Chain-Type Data and Macro Model Properties:
the DRI/McGraw-Hill Experience

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ABSTRACT

In January 1996, the Bureau of Economic Analysis switched from fixed-weighted GDP to chain-type GDP as its featured measure of real output, because fixed weights are appropriate only when the relative price structure of the economy does not change over time. This paper shows that the better microeconomic properties of the chain-type measures also help the simulation properties of a macroeconomic forecasting model. In particular, Laspeyres-based models often violate the assumption that marginal revenue equals marginal cost. This can produce unrealistic income and multiplier responses, which are eliminated by using chain-type output.

In early 1996, the Bureau of Economic Analysis (BEA) shifted its emphasis from fixed-weighted measures of real output and prices to chain-type annual-weighted measures. At the time, the new measures were panned in the popular press as adding pointless complexity to the GDP calculation and undue pessimism about recent economic performance (Cooper and Bernstein, 1995; Roach, 1995; Spiers, 1995).

While it is true that the new measures are mathematically more complex than the old, this paper shows that a model constructed using chain-type output provides superior simulation properties to a model constructed using the old fixed-weighted output measures. Using fixed-weighted output introduces distortions serious enough to make the results of certain policy experiments unreliable.

The first section of the paper lays out the basic mathematics of fixed-weighted and chain-type data. The second section describes how to create chain-type data for aggregates not published by the BEA, and a useful approximation to chain-type aggregation. These sections are fairly technical, and will be of interest mainly to readers actually constructing economic models.

The next two sections contain the main results of the paper: the comparative impacts of fixed and chain weights on the simulation properties of macroeconomic models. Section 3 shows that a demand shock to a component of GDP with a price deflator significantly different from that for overall GDP will produce unrealistic income responses in a model based on fixed-weighted output, but not in a chain-based model. The fourth section demonstrates that the use of fixed weights also distorts the multiplier properties of an economic model. It also shows that the traditional multiplier is somewhat misleading, and that in calculating the multiplier, real changes in output and autonomous demand should by multiplied by the corresponding price indexes.

Section 5 suggests how the BEA could improve how it presents the aggregate inventory change data. The final section offers concluding remarks.

1. THE MATHEMATICS OF CHAIN-TYPE DATA

A. Annual Data

This part of Section 1 outlines the mathematical formulas used to create annual Fisher ideal (chain-type) aggregates from annual detail on output and prices. Much of this material is drawn from articles published in the Survey of Current Business (Young, 1992; Young, 1993). Summaries of DRI/McGraw-Hill’s handling of this data can be found in Lasky (1995) and Lasky (1996).

Assume that, at the finest level of detail, there are many goods and services for which the Bureau of Economic Analysis (BEA) calculates output and price indexes. Let Xj_t be nominal demand for item j in period t. (Inventory change can be positive or negative.) Let PXj_t be the price index for item j in period t. Then XRj_t, defined as Xj_t/PXj_t, is the real demand for item j in period t. If prices are indexed to 1992=1.0, i.e., PXj_{92} =1, then the units of XRj_t are 1992 dollars, and $XRj_{92} = Xj_{92}$. At this finest level of detail, Xj, PXj, and XRj are identical in the Laspeyres (fixed-weight) and Fisher ideal (chain-type) systems of aggregation. Chaining and fixed-weighting are simply different methods of aggregating the XRj_t.

In Laspeyres aggregation, real output detail is added up using base year price weights. For base year B, the ratio of current year to prior year output is given by

$$(1.1) \qquad \frac{XR_{F,t}}{XR_{F,t-1}} = \frac{\sum_j PXj_B\, XRj_t}{\sum_j PXj_B\, XRj_{t-1}},$$

where the subscript F denotes fixed-weight aggregation. If XR_F is real GDP or final sales, the XRj_t corresponding to imports are negative numbers. This is also true for chain-type aggregation.

The great virtue of fixed weighting is that this formula simplifies to

$$(1.2) \qquad X_{F,t} = \sum_j PXj_B XRj_B,$$

allowing aggregate output to be calculated by simple addition. Unfortunately, note Landefeld and Parker (1995), "output measures that use fixed price weights of a single period tend to misstate growth as one moves further from the base period." This problem results from "substitution bias," which occurs because the items for which output grows most rapidly tend to be those for which prices are growing most slowly, or declining. In general, growth rates tend to be biased upward after the base year.

In Fisher ideal output aggregation, the output detail is weighted together using a mix of current-year and prior-year price weights. The ratio of the current-year to the prior-year value for a chain-type aggregate (XR_C) is the geometric mean of the ratios of current-year to prior-year fixed-weighted output using current-year price weights (a Paasche output index) and prior-year price weights:

$$(1.3) \qquad \frac{XR_{C,t}}{XR_{C,t-1}} = \sqrt{\frac{\sum_j PXj_t\, XRj_t}{\sum_j PXj_t\, XRj_{t-1}} \times \frac{\sum_j PXj_{t-1} XRj_t}{\sum_j PXj_{t-1} XRj_{t-1}}},$$

or

$$(1.4) \qquad XR_{C,t} = XR_{C,t-1}\sqrt{\frac{\sum_j PXj_t\, XRj_t}{\sum_j PXj_t\, XRj_{t-1}} \times \frac{\sum_j PXj_{t-1} XRj_t}{\sum_j PXj_{t-1} XRj_{t-1}}}.$$

To get output starting from years other than t-1, such formulas are "chained" together. For example, given aggregate output in year t-2, aggregate output in year t is given by

$$XR_{C,t} = XR_{C,t-2}\sqrt{\frac{\sum_j PXj_t\, XRj_t}{\sum_j PXj_t\, XRj_{t-1}} \times \frac{\sum_j PXj_{t-1} XRj_t}{\sum_j PXj_{t-1} XRj_{t-1}}} \times \tag{1.5}$$

$$\sqrt{\frac{\sum_j PXj_{t-1} XRj_{t-1}}{\sum_j PXj_{t-1} XRj_{t-2}} \times \frac{\sum_j PXj_{t-2} XRj_{t-1}}{\sum_j PXj_{t-2} XRj_{t-2}}}.$$

A chained 1992 dollar series is created by setting real output equal to nominal output in 1992, and then chaining backwards and forwards.

The formula for the chain-type price index takes equation 1.4 and reverses the X and P:

$$PX_{C,t} = PX_{C,t-1}\sqrt{\frac{\sum_j XRj_t\, PXj_t}{\sum_j XRj_t\, PXj_{t-1}} \times \frac{\sum_j XRj_{t-1} PXj_t}{\sum_j XRj_{t-1} PXj_{t-1}}}. \tag{1.6}$$

By multiplying equation 1.4 by equation 1.6, and realizing that

$$PX_{C,t}\, XR_{C,t} = \sum_j PXj_t\, XRj_t$$

in the base year, one can see that chain-type aggregate output times the chain-type aggregate price index equals aggregate nominal output. Another way to say this is that the implicit price deflator, defined as nominal output divided by real output, equals the price index. This is not true for fixed-weighted concepts: the product of fixed-weighted real GDP and the fixed-weighted GDP price index generally does not equal nominal GDP, except in the base year. Nor does this property necessarily hold for quarterly chain-type data, as we shall see below.

B. Quarterly Data

This portion of Section 1 outlines the mathematical formulas the BEA uses to create quarterly chain-type aggregates from quarterly detail on output and prices. In producing quarterly chain-type quantity data, the BEA uses annual prices rather than quarterly prices as weights, "because annual prices are more stable and contain less statistical noise than quarterly prices." (Young, 1993) The BEA calls its quarterly aggregates "chain-type annual-weighted" measures of output and prices.

The first two quarters of the year use current-year and prior-year price weights, while the third and fourth quarters use current-year and next-year price weights. Let $XRj_{[z:x]}$ represent chained 1992 dollar demand for good or service j in quarter x of year z, $XR_{C,[z:x]}$ represent chained 1992 dollar GDP in quarter x of year z, and $PXj_{[z]}$ represent the price index for j in year z. Then, real GDP in the first quarter of a year is related to real GDP in the fourth quarter of the prior year by

$$(1.7)\quad XR_{C,[z:1]} = XR_{C,[z-1:4]}\sqrt{\frac{\sum_j PXj_{[z]}XRj_{[z:1]}}{\sum_j PXj_{[z]}XRj_{[z-1:4]}} \times \frac{\sum_j PXj_{[z-1]}XRj_{[z:1]}}{\sum_j PXj_{[z-1]}XRj_{[z-1:4]}}}.$$

For other quarters of the year, we have

$$(1.8)\quad XR_{C,[z:2]} = XR_{C,[z:1]}\sqrt{\frac{\sum_j PXj_{[z]}XRj_{[z:2]}}{\sum_j PXj_{[z]}XRj_{[z:1]}} \times \frac{\sum_j PXj_{[z-1]}XRj_{[z:2]}}{\sum_j PXj_{[z-1]}XRj_{[z:1]}}},$$

$$(1.9)\quad XR_{C,[z:3]} = XR_{C,[z:2]}\sqrt{\frac{\sum_j PXj_{[z+1]}XRj_{[z:3]}}{\sum_j PXj_{[z+1]}XRj_{[z:2]}} \times \frac{\sum_j PXj_{[z]}XRj_{[z:3]}}{\sum_j PXj_{[z]}XRj_{[z:2]}}},$$

and

$$(1.10)\quad XR_{C,[z:4]} = XR_{C,[z:3]}\sqrt{\frac{\sum_j PXj_{[z+1]}XRj_{[z:4]}}{\sum_j PXj_{[z+1]}XRj_{[z:3]}} \times \frac{\sum_j PXj_{[z]}XRj_{[z:4]}}{\sum_j PXj_{[z]}XRj_{[z:3]}}}.$$

The BEA makes adjustments to the quarterly data thus constructed in order to get the quarterly figures to average out to the chained annual data.

Calculating a single time series from a combination of annual and quarterly data using four different formulas is no easy task. Fortunately, the four formulas can be combined into a single equation which uses only quarterly data. First, define the quarterly variable PXj^A_t as the price index for the year containing period t. I.e.,

$$PXj^A_{[Z:1]} = PXj^A_{[Z:2]} = PXj^A_{[Z:3]} = PXj^A_{[Z:4]} = PXj_{[Z]} = \sum_{x=1}^{4} PXj_{[Z:x]} / 4.$$

Then,

$$(1.11)\quad XR_{C,t} = XR_{C,t-1}\sqrt{\frac{\sum_j PXj^A_{t+2}XRj_t}{\sum_j PXj^A_{t+2}XRj_{t-1}} \times \frac{\sum_j PXj^A_{t-2}XRj_t}{\sum_j PXj^A_{t-2}XRj_{t-1}}}.$$

Paralleling the annual data, an analogous formula can be used to aggregate prices:

$$(1.12)\quad PX_{C,t} = PX_{C,t-1}\sqrt{\frac{\sum_j XRj^A_{t+2}PXj_t}{\sum_j XRj^A_{t+2}PXj_{t-1}} \times \frac{\sum_j XRj^A_{t-2}PXj_t}{\sum_j XRj^A_{t-2}PXj_{t-1}}}.$$

Notice that, unlike the case of annual data, it will not necessarily be true that nominal GDP exactly equals the product of chain-type real GDP and the chain-type GDP price index, although they will generally be very close.

Unfortunately for this method of creating quarterly data, we run out of price data before we reach the end of the historical interval. Beginning with the third quarter of the last year for which annual data is available, equation 1.11 requires price data for the following year, data which is not yet available. For example, to calculate third quarter 1996 real chained GDP, one needs the annual price level for 1997, which becomes available only in July 1998.

Until August 1997, the BEA reverted to fixed-weight, or Laspeyres, measures for both output and prices over this interval, using annual price weights from the most recent year which had undergone an annual revision. Real GDP was calculated using equation 1.1, with B defined as the last year which had undergone an annual revision. The period during which real GDP was constructed using Laspeyres output and price indexes was often referred to as the "Laspeyres tail."

The use of Laspeyres weighting over the last several quarters of history introduces, on a small scale, the problems associated with fixed weights. The growth rates of both output and prices will in general be overstated due to substitution bias. Consequently, when Laspeyres weighting was used in the "tail" period, the product of real output and its price index grew more rapidly than nominal output. As a result, implicit price deflators generally grew more slowly than the corresponding price indexes.

With its 1997 annual revision, the BEA switched to a new methodology for computing quarterly output and prices in the tail period. Estimates now use weights from the current and preceding quarters only, a "quarterly Fisher" index (Parker and Seskin, 1997). The same formulas used to calculate annual output and prices, equations 1.4 and 1.6, are thus used to calculate quarterly output and prices in recent quarters.

2. USING CHAIN-TYPE DATA

A. Creating Alternative Chain-Type Aggregates

In many circumstances, one may want to construct alternative aggregates to those published by the BEA. For example, real consumption of furniture and household equipment excluding personal computers is a variable in the DRI/McGraw-Hill model of the U.S. economy, but is not calculated by the BEA. In the old days of fixed weights, the creation of alternative output indexes was a simple matter of addition or subtraction. With the new chain-type measures, creating price and output measures for alternative concepts has become more difficult. This section discusses how DRI uses the formulas presented in the prior two sections to create chain-type output and prices for alternative aggregates.

To produce a time series for real consumption of furniture and household equipment excluding personal computers, we first need quarterly real chain-type output data for two series: real consumption of furniture and household equipment, and of personal computers. Then, we need annual price data for these two series. For series for which BEA only publishes quarterly price data, we can use annual averages. The annual price data must then be distributed to a quarterly frequency using a step function, in which each quarter of a year receives the average value for the year.

We now have all the data needed to solve equation 1.11, except a lagged value for the desired series itself. (Real chain-type consumption of personal computers enters equation 1.11 as negative numbers.) So, pick an arbitrary seed value for the first quarter for which historical data is available for all the other variables in equation 1.11. Then solve equation 1.11 through 1992, the base year. The resulting series will have the correct growth rates, but the wrong level.

To adjust the level of the series, remember that in the base year, 1992, chained 1992 dollar output must equal nominal output. Since equation 1.11 is multiplicative, multiply the seed value by the ratio of the desired average 1992 value of the series (nominal output) to the average 1992 value of the series obtained by starting with the seed value. Then re-solve equation 1.11 through the last quarter prior to the beginning of the Laspeyres tail, currently the second quarter of 1996. Then solve equation 1.4 from the third quarter of 1996 through the final quarter of history, using quarterly prices for the PXj_B.

B. Alternative Chain-Type Aggregates: Annual Vs. Quarterly Price Weights

As an approximation to the BEA's methodology, a user could also create alternative quarterly aggregates using a quarterly Fisher, i.e., with quarterly instead of annual price weights. Instead of using equation 1.11 prior to the tail period and equation 1.4 during the tail period, one would just use equation 1.4 throughout the entire historical interval. As above, one would first pick an arbitrary seed value and solve the equation through the base year, then alter the seed value and re-solve.

In general, movements in a time series created using the quarterly Fisher approximation parallel movements in a time series created using the BEA's annual price weight methodology. For example, in the case of imports of capital goods excluding motor vehicles and computers (derived by "subtracting" computer imports from non-automotive capital goods imports), the quarterly Fisher is never more than 0.7% bigger than the annual-weighted series, and never more than 0.4% smaller. In contrast, the series obtained by simple subtraction--chained 1992 dollar imports of non-motor vehicle capital goods minus chained 1992 dollar imports of computers and peripherals--starts out 25% below the chain-type annual weighted series in 1967 (the earliest year for which the necessary data is available), rises to equal it in 1992, and falls 7% below it again by mid-1996 (Exhibit 2.1). A similar story unfolds when we subtract computers and autos from investment in producers' durable equipment (Exhibit 2.2). Clearly, the quarterly Fisher approximation is far superior to simple addition or subtraction.

However, the quarterly growth rates obtained using quarterly price weights can differ noticeably from growth rates obtained using the BEA's annual weight methodology (Exhibits 2.3 and 2.4). Differences tend to be largest in the third quarter of the year, when the differences between quarterly and annual price weights are the largest. For example, in the third quarter of 1974, the growth rate of annual-weighted non-auto noncomputer PDE exceeds the quarterly-weighted growth rate by a percentage point. The reason is that the quarterly-weighted series subtracts out a large rise in real computer spending using computer prices for the middle two quarters of 1974, which on average are larger than average computer prices for 1974-75. (Remember that computer prices tend to decline over time.) Since the BEA's overall PDE series "adds in" this increase in real computer spending to overall PDE using 1974 and 1975 prices, the quarterly Fisher, by using too large a price weight, subtracts out too much.

C. Forecasting With a Chained Laspeyres

To perform chain-type aggregation in the forecast, DRI uses an approximation to the Fisher ideal, called a "chained Laspeyres," in which each quarter's GDP is calculated using a fixed-weight quantity index with the prior quarter as the base period:

$$(2.1) \quad XR_{L,t} = XR_{L,t-1} \frac{\sum_j PXj_{t-1} XRj_t}{\sum_j PXj_{t-1} XRj_{t-1}}.$$

This is equivalent to replacing the PXj_t with PXj_{t-1} in equation 1.4, the equation for the quarterly Fisher.

Recognizing that

$$(2.2) \quad \frac{1}{PX_{t-1}} = \frac{XR_{L,t-1}}{\sum_j PXj_{t-1} XRj_{t-1}},$$

this formula can be rewritten as

$$(2.3) \quad XR_{L,t} = XR_{L,t-1} + \sum_j \frac{PXj_{t-1}}{PX_{t-1}} \left(XRj_t - XRj_{t-1} \right).$$

This is the form in which DRI implements the chained Laspeyres.

The advantage of the chained Laspeyres over the quarterly Fisher is that it restores some additivity to real GDP, enabling users to easily apportion growth among its components. The contributions to growth from each of the j components of X sum to the total change in real X. In contrast, in the quarterly Fisher, the contributions to growth of each of the j components of X do not sum to the total change in real X, and the contributions themselves are messy nonlinear functions.

The chained Laspeyres has another useful property: the growth rate of aggregate output is a weighted sum of the growth rates of its components, with the nominal shares of the prior period as weights. Mathematically,

$$(2.4)\qquad \%ch(XR_t) = \sum_j \left[\%ch(XRj_{t-1}) \frac{Xj_{t-1}}{X_{t-1}} \right],$$

where %ch(X) denotes percent growth of X. (For the Xj corresponding to inventory change, %ch(XRj) is the rate of growth of the real stock of inventories, and Xj is the nominal stock of inventories.) This relationship holds approximately for the BEA's chain-type history as well.

3. COMPARATIVE SIMULATION PROPERTIES: INCOME RESPONSE

Much of the discussion comparing Fisher ideal and Laspeyres quantity indexes has focused on their usefulness as measures of consumer utility or the standard of living, e.g., Triplett (1992). According to basic economic theory, consumers adjust their consumption so that price equals marginal utility. Relative prices thus reveal the relative value consumers place on the marginal unit of each type of good or service purchased. Because Fisher ideal indexes use relative prices at the time items are purchased to weight different items, they are likely to come closer than Laspeyres indexes to measuring the value attached to total output.

From the macroeconomic modeler's point of view, however, a more important relationship than "price equals marginal utility" is "price equals marginal cost" (or "marginal revenue equals marginal cost"). According to basic economic theory, the relative prices of different goods and services provide measures of the relative cost of the inputs required to produce them. A measure of GDP which fails to relate price to marginal cost will produce unrealistic income and employment responses to changes in demand. This section shows how the response of income to a change in demand differs in Laspeyres-based and Fisher-based models.

A. Overview of the Output-Income Link in the DRI Model

Exhibit 3.1 summarizes the structure of the DRI/McGraw-Hill model of the U.S. economy. The choice of which measure of real GDP to use directly affects the links between the domestic spending and domestic income sectors of the model. Changes in either of these sectors will, in turn, spill over to the other sectors of the model. For example, a larger income response to a change in one of the components of GDP will also mean a larger interest rate response. A broad overview of the DRI model structure is contained in Brinner and Lasky (1997).

This section summarizes the portions of the model relevant to an analysis of the impact of individual components of GDP on labor income. Mathematically, the effect of additional nominal purchases of good or service X on nominal labor compensation in the DRI model can be thought of in the following chain of causality:

$$(3.1)\quad \frac{dW}{dX} = \frac{\partial XR}{\partial X}\frac{\partial GDPR}{\partial XR}\frac{\partial L}{\partial GDPR}\frac{\partial W}{\partial L},$$

where W is aggregate labor compensation, XR is real demand for item X, X is nominal demand, GDPR is real GDP, and L is aggregate labor hours. An increase in nominal purchases of X corresponds to an increase in real purchases of X, which boosts real GDP, in turn increasing the demand for labor hours, causing businesses to pay out more in labor compensation. Exhibit 3.2 presents a schematic diagram of this portion of the DRI model. As a simplifying assumption, this discussion assumes wage rates and prices are exogenous to changes in demand or income.

The first right-hand-side term in equation 3.1 translates the nominal change in X into a real change in X, and is simply the reciprocal of the price of X, PX. In fact, real output, rather than nominal output, is the ultimate building block of the DRI model, so one would actually change X by changing XR, rather than the other way around. For expositional purposes, however, it is more convenient to keep the relationship between real and nominal X on the right-hand side of the equation.

The second term on the right-hand side of equation 3.1--∂GDPR/∂XR, the impact of a change in real X on real GDP--is the key to the difference between simulation results in chain-based and Laspeyres-based models, and is discussed below. The third right-hand side term in equation 3.1--the impact of real GDP on labor hours, ∂L/∂GDPR--is short-hand for what, in the DRI model, is actually several equations. Through Okun's Law, the ratio of real to potential GDP determines the unemployment rate. This ratio is also used to determine weekly hours per worker. The unemployment rate, in turn, along with demographics and the real after-tax wage, determines the labor force. Together, the unemployment rate, the labor force, and weekly hours determine the denominator of output per hour. Total hours worked are then determined by dividing output by labor productivity.

Let L = A * GDPR / J, where A is a constant and J is average labor productivity. Then,

$$(3.2)\quad \frac{\partial L}{\partial GDPR} = \frac{A}{J}(1-e),$$

where e is the elasticity of productivity with respect to real GDP (e = ∂J / ∂GDPR * GDPR / J). Because of the lag structures of the equations determining this relationship in our model, especially in the Okun's Law equation, the adjustment of labor hours to GDP is not immediate. In equation 3.2, this means the parameter e is positive in the short run, but zero in the long run. In other words, ∂L is a function of both current and lagged values of ∂GDPR. The implied procyclical response of productivity to real output, often called short-run increasing returns to labor, has been long observed in the U.S. and in other countries (Okun, 1962; Brechling and O’Brien, 1967; Gordon, 1986).

In analyzing the final term of equation 3.1, marginal labor compensation per hour worked, it should be remembered that prices are modelled as proportional to unit labor costs (compensation per hour divided by labor productivity), all else equal. I.e.,

$$(3.3) \qquad PGDP = B\frac{W/L}{J},$$

where B is a constant and PGDP is the price index for GDP. Solving for labor compensation, W, and then taking the derivative with respect to labor hours, L, we obtain

$$(3.4) \qquad \frac{\partial W}{\partial L} = \frac{1}{B} PGDP \cdot J.$$

Starting with equation 3.1, and substituting 1/PX for ∂XR/∂X, equation 3.2 for ∂L/∂GDPR and equation 3.4 for ∂W/∂L, we obtain

$$(3.5) \qquad \frac{dW}{dX} = \frac{1}{PX}\frac{\partial GDPR}{\partial XR}\frac{A}{J}(1-e)\frac{1}{B}PGDP \cdot J = \frac{PGDP}{PX}\frac{\partial GDPR}{\partial XR}\frac{A}{B}(1-e).$$

The labor productivity terms cancel out: while greater productivity reduces the additional number of labor hours required to produce an additional unit of X, it boosts the amount of (real) labor compensation paid per hour.

B. Income Responses With a Fisher Ideal Output Index

The only remaining step in determining the income response to additional demand is to evaluate the term ∂GDPR/∂XR, the marginal contribution of an extra unit of real X to real GDP for a Fisher ideal measure of real GDP. Starting from equation 1.4, using annual data, it can be shown that

$$(3.6) \qquad \frac{\partial GDPR_t}{\partial XR_t} = \tfrac{1}{2}\frac{PX_t}{PGDP_t} + \tfrac{1}{2}\frac{PX_{t-1}}{PGDP_{t-1}}\sqrt{\left(\frac{\sum PX_t XR_t}{\sum PX_t XR_{t-1}}\right)\Bigg/\left(\frac{\sum PX_{t-1} XR_t}{\sum PX_{t-1} XR_{t-1}}\right)}.$$

The square root term at the end of the equation is the ratio of a Paasche output index with base year t to a Laspeyres output index with base year t-1. For the U.S. economy over the past 36 years, the square root term has ranged between 0.9993 and 1.0008, using annual data for the five major components of GDP. So, as a very close approximation, we can write equation 3.6 as

$$(3.7) \qquad \frac{\partial GDPR_t}{\partial XR_t} = \tfrac{1}{2}\frac{PX_t}{PGDP_t} + \tfrac{1}{2}\frac{PX_{t-1}}{PGDP_{t-1}}.$$

That is, the impact of an extra unit of X on real GDP is proportional to a two-year moving average of its price relative to that of GDP.

Substituting equation 3.7 into equation 3.5, we find

$$(3.8) \qquad \frac{dW_t}{dX_t} = \left(\frac{1}{2} + \frac{1}{2}\frac{PGDP_t/PGDP_{t-1}}{PX_t/PX_{t-1}}\right)\frac{A}{B}(1-e).$$

The primary thing to notice about this equation is that because $PGDP_t/PGDP_{t-1}$, PX_t/PX_{t-1}, A, B, and e are all independent of the base year chosen for Fisher ideal measures of output, that dW_t/dX_t is also independent of the base year chosen. The income response to an extra unit of output thus depends only on economic fundamentals. Of less importance but still noteworthy, items with relative prices which decline over time, i.e., $PX_t/PX_{t-1} <$ $PGDP_t/PGDP_{t-1}$, will have somewhat larger income impacts than items with relative prices which are stable or rising over time.

Now, assume that there are only two types of income: labor compensation, and other income, OT. If there is no statistical discrepancy between income and output, then gross domestic income equals GDP: GDP = W + OT. Because $\partial GDP/\partial X = 1$, it follows that

$$(3.9) \qquad \frac{dOT_t}{dX_t} = 1 - \left(\frac{1}{2} + \frac{1}{2} \frac{PGDP_t / PGDP_{t-1}}{PX_t / PX_{t-1}} \right) \frac{A}{B} (1-e).$$

The impact of additional production of X on nonlabor income is also independent of the base year chosen.

C. Income Responses With a Laspeyres Output Index

In the Laspeyres case, we can express the impact of a change in nominal demand for X on nominal wages W using the same general expression obtained earlier:

$$(3.5) \qquad \frac{dW}{dX} = \frac{PGDP}{PX} \frac{\partial GDPR}{\partial XR} \frac{A}{B} (1-e).$$

The only term which differs from the Fisher ideal output case is the impact of real X on real GDP, $\partial GDPR/\partial XR$, which now equals unity. The impact of real X on real GDP no longer depends on the relative prices of X and GDP. Equation 3.5 becomes

$$(3.9) \qquad \frac{dW}{dX} = \frac{PGDP}{PX} \cdot \frac{A}{B} (1-e).$$

The impact of a change in demand for X on labor compensation depends on the relative price indexes of GDP and X. That is, the impact of X on W depends on how far this price ratio has moved since the base year, and thus on the base year itself.

The problem is that the labor cost of producing an item--$(\partial W/\partial L)$ $(\partial L/\partial GDPR)$ $(\partial GDPR/\partial XR)$--becomes disconnected from its price, violating price equals marginal cost. In the Laspeyres case, marginal cost depends not on the price of the item, but on the price of GDP. The same dollar, spent on items with different price indexes, produces different changes in labor compensation, even if the multiplier effects and capital intensities of production of the items are identical.

The impact of changes in X on nonlabor income also varies with the relative price of X:

$$(3.10) \qquad \frac{dOT}{dX} = 1 - \frac{PGDP}{PX} \frac{A}{B} (1-e)..$$

One problem with Laspeyres output now becomes obvious: for large enough values of PGDP/PX, an increase in demand for X can actually cause nonlabor income to fall. This situation is particularly likely in the case of computers after the base year.

Its disregard for a basic microeconomic principle--price equals marginal cost--causes a Laspeyres output index to exhibit unrealistic simulation properties in a macroeconomic model. The response of labor income to a change in nominal demand for a component of GDP will change over time with the reciprocal of that component's relative price. After the base year, an extra dollar spent on a good or service whose relative price is generally declining, like durable goods, will produce a larger increase in labor income than it will before the base year. Conversely, an extra dollar spent on a good or service whose relative price is rising over time, like consumer services, will give a smaller boost to labor income after the base year than before. In addition, if the capital/labor ratio used in the production of two items--one with declining price and one with rising price--were the same, an extra dollar spent on the declining-price item would give a smaller boost to labor income than a dollar spent on the rising price item prior to the base year, but a larger boost after the base year.

D. Example: Income Responses in the DRI Model

To demonstrate the differences between Laspeyres-based and Fisher-based models, we compare the income response to a demand shock in the 95A version of the DRI U.S. model, the last model estimated before the BEA's switch to chain-type output, to the income response in the 96B model, the first model fully based on the chain-type data.[1] All model simulations were performed using DRI's Model386 software.

For each model, two simulations incorporating a $10 billion increase in nominal export demand were run. In the first, the add factor for real service exports was boosted by an amount equal to the desired nominal increase divided by its price deflator or price index, i.e., an ex ante exogenous increase in nominal service exports of $10 billion. (For each quarter, the change in the add factor was thus inversely proportional to the price index.) In the second simulation, the add factor for real exports of computers and peripherals was increased by an amount equal to $10 billion divided by its price deflator or price index.

In the chain-based model, the resulting increase in labor compensation for the two categories of exports is similar. Exhibit 3.3 shows the increase in nominal compensation from the baseline as a share of the increase in nominal GDP from the baseline. In the first quarter of each simulation, the increase in labor compensation is just 20% as large as the increase in GDP, due to the lags in the response of labor hours to real GDP, i.e., the large size of the parameter e in the first quarter of a change in demand. Within a year, however, workers are receiving 50% of the increase in GDP, and their share continues to grow as e goes to zero.

In fact, the ratio of the increase in compensation to the increase in GDP actually overshoots its long run value in the fourth year of the simulation. This occurs because nominal monetary reserves are assumed to remain at their baseline level, so the extra inflation caused by a stronger economy reduces real reserves, pushing up interest rates and

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[1] Changes to the DRI model made between the 95A and 96B versions are discussed in Lasky (1996) and Lasky and Kunkel (1996).

causing a slowdown in GDP growth from baseline levels in the fourth year of the simulations. As in the first two years of the simulation, the change in labor hours lags the change in GDP. This time, however, both are falling, so the percentage increase in labor compensation is temporarily larger than the percentage increase in GDP.

In the long run, the ratio of the rise in labor compensation to the rise in GDP roughly matches compensation's 60% share of GDP. The results confirm that in a chain-based model, the income responses of different types of shocks are similar: higher computer exports boost compensation by about the same amount as higher service exports. Both labor compensation and other income are boosted above baseline levels.

In a Laspeyres-based model, the results for service exports are similar to those obtained in a chain-based model (Exhibit 3.4). In the first quarter of the simulation, compensation rises only about one-fifth as much as GDP, because of the lags in the relationship between real GDP and labor hours. Eventually, however, the rise in nominal compensation is roughly 70% as large as the increase in nominal GDP. The overshooting during the fourth year is less pronounced in the 95A version than in the 96B version of the DRI model, indicating that even apart from the change in GDP aggregation, the models are similar but not identical.

Income behaves very differently in the Laspeyres-based model, however, when $10 billion is added to computer exports instead of service exports. As noted above, the response of labor income is too large, because the impact of additional real computer exports on real GDP does not correspond to the cost of producing those exports. The assumption that each dollar of real GDP requires the same resources to be produced, which is a good approximation for chain-type GDP, is far from true for fixed-weight GDP. Instead, the same $10 billion, added to nominal computer exports instead of nominal service exports, causes a much larger change in real GDP, requiring a much larger increase in labor hours and thus labor income.

So, by the beginning of the fourth year of the simulation, the increase in labor compensation actually outstrips the increase in nominal GDP.[2] The converse indicated by equation 3.10 is also true: the sum of non-labor categories of income actually declines below baseline levels. In the DRI model, profits are calculated as a residual after all the other categories of income are subtracted from gross domestic income, so it is profits which plunge in response to the overly large increase in labor compensation. Thus, use of a Laspeyres-based model yields the unlikely result that a rise in computer exports, or in fact increased computer production of any kind, triggers a substantial decline in corporate profits.

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[2] The only reason this does not remain true during the rest of the simulation is that the higher real GDP leads to a general inflation. Since labor compensation is smaller than GDP, and the general inflation pushes both up by roughly the same percentage amount, the resulting absolute increase in nominal GDP exceeds the resulting absolute increase in nominal labor compensation.

4. COMPARATIVE SIMULATION PROPERTIES: THE MULTIPLIER

In the typical macroeconomic forecasting model, an exogenous increase in one of the components of GDP leads to a larger increase in total GDP. The additional income generated by higher production of the affected component of GDP goes to workers, who then spend the higher income on goods and services, giving a boost to GDP beyond the initial increase in production. A portion of the income generated by the production of these goods and services is then spent on more goods and services, adding further to GDP, and so on. In addition, investment rises as businesses enlarge capacity in order to produce the additional goods and services--the accelerator effect. On the other hand, some of the additional demand may leak out through imports or taxes, reducing the impact on overall GDP.

These relationships are often summarized in the "multiplier." A leading textbook defines the multiplier as "the amount by which equilibrium output changes when autonomous aggregate demand increases by one unit," where autonomous demand is defined as demand which is independent of the level of income (Dornbusch and Fischer, 1984). This section examines the multiplier properties of Laspeyres-based and chain-based forecasting models. Once again, the failure to equate price with marginal cost causes problems for the Laspeyres-based models.

A. Definitions

The first question is what "units" should be used in measuring the change in output: nominal dollars or real dollars, base year dollars or chained base year dollars? (Base year dollars measure output in base year prices, while chained base year dollars refer to a Fisher ideal index multiplied by base year nominal output.) In a model where prices are endogenous, the use of nominal rather than real dollars runs into an immediate problem if applied in a simple-minded manner. After a few quarters, a significant portion of the overall change in nominal GDP may be due to an endogenous change in prices, so dividing the total change in nominal GDP by the change in nominal autonomous demand will produce an upwardly biased measure of the multiplier.

Why not use base year dollars or chained base year dollars? The problem with chained base year dollars is that a chained base year dollar of GDP is generally not the same thing as a chained base year dollar of any of its components. If it were, chained GDP would be additive. Instead, as seen in equation 2.3, changes in the different components of GDP are added together based on relative prices. Dividing the change in chained dollar GDP by the initial change in chained dollar autonomous demand is like dividing apples and oranges by apples.

Base year dollars suffer from a more subtle problem. When real output is expressed in base year dollars, the changes in real GDP and autonomous demand are valued according to what they would have been worth in the base year. Because relative prices will generally be different at the time of the hypothetical change in GDP than they were in the base year, however, this means that the changes in GDP and autonomous demand are not measured using a common utility or production function. So, for example, although an extra 1992 dollar of autonomous demand in 1998 might produce an extra two 1992 dollars of GDP in 1998, we have no reason to think that the two-dollar change in GDP provides twice as much extra utility as the one-dollar change in autonomous demand.[3] Base year dollars measure marginal utility in the base year, not in any other year. Once again, we are comparing apples and oranges.

The solution is to convert the real changes into nominal changes, and then to make the comparison. That is, multiply the change in real GDP by the GDP deflator, and divide this quantity by the product of the change in autonomous demand and the deflator for autonomous demand. If output is distributed efficiently, i.e., price equals marginal utility, then each extra dollar of nominal autonomous demand provides the same utility as each extra dollar of nominal GDP. In addition, if production occurs efficiently, i.e., price (or marginal revenue) equals marginal cost, then the production of each extra dollar of nominal autonomous demand requires the same amount of resources as the production of each extra dollar of nominal GDP. So, we are comparing apples with apples.

Mathematically, if A is nominal autonomous demand, AR is real autonomous demand, GDP is nominal GDP, GDPR is real GDP, the subscript 0 denotes baseline equilibrium values and the subscript 1 denotes values in an alternative equilibrium, with a different level of autonomous demand, then we can define the multiplier as

$$\text{(4.1)} \qquad \begin{aligned} \text{Multiplier} &= \frac{\left(GDP_0/GDPR_0\right)}{\left(A_0/AR_0\right)}\frac{dGDPR}{dAR} \\ &= \frac{\left(GDP_0/GDPR_0\right)\left(GDPR_1 - GDPR_0\right)}{\left(A_0/AR_0\right)\left(AR_1 - AR_0\right)}. \end{aligned}$$

This multiplier matches that obtained by dividing the nominal change in GDP by the nominal change in autonomous demand in a model in which prices are exogenous.

In analyzing the multiplier, it will be useful to discuss the direct impact of autonomous demand on GDP, excluding second and higher round effects. So, define the "instantaneous multiplier," the direct impact of autonomous demand on GDP, as

$$\text{(4.2)} \qquad \text{Instantaneous Multiplier} = \frac{\left(GDP_0/GDPR_0\right)}{\left(A_0/AR_0\right)}\frac{\partial GDPR}{\partial AR}.$$

B. Chain-Based Model Results

The first thing to notice in a chain-based model is that the instantaneous multiplier is approximately one. That is, assuming exogenous prices and no change in GDP except for the change in autonomous demand, the change in nominal GDP would approximately

[3] Technically, this is true because a Laspeyres output index is not a superlative index (Diewert, 1995).

equal the change in nominal autonomous demand. This can be seen by inserting equation 3.7 into equation 4.2.

Because the instantaneous multiplier is always unity in a chain-based model, any differences in the multiplier across different components of autonomous demand or over time are caused by different later-round effects. Across different components of autonomous demand, the multiplier will vary with such factors as capital intensity of production and import intensity of demand. For example, few of the inputs to medical services are imported, while a sizable proportion of computer components are. So, for a simulation beginning in the first quarter of 1997, the first-quarter multiplier for consumption of medical care services, 1.17, exceeds that for computer exports, 0.89 (Exhibit 4.1). For computers, the increase in imports more than offsets the extra consumption caused by the rising incomes of workers producing computers.

The multiplier grows over the first few quarters of the simulation because changes in employment, and thus in consumers' incomes, lag changes in GDP. After about a year, higher interest rates begin to crowd out investment, interest-sensitive consumption and net exports, and the multiplier begins to decline. The multiplier for medical services peaks at 1.74 in the fourth and fifth quarters of the simulation, while the multiplier for computers peaks at 1.59 in the fourth quarter of the simulation. The rise in the multiplier between the first and fourth quarters is larger for computer exports than for medical services because the accelerator effect is larger. The computer industry is more capital-intensive than the medical care industry, so the boost to investment is larger.

The results change in a predictable manner for simulations beginning ten years later: as import penetration of the U.S. economy continues to grow, the multipliers fall. Since virtually no medical services are imported, the first quarter multiplier for an increase in consumption of medical services is the same in 2007 as in 1997. In 2007, however, a larger share of the additional income of health care workers is spent on imports than in 1997, so the multiplier peaks at 1.66 after four quarters, instead of 1.74 after five quarters.

A higher share of computer components is imported in 2007 than in 1997, so the first quarter multiplier for computer exports in 2007 is just 0.72, compared with 0.89 in 1997. In addition, a higher share of the induced consumption and investment goes to imports as well, so the multiplier for computer exports peaks at just 1.05 in the third quarter of the simulation, compared with 1.59 in the fourth quarter of the 1997 simulation.

C. Laspeyres-Based Model Results

The same factors which determine the multiplier in a chain-based model--import penetration and the marginal propensity to consume out of disposable income--also affect the multiplier in a Laspeyres-based model. In addition, however, the Laspeyres-based multiplier is inversely related to the ratio of the deflators of autonomous demand and GDP. The instantaneous multiplier now equals the ratio of the GDP deflator to the deflator for autonomous demand (plug $\partial GDPR/\partial AR=1$ into equation 4.2), and the overall multiplier is strongly related to the instantaneous multiplier. Thus, the multiplier for a particular category of autonomous demand will depend on both the base year chosen and whether that category's real price deflator (its price deflator divided by the GDP price deflator) has risen or fallen since the base year.

In the 95A version of the DRI model, in which real output is expressed in fixed-weight dollars, an exogenous increase in medical care services has a multiplier of just 0.91 in the first quarter of 1997 (Exhibit 4.2). The difference between this and the 1.17 multiplier obtained in the chain-based model is due almost entirely to the difference between chain and fixed weights: the instantaneous multiplier is just 0.80, so the ratio of the overall multiplier to the instantaneous multiplier, 1.14, is very close to that in the chain-based model. The overall multiplier peaks at 1.71, more than twice the size of the instantaneous multiplier, implying that second- and later-round effects are somewhat larger in the 95A model than in the 96B model.

How can the instantaneous multiplier differ from one? Nominal GDP is still additive in a Laspeyres-based model, so, if prices were exogenous, would not an extra $10 billion in nominal autonomous demand add $10 billion to nominal GDP? The answer is that, even if prices of the detailed components of GDP were exogenous, the price deflator for fixed-weight GDP would not be. It is affected by the composition of overall demand, rising when demand for a GDP component with a relatively high price index rises, and falling when demand for a GDP component with a relatively low price index rises. The GDP identity--nominal GDP equals real GDP times the GDP deflator--is restored by a change in the GDP deflator. So, the difference between the $10 billion nominal increase in health care spending and the $8 billion "instantaneous" increase in nominal GDP ($10 billion times the 0.80 instantaneous multiplier) is accounted for by a rise in the GDP deflator which adds $2 billion more to nominal GDP. This increase in the GDP deflator would occur even if no prices in the economy had changed. It would not occur, however, if the base year were switched to 1997, because the deflator for medical services would be the same as that for GDP.

The effects of moving farther away from the base year can be seen by repeating this simulation with a start date of 2007 instead of 1997. Because the real price of medical services is expected to rise steadily over this interval, the instantaneous multiplier falls to just 0.67 by 2007. The overall multiplier falls to 0.75, even though the ratio of the overall multiplier to the instantaneous multiplier is the same as in the 1997 simulation. The multiplier is smaller throughout this simulation than the earlier one, peaking at just 1.39 in the sixth quarter.

Just as the multiplier for medical services is too small in a Laspeyres-based model because the deflator for medical services is larger than that for GDP, so the multiplier for computer exports is too large, because its deflator is smaller than that for GDP. In 1997, the instantaneous multiplier for exports of computers and peripherals is 4.60, because the deflator for 1987-dollar GDP is 4.6 times as large as the deflator for computer exports. A large drop (0.3%) in the GDP deflator is thus needed to maintain the nominal GDP identity. The overall multiplier is a smaller 3.27, because a large portion of the components used to manufacture exported computers are imported. As employment, incomes and investment rise in response to the increase in computer production, the multiplier builds to 5.65 in the fifth and sixth quarters of the simulation. This is more than three times as large as the peak multiplier for a rise in medical care service consumption. In the chain-based model, the peak multipliers differ by less than 10%.

As the prices of computers and overall GDP continue to move farther apart, the instantaneous multiplier rises to 7.75 in 2007, nearly 70% larger than the instantaneous multiplier in 1997. Because of the increased import intensity of the economy, however, the overall multiplier is 4.92, just 51% larger than in 1997. The multiplier peaks at 7.45 in the fifth quarter of the simulation (the first quarter of 2008), more than five times the size of the peak multiplier for medical care services. In the chain-based simulations, in contrast, the peak computer export multiplier is actually 37% smaller than the peak medical services multiplier.

D. Conclusions

In both chain-based and Laspeyres-based models, the multiplier for a given component of autonomous demand depends on economic factors such as the marginal propensity to consume, investment behavior, and imports' share of both GDP and that component of autonomous demand. In Laspeyres-based models, however, an important additional factor affects the multiplier: the ratio between the deflator for GDP and the deflator for the component of autonomous demand which is being increased. Since this ratio depends on the base year chosen, the multiplier also depends on the base year chosen. The results obtained from a Laspeyres-based model become more distorted the farther one gets away from the base year.

The multipliers used in this analysis were calculated by converting both the real change in autonomous demand and the resulting real change in GDP into current dollars before dividing the change in GDP by the change in autonomous demand. Obviously, the results would be quite different if we divided the real change in GDP by the real change in autonomous demand, both defined in base year dollars. While this more traditional multiplier is a bit easier to calculate, it has no economic justification.

5. SUGGESTED IMPROVEMENT

Currently, the inventory change aggregates are not calculated according to equation 1.11:

$$(1.11)\quad XR_{C,t} = XR_{C,t-1}\sqrt{\frac{\sum_j PXj^A{}_{t+2}\,XRj_t}{\sum_j PXj^A{}_{t+2}\,XRj_{t-1}} \times \frac{\sum_j PXj^A{}_{t-2}\,XRj_t}{\sum_j PXj^A{}_{t-2}\,XRj_{t-1}}}.$$

The problem is that the detailed components of inventory change can be positive or negative. If an odd number of the four summation terms under the square root in equation

1.11 is negative, then the equation cannot be solved. This occurs in at least one quarter of history for most of the inventory change aggregates, meaning they must be calculated some other way. So, instead of using equation 1.11 to chain-weight the detailed components of inventory change, the BEA uses equation 1.11 to chain-weight the detailed components of total inventory stock, and then publishes the change in this measure as total inventory change. In calculating real GDP using equation 1.11, however, the BEA continues to use the detailed components of inventory change.

The inadequacy of this approximation for aggregate inventory change can be seen from the large errors encountered when using this data to construct broader aggregates. For example, real final sales, i.e. real GDP less inventory change, can be closely approximated by chain-weighting the aggregate data for personal consumption, fixed private investment, government consumption and investment, exports, and imports. The average absolute error over the historical interval prior to the tail period is just $0.1 billion. However, when GDP is approximated using the same five components and inventory change, the average absolute error jumps to $1.2 billion over the historical interval (1947 to 1996), and $1.5 billion since 1970. Clearly, the published aggregate inventory change data are a poor summary statistic for the impact of inventory change on GDP.

A reasonable solution is to use some sort of Taylor's series expansion as an approximation when equation 1.11 cannot be computed. For annual data, one could start with equation 3.7. The change in total real inventory change IRCH, would be a function of the change in the different components of inventory change IjRCH:

$$(5.1) \qquad IRCH_t = IRCH_{t-1} + \sum_j \left[(IjRCH_t - IjRCH_{t-1}) \left(\frac{PIjCH_t}{PICH_t} + \frac{PIjCH_{t-1}}{PICH_{t-1}} \right) \Big/ 2 \right],$$

where PIjCH is the price index for inventory change category j, and PICH is the price index for total inventory change. If $PICH_t$ cannot be calculated using equation 1.12, one could iterate on $IRCH_t$ and $PICH_t$, or use $PICH_{t-1}$. For quarterly data, the price weights could be annual or quarterly.

6. CONCLUSIONS

Sections 3 and 4 of this paper show the macro model consequences of using a measure of real output which is not consistent with microeconomic theory. Fixed-weighted output measures generally violate the theoretical result that marginal revenue equals marginal cost, so simulation properties and forecasting equations which depend on this property do not hold in models which rely on fixed-weighted output. Employment responses, multiplier responses, and the distribution of income among profits, wages and other components are all distorted by the use of fixed-weighted output. The deterioration in model performance--both in simulation properties and equation forecast errors--becomes greater the farther one gets from the base year. Multiplier properties are worst for the categories of autonomous demand whose price indexes differ the most from the overall GDP price index.

These problems not only make life more difficult for the economic forecaster, but they render any policy analysis performed using a Laspeyres-based model subject to error. Even if the policy does not focus on computers, the area of greatest difficulty, errors may still be large. For example, the impacts of medical care spending on GDP, employment, and wage income are all understated after the base year in a Laspeyres-based model. By 2007, as Section 4 showed, the use of fixed-weighted output reduces these impacts by almost 20%. Since changes to Medicare and Medicaid are likely to be included in any alternative budget proposal, a Laspeyres-based model would produce misleading estimates of the impact of such proposals. For example, the estimated impact of reductions in Medicaid and Medicare on jobs and income would be artificially low, as would the estimate of the amount by which interest rates would fall in response to such cuts.

To sum up, chain-type GDP not only gives a more accurate measure of output from the standpoint of measuring economic well-being, but also gives the economic modeler a better tool for performing simulation work and forecasting. There are still some bugs to be worked out, like the published measures of inventory change, and there are still many issues to be dealt with in measuring quality change, but overall, the switch from fixed to chain weights gives us a better measure of output than we had before.

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