 2	Version 2	Version 2

C votes in favor of Version 1

C votes in favor of Version 2

Note the difference in the outcome matrix from before. Now Version 2 dominates Version 1 for all players. So the subgame perfect equilibrium is (2,2,2) when no dominated strategies are allowed.

13.16



There are 2 subgames: the whole game and the post-entry game.

13.17

Firm 1's strategies specify whether or not to enter and the price that is going to be posted upon entry. Firm 2's strategies include only one component - how to price in the event of entry. A subgame perfect equilibrium requires that, first, the price competition to be self-sustaining and, second, that this be true of all nodes.

13.18

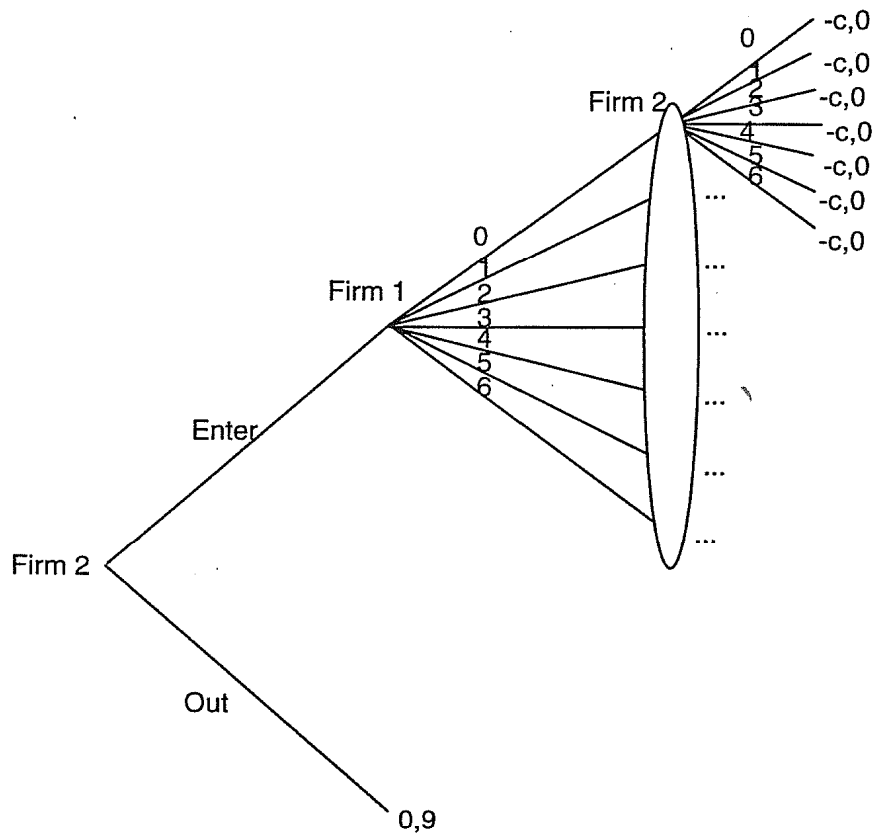
In a subgame perfect equilibrium, the market price must be 1 if firm 2 enters the market because (1,1) is the only Nash equilibrium in the post-entry game.

13.19

Upon entry, prices turn out to be low, Firm 2 will enter if the profits to this unique Nash equilibrium exceed the costs of entering this market.

13.20

The extensive form of the game is



Since the entry is a sunk cost, it does not affect any decisions made by the firms. So $(1, 1)$ is still the only Nash equilibrium in the post entry game. So when $c \leq 2.5$, firm 2 enters the market, the subgame perfect equilibrium is (Enter and 1, 1). When $c > 2.5$, the subgame perfect equilibrium is (Out and 1, 1).

14.11 The two stage-game (PS) NE are (Up, Left East) and (Down, Right, West),

14.12 Since (U,L,E) and (D,R,W) are NE in the stage game, it is credible to specify that they will be played in the last stage.

In the first stage, let us check each player's deviation opportunities – from the prescribed strategies:

Player 1: By playing D – against (L,W) – he gets 0 followed by 0; by sticking to the strategy and playing U in the first stage, he gets 0 followed by 1. Player 1 will stick to the strategy.

Player 2: By playing R – against (U,W) – he gets 0 followed by 0; by sticking to the strategy and playing L in the first stage, he gets 0 followed by 1. Player 2 will stick to the strategy.

Player 3: By playing E – against (U,L) – he gets 1 followed by 0; by sticking to the strategy and playing W in the first stage, he gets -1 followed by 1. Player 3 does better by deviating and will therefore not stick to the strategy.

The strategy prescribed is NOT a SPE.

14.13 There are four SPE based on stage-game NE. We go on to look for other SPE.

In the second stage, we are confined to use a stage-game NE. So we have players play (U,L,E) if they conform in the first stage; (D,R,W) if someone deviates. Note (U,L,E) brings the player payoffs (1,1,1); (D,R,W) brings (0,0,0). It means we can support in the first period action profiles which do not give any individual player chances to gain more than 1 in the first period.

In addition to (U,L,W) followed by (U,L,E), two other PS equilibrium paths can be supported: (U,R,E) followed by (U,L,E); and (D,L,E) followed by (U,L,E). The punishment part of the strategies are the same as that in Exercise 14.12.

Additional Exercises

- (a) The subgame perfect equilibria are (WY, AC) and (ZX, BC). The Nash equilibria are (WY, AC), (ZX, BC), (WY, AD), (ZY, BC), and (WX, BD).
 - (b) The subgame perfect equilibria are (UE, BD) and (DE, BC). The Nash equilibria are (UE, BD), (DE, BC), (UF, BD), and (DE, AC).
- (U, L) can be supported as follows. If player 2 defects ((U,M) is played) in the first period, then the players coordinate on (C, R) in the second period. If player 1 defects ((C, L) is played) in the first period, then the players play (D, M) in the second period. Otherwise, the players play (D, R) in the second period.