A simple analytic model is presented relating local sediment transport capacity to variance in the transverse shear stress distribution in a stream channel. The model is used to develop a physically based conceptual model for the initiation of meandering in straight, bedload-dominated streams as a result of a feedback mechanism. The feedback maximizes the cross-sectional shear stress variance and – in order to achieve stability – ultimately minimizes the energy slope at repeated locations along the channel, subject to steady-state mass flux and the stability of the channel boundary. These locations develop into pools in a fully developed meandering channel; they represent attractor states wherein sediment continuity is satisfied using the least possible energy expenditure per unit length of channel. However, since the cross-sectional geometry of a pool (and the adjacent bar) is asymmetric, these attractor states are only conditionally stable, requiring strong, curvature-induced secondary circulation to maintain their asymmetry. Between two successive pools, a stream occupies a metastable, higher energy state (corresponding to a riffle) that requires greater energy expenditure per unit length of channel to transport the same volume of sediment. The model we present links processes at the scale of a channel width to adjustments of the channel sinuosity and slope at the scale of a channel reach. We argue that the reach-scale extremal hypotheses employed by rational regime models are mathematical formalisms that permit a one-dimensional theory to describe the three-dimensional dynamics producing stream morphology. Our model is consistent with the results from stream table experiments, with respect to both the rate of development of meandering and the characteristics of the equilibrium channel morphology. Copyright © 2006 John Wiley & Sons, Ltd.

Keywords: sediment transport; meander development; bank stability; fluvial system; extremal hypotheses
according to well-known Newtonian principles and gives rise to these geometric properties. While this argument is not made by way of analogy with thermodynamics, the approach is broadly equivalent: our conceptual model is an attempt to describe the large-scale manifestation of Newtonian physics in much the same way as thermodynamic diffusion describes the outcome of many individual molecular collisions.

Following on from the pioneering work on rational regime theory (Yang, 1976; Kirkby, 1977; Chang, 1979; White et al., 1982; Davies and Sutherland, 1983; Millar and Quick, 1993), Eaton et al. (2004) proposed that stable channel morphologies arise from the maximization of flow resistance within what they define as the fluvial system ($f_{\text{sys}}$), subject to the limits of bank stability and the ability to pass the imposed sediment supply with the available discharge. This conjecture can be shown to be similar to the minimum energy argument advanced by Huang et al. (2004), though they pay no attention to the important constraint of bank erodibility. However, both propositions are essentially probabilistic considerations of the problem constructed at the reach scale. Nothing is said about the process–form interactions occurring at the sub-reach scale, at the level of individual bars, ripples and pools. The striking regularity of the meanders produced in the laboratory, and the organized way in which equilibrium evidently is achieved (Eaton and Church, 2004), does not seem to support the position that channel change is simply a random process, guided in only a general way by a drift towards stability: rather, fluvial systems appear to be self-organized, exhibiting emergent behaviour as a result of interactions among random processes occurring at the grain scale.

Linear stability models (LSMs) of alternate bar development (e.g. Engelund and Skovgaard, 1973; Parker, 1976; Fredsoe, 1978) represent an attempt to model the apparent regularity exhibited by meandering channels. The form of the bed adjustment (the perturbation, in the language of stability analysis) is specified at the outset: usually a double harmonic perturbation of diminishingly small amplitude is imposed. These models, and the more recent LSMS that couple the alternate bar-scale analysis with an analysis of the channel alignment using some form of bank erosion relation (Blondeaux and Seminara, 1985; Johannesson and Parker, 1989; Sun et al., 2001), consistently predict that uniform straight channels are unstable, once perturbed, and produce reasonable predictions of stream channel characteristics. But, in order to achieve solutions, these models must constrain the nature of the initial perturbation and simplify the relation between transport dynamics and channel geometry. As a result, while they are very informative with respect to the existence and persistence of instabilities producing meandering stream morphology, they are not well suited to developing the mechanistic links between reach-scale properties and the patterns of sediment transport.

Our conceptual model, based on the feedback process described in this paper between the asymmetry of the cross-sectional shear stress distribution and the local transport capacity, attempts to explain why uniform shear stress distributions are nearly always unstable, once perturbed.

The regime argument put forward by Eaton et al. (2004) recognizes three separate scales of flow resistance (grain scale, bedform scale and reach scale). It is hypothesized that, as a system attains stability, it will minimize the energy available to deform the system by adjusting one or more of these flow resistance components so as to optimize the dissipation of energy as frictional resistance to flow. The adjustment of each component must be the result of some process–form interaction. For example, the relation between sediment supply and surface texture and structure, and thereby the grain-scale flow resistance, is well known (Dietrich et al., 1989; Church et al., 1998). When stream banks are relatively erodible, it appears that the reach-scale flow resistance (i.e. the sinuosity of the thalweg) is preferentially adjusted (Eaton and Church, 2004), and the flow resistance maximization is manifested by minimizing the channel slope. The conceptual model presented herein primarily describes this reach-scale adjustment, but it must be remembered that adjustments of the grain-scale and the bedform-scale flow resistance components will modulate the reach-scale response. This is perhaps the defining difference between our model of meander initiation and a more complete model of stream channel dynamics.

We do not, then, claim that the model described here incorporates all necessary and sufficient conditions to describe alluvial channel behaviour. We also wish to emphasize that the model is primarily concerned with initiation of meandering in single-thread channels, since the added degree of freedom introduced by channel bifurcations and confluences requires a more sophisticated modelling approach than is consistent with the use of regime theory in this paper. We will: (i) present a theoretical basis for the hypothesized feedback founded on a simplified representation of the shear stress distribution to estimate the sediment transport capacity; and (ii) present evidence from the experiments discussed in Eaton and Church (2004) that demonstrates the action of the hypothesized feedback mechanism to produce equilibrium channels.

Transport Capacity for Asymmetric Shear Stress Distributions

The feedback initiating meandering and slope reduction (SMB) operates locally, at the level of the channel cross-section. Once initiated, it propagates downstream. The feedback is described using a simplified shear stress
A conceptual model for meander initiation is presented, focusing on the initiation process of meanders. The model is based on the transport capacity and shear stress distributions. Shear stress distributions are characterized by a width (W), mean shear stress (τ̄), and a parameter (β) representing the variance of the shear stress distribution. The coordinate x is defined as the proportion of the distribution (P) less than a given shear stress value (P varies from 0 to 1), scaled by the channel width (x varies from 0 to W).

In the limit case when $\beta = 0$, the distribution is uniform, corresponding to a transport capacity $TC$. Once the characteristic grain size ($D$) is known, the transport capacity is estimated using a simplified form of the Meyer-Peter and Müller equation (Meyer-Peter and Müller, 1948).

$$TC = W\xi[\tau - \tau_c]^{3/2}$$ (1)

wherein $\xi$ is a constant, $\tau_c$ is the critical shear stress for entrainment, given by $\tau_c = 0.045(\gamma_s - \gamma)D$ (cf. Meyer-Peter and Müller, 1948), $\gamma$ is the unit weight of water and $\gamma_s$ is the unit weight of the sediment.

For non-uniform distributions ($\beta > 0$) the sediment transport capacity increases. Ferguson (2003) presents a more complete treatment of the difference between the quantities defined by Equations 1 and 2 in which he employs a shear stress distribution represented by two planar surfaces. A shape parameter, $b$, is varied between the limits 0 and 1, representing distribution shapes ranging from uniform to a convex dogleg, respectively.

Ferguson observes that the distribution of shear stress in one particular test case (Fraser River, British Columbia) is consistent with a simple distribution ($0.4 \leq b \leq 0.7$). It may be inferred, then, that the simple shear stress distributions assumed in this analysis are adequately realistic.
The nature of the feedback process depends on how the ratio \( \frac{TC}{\bar{TC}} \) behaves. When \( \frac{TC}{\bar{TC}} > 1 \), a local excess transport capacity results, which will necessarily deform the channel through net erosion until the disparity is reduced. For all \( \beta \) in the range \( \beta_{\text{max}} > \beta > 0 \), this ratio becomes infinite in the limit \( (\bar{\tau} \to \tau) \), since \( \bar{TC} \) approaches 0 but \( TC \) remains finite, if small. At the other extreme \( (\bar{\tau} \to \infty) \), the ratio approaches 1 (no effect). The behaviour between these limits depends on the channel width \( W \) and the distribution variance \( \beta \).

If feedback strength is expressed as \( \frac{(TC - \bar{TC})}{\bar{TC}} \), excess shear stress is expressed as \( (\bar{\tau} - \tau_{c}) \tau_{c} \), and shear stress distribution variance is expressed as a proportion of the theoretical maximum \( \beta/\beta_{\text{max}} \), then the dependence on \( W \) becomes implicit. The strength of the feedback tendency is functionally related to excess shear stress for given values of \( \beta/\beta_{\text{max}} \), as shown in Figure 2.

![Figure 2](image)

**Conceptual Model of Meander Initiation**

The feedback which is the basis for the meander initiation model is based on the idea that: (i) a disparity between \( TC \) and \( \bar{TC} \) produces net erosion centred around the position of maximum shear stress; and (ii) this local erosion will increase the variance of the shear stress distribution, thus increasing the disparity between \( TC \) and \( \bar{TC} \), and producing a positive feedback. This ultimately necessitates a change in the mean shear stress via a channel slope adjustment in order to establish a stable, meandering channel pattern.

**Characteristics of the positive feedback mechanism**

Two key points must be made before describing in detail the feedback mechanism. First, it is evident in Figure 2 that, as the mean shear stress approaches \( \tau_{c} \), the effect of the distribution variance \( (\beta/\beta_{\text{max}}) \) on the local transport capacity is proportionally greater. For example, a mean shear stress 50 per cent greater than \( \tau_{c} \) has a transport capacity only 30 per cent greater than \( \bar{TC} \), assuming a relatively high variance \( \beta/\beta_{\text{max}} = 0.5 \). For the same distribution variance, a shear stress only 10 per cent larger than \( \tau_{c} \) has a transport capacity that is fully 300 per cent greater than \( \bar{TC} \). Second,
provided the mean shear stress ($\tau$) does not change (which amounts to holding the channel slope constant), the feedback intensifies as $\beta/\beta_{\text{max}}$ increases. Once the feedback starts, it quickly increases the local disparity between the reach-average sediment supply and the local transport capacity, $TC$. For a constant excess shear stress, $TC$ increases quickly with increasing $\beta/\beta_{\text{max}}$ (Figure 2). For example, if the excess shear stress is 25 per cent, then $TC$ is 5 per cent greater than $\overline{TC}$ when $\beta/\beta_{\text{max}}$ is about 1/8, and 80 per cent greater when $\beta/\beta_{\text{max}}$ is about 1/2.

Both of these effects are likely to be generally true of any reasonable shear stress distribution model. They certainly apply to the shear stress distribution models introduced by Ferguson (2003). However, it is appropriate to emphasize that the driving variable is ($\tau - \overline{\tau}$) not simply $\tau$, and that sorting of bed sediment under partial transport conditions can inhibit the sort of feedback that we invoke. Sorting amounts to a form of grain-scale flow resistance response in the framework described by Eaton et al. (2004), and the topic is relatively well studied (Parker et al., 1982; Parker and Klingeman, 1982; Dietrich et al., 1989; Wilcock and McArdell, 1993; Paola and Seal, 1995). So, the formative flows responsible for the construction of riffle and pool sequences must be capable of moving nearly all of the grains found on the bed (locally, if not everywhere) in something approaching full mobility (Wilcock and McArdell, 1993) in order to prevent the development of an armour layer sufficient to suppress the feedback. The area subject to fully mobile transport need not be extensive to initiate meander development; it may be sufficient to have very localized fully mobile transport, which can occur in laboratory experiments, at least, under flow conditions not very different from threshold.

**Linking the positive feedback to planform adjustment and stability**

*Simplifying assumptions.* Since we are concerned primarily with the relative (in)stability of a flat-bottomed, straight channel we can usefully apply the uniform flow approximations, wherein the distribution of shear stress ($\tau$) has a direct correspondence to depth distribution (after Lane, 1955). Visual inspection during stream table experiments (Eaton and Church, 2004) suggests that the distribution of sediment transport is highly correlated with the local water depth, particularly during the phase of meander initiation. Once a channel develops significant cross-sectional asymmetry, the role of three-dimensional flow structure becomes more important, but by that time the initial morphologic structure has been set (i.e. the meanders have been initiated), which conditions the further development of the channel morphology.

In addition to disregarding the three-dimensional flow structure, we ignore the accelerations and decelerations of the flow that may produce spatial variations in velocity head (and thereby the energy gradient), and assume that the mean velocity is similar everywhere. We justify this assumption on the observation that the velocity for riffle and pool cross-sections in laboratory experiments (Eaton and Church, 2004) were not systematically different, and on the reported convergence of velocities at near-bankfull conditions in the field (Carling, 1991; Clifford and Richards, 1992).

We also note that the maximum possible reach average sediment throughput ($Q_b$) for the system is equal to $\overline{TC}$ since sections with uniform shear stress distributions act as chokes within the system. It must also be equal to the sediment supply rate if a system is in equilibrium.

*The feedback process.* Assuming an initially uniform shear stress distribution, a small local perturbation is imposed in the form of bed erosion (see Figure 3 (first channel configuration) and Figure 4). At this stage, the thalweg remains straight, and the water surface slope remains unchanged. However, for flow conditions that are nearly uniform, a random local increase in depth will increase $\beta$, thereby increasing $TC$ at the site of the perturbation. We assume that this process does not modify $\overline{d}$ and $W$ appreciably. Nevertheless, a perturbation that increases the variance, $\beta$, will quickly amplify through localized net erosion at an increasingly rapid rate. The system is no longer stable. Excess transport capacity is localized around the initial perturbation and net scour continues, increasing the maximum depth ($Y_\text{max}$) and $\beta$. This net change is primarily vertical, the banks remaining stable.

Downstream, where the shear stress distribution remains uniform, deposition necessarily occurs (see Figure 3, second channel configuration). The deposition will affect $\beta$ there and initiate another perturbation. Since entrained sediment will be deposited on the same side of the channel, the new perturbation will occur on the opposite side of the channel. That is, divergence of the sediment flux modulates the shear stress distribution and replicates the initial perturbation. The longitudinal scale of the flux divergence, which presumably is related to the step length for sediment transport at formative discharge (Pyrcz and Ashmore, 2003), sets the wavelength for the emergent channel bars. The characteristic wavelength is considered in more detail below.

The rate of morphologic change accelerates rapidly, since the initial increase in $TC$ will cause further net erosion, around the deepest part of the channel, increasing $\beta$ and $TC$ in the positive feedback loop shown at the top of Figure 4. At some point, the vertical degradation will cause the adjacent stream bank to fail by over-steepening it, introducing a lateral component to the adjustment and producing the third configuration in Figure 3. The onset of bank erosion is critical to the development of meanders (Friedkin, 1945), at which time the flux divergence increases rapidly (Dietrich...
and Smith, 1983), and lateral adjustment comes to dominate the system. Channel slope begins to adjust and a negative feedback is initiated (bottom of Figure 4), through which channel sinuosity increases and slope decreases. Stability results from the reduction in channel slope (fourth configuration, Figure 3). At this point, the assumptions of our initial analysis clearly are no longer valid, but the meander form has been initiated and further development is strongly conditioned by the initial morphologic change.

Longitudinal variations in energy gradient must develop within the system in order to maintain a constant $TC$ in a channel where $\beta$ varies. The slope over the apex of the developing bend (i.e. the zone of most intense positive feedback – which will be referred to as an attractor) is quickly reduced, and the slope over the crossover (where $\beta$ is nearly 0) will increase due to the drawdown effect of pool deepening (downstream of the crossover) and the deposition of sediment eroded from the pool (on the upstream side of the crossover). At equilibrium, the shallower, more uniform crossover sections cannot have the same local slope as the apex sections if they are to continue to transport, on average, the same throughput sediment load.

**Meander wavelength.** There remains the issue of the wavelength for the adjustment. In the field, meander wavelength is remarkably consistent across a range of channel scales, ranging from about $5W$ to $15W$ (Leopold et al., 1964; Yalin, 1971; Rhoads and Welford, 1991). In laboratory experiments (e.g. Garcia and Nino, 1993), the wavelength of alternate bars developed between fixed, straight banks is nearly constant, despite a range of flow depths and channel gradients. On all evidence, the critical length scale appears to be the width of the channel. Yalin (1971) argues that this is the result of some spatial sinusoidal autocorrelation with a characteristic length scale equal to the channel width. He attributes the autocorrelation to channel width-scale turbulent eddies, but this seems an unlikely control given the circulation cell flow structure typical of meandering rivers (Hey, 1976; Dietrich and Smith, 1983).

We relate the characteristic meandering wavelength to the diffusion of sediment, eroded at the initial perturbations, across the channel. Experimental evidence reported by Nikora et al. (2002) suggests that the rate of lateral sediment diffusion is consistent with the lower range of observed meander wavelengths (c. 8W). This sediment diffusion process has also been observed in experiments studying bar development, wherein bedload sheets have been observed to migrate downstream and expand across the channel (Fujita, 1989; Fukuoka, 1989; Pyrce and Ashmore, 2003).
This sediment transport-based view of meander initiation and propagation is consistent with Parker’s (1976) analysis, which concludes that sediment transport is the primary cause of alternate bar-type instability, and with Smith’s (1998) observation that channel width-scale bedload deposition is the key mechanism giving rise to meandering in his stream tray experiments.

Though experimental evidence provides some sense for the rate of diffusion, we do not fully understand the physical processes controlling the diffusion (Nikora et al., 2002). Given that cross-stream velocity components in straight channels are typically on the order of a few per cent of the downstream flow velocity (Tracy, 1965), it seems unlikely that lateral momentum diffusion is responsible for sediment wave diffusion. We note, however, that, in a fluid, a change in the flow structure (and hence the sediment flux) can be transmitted at a maximum speed equal to the celerity of a gravity wave ($v_g$). Accordingly, we suppose that the sediment wave emanating from the perturbation can propagate laterally by influencing the flow structure at a speed ($v_p$) that is proportional to $v_g$ (cf. Werner, 1951; Anderson, 1967), thus:

$$v_g = \sqrt{gd} \quad \text{and} \quad v_p = \alpha v_g$$
where $\alpha$ is a parameter between zero and unity. The angle at which the disturbance propagates across the channel is given by the ratio ($v_p/v$), where $v$ is the average stream velocity, or, equivalently, by ($\alpha/Fr$), where $Fr$ is the Froude number, since $Fr = v/v_e$.

Figure 5 shows a schematic representation of our conceptual view of the diffusion process. As the edge of the sediment wave approaches the opposite bank, flow diverging over the edge of the wave front encounters the bank, and strong turbulent circulation cells develop. This stalls the advance of the wave front. Although sediment can be transported across the front of the sediment wave, it is no longer accreted to the front of the bedform. Downstream, the development of strong secondary circulation adjacent to the bank forces the flow to converge and limits the lateral bedform growth, thereby distorting the wave, increasing $\beta$ and triggering another local scour and a new sediment wave that diverges across the channel in the opposite direction. This view assumes that sediment is transported in a series of nearly closed erosion and deposition cells with length scales of $0.5\lambda$, which Pyrce and Ashmore (2003) demonstrate to be the case.

The point at which the proximity of the bank first produces an effect sufficient to stall the advance of the wave front corresponds to the location of the root of what becomes the crossover riffle (position ‘b’ in Figure 5). The interplay between the tendency for the sediment wave to diffuse laterally and increasingly strong secondary circulation between the wave front and the channel bank results in the equilibrium wave front configuration and the associated alternate bar deposit (Figure 5B). If we assume that the root of the riffle is approximately midway between the upstream and downstream pools ($\overline{ab} = \overline{bc}$), then the pool-to-pool spacing is $\overline{ac}$, and therefore the wavelength for the alternate bars is $2(\overline{ac})$. Thus, we can combine the divergence angle ($\alpha/Fr$) with the channel geometry parameters ($W, \lambda$) such that, from geometrical similarity:

$$\lambda = \left(\frac{4Fr}{\alpha}\right)W$$

(5)

$W$ is the obvious natural scale that constrains wave diffusion and $Fr$ reflects the rate at which a disturbance in the fluid can be transmitted laterally. The details of the physical processes governing the lateral propagation of a bedload sheet are parameterized in the coefficient, $\alpha$, which can be expected to vary with channel shape, the degree of divergence between the sediment flux and the velocity field and/or relative roughness. An LSM-derived equation for predicting wavelength, presented by Parker and Anderson (1975) and modified by Ikeda (1984), can be expressed in a similar form:

$$\lambda = \left(\frac{5Fr}{\kappa}\right)W, \quad \text{where } \kappa = \sqrt{\frac{S}{d}}$$

(6)

Equation 6, then, can be interpreted as an expression of the dynamics shown in Figure 5, implying that our conceptual view of the processes controlling meander wavelength is consistent with the scaling predicted by linear stability analysis.
While it is difficult (perhaps impossible) to derive $\alpha$ theoretically, we can at least estimate $\alpha$ based on data from the field and from laboratory experiments. Lewin (1976) provides a field example of meander initiation in a gravel-bed river that was artificially straightened ($Fr \approx 0.85$). The initial characteristic wavelength for the bars that developed was about $6.1W$, while the ultimate stable wavelength, once cut-banks and point bars developed, was about $6.75W$, suggesting $\alpha \approx 0.5$. Data from laboratory experiments generally support this estimate. The wavelengths for the constant sinuosity experiments reported by Eaton and Church (2004) – for which we know the effective width tolerably well – are on average about $7.3W$. The Froude number is about 0.83, implying $\alpha \approx 0.46$. However, data from experiments reported by Garcia and Nino (1993) indicate that $\alpha$ is not constant; rather, it varies with $Fr$ (Figure 6). The data from all of the laboratory experiments exhibit a consistent general trend, despite the fact that the wavelengths for Garcia and Nino’s (1993) experiments are significantly higher ($\lambda \approx 9W$, on average) than those reported by Eaton and Church (2004). A similar behaviour that may be analogous is the development of dune fields downstream of initial perturbations, as reported by Venditti et al. (2005) (Figure 7). The angle of cross-channel propagation at $Fr = 0.33$ for the bedforms implies $\alpha = 0.25$ (and from Equation 5, $\lambda = 5.5W$).

The results are generally consistent with the $\lambda = 2\pi W$ scaling suggested by Yalin (1971), and conform to the lower range of the reported wavelengths for fully developed meanders in natural channels. Since the argument above is
based on the maximum possible rate of cross-channel diffusion, it is expected that the analysis should represent the lower bound, and that other factors, such as the role of secondary circulation and bank erosion, could easily distort the alternate bar wavelengths predicted by Equation 5. According to Parker (1976), the development of significant secondary circulation is a trailing phenomenon that enhances the development of meandering, and informs the ultimate stable channel morphology. That secondary circulation is not incorporated into our conceptual model is not a serious limitation, then, since we are concerned with understanding the initiation of the meander structure, not with predicting the ultimate meander geometry.

**On the persistence of the instability**

The LSMs presented by Engelund and Skovgaard (1973) and by Parker (1976) show that, provided the bed is actively transporting sediment, some sort of instability always grows. Fredsoe (1978) predicts general instability for $W/d$ greater than 8. However, since they assume that the perturbation, though infinitesimal, is ubiquitous, and that bars will emerge spontaneously everywhere, they cannot be used to investigate the processes that actually produce the instability. We have suggested a mechanism that produces a perturbation of similar geometry with an appropriate length scale, which diffuses downstream from an initially localized perturbation via sediment flux divergence.

An important question remains: what is the initial cause of the local perturbation? One plausible cause is the effect of a channel wall on the flow structure and turbulence intensity. It has long been known that, in a straight channel, the turbulence intensity is greatest adjacent to the side-wall (Gessner and Jones, 1965), and Tominaga et al. (1989) relate the production of vorticity there to secondary circulation cells adjacent to the bank. These cells scale with the water depth and the relative roughness of the banks, consistent with results reported by Einstein and Shen (1964), who observed alternate bar formation only in the presence of rough channel banks. The cells are not large enough to affect the entire channel cross-section, but they can generate an initial perturbation of the sort we invoke above. Allen (1994) and Smith (1996) summarize the turbulent structure along the banks as a series of corkscrew vortices that are periodically disrupted by a horseshoe vortex, which may coalesce and grow in scale. These two scales of periodic behaviour are implicit in the time-averaged secondary flow cell, and may be responsible for the initiation and maintenance of a perturbation for long enough that the feedback between cross-sectional shear stress variance, flow structure and transport capacity can begin. However, the feedback will amplify only at flows that can produce full mobility in the developing pools.

The cause and persistence of a local perturbation must fundamentally lie in the interaction between the probability that a flow excursion will produce local erosion, and the probability that an altered sediment supply from upstream due to some other perturbation will interact with it, thereby suppressing or enhancing the net erosion. That is, random processes interact with each other, producing the self-organization (the alternating series of riffles, pools and bars) characteristic of most bedload-dominated streams. The outcome is related to the step length for sediment transport. While step length may be set, essentially, by the spatiotemporal structure of the flow over a nearly uniform bed or, later, by the larger scale structure of pools and bars (Pyrce and Ashmore, 2003), it is appropriate to view it as a probabilistic statement about mean distances of movement, and thus related primarily to the mean boundary shear stress.

**Scales of stability for developed meanders**

In this section we relate the processes and feedbacks in our model of meander initiation to the structure and function of fully developed meanders. At the reach scale, stability has been argued to result from the maximization of the system-scale flow resistance and the consequent minimization of the energy available to deform the system (Eaton et al., 2004). However, at the scale of morphologic units, stability is conditional, and there is a continual oscillation between various states along the stream channel and over time (Figure 8). The minimum variance sections (i.e. the riffles) are metastable, at best, as a result of the susceptibility of nearly uniform shear stress distributions to perturbations. The maximum variance sections (i.e. the pools, or attractors) are also unstable. Transitions to maximum variance states are achieved by changing the local channel path length and thereby lowering the local channel gradient. The resultant channel shape becomes intimately linked with the centripetal acceleration of flow around a bend, and the resultant secondary circulation currents (Thompson, 1986). When the centripetal acceleration ceases and the secondary currents weaken, sediment that is tracking along the bar face descends and deposits into the thalweg, reducing the cross-channel shear stress variance and necessitating an increased slope to transport the incoming sediment (cf. scouring and filling sections; Andrews, 1979; Lisle, 1979). The maximum variance solutions are stable, conditional on the existence of sufficient centripetal acceleration to maintain their asymmetric cross-sectional form.

The pools can be considered to be attractor states since, due to their greater shear stress variance, they can do the same geomorphic work while expending less energy, as described above. The spatiotemporal pattern of channel
adjustment involves the attainment of a maximum variance state on one side of the channel (a triangular shear stress distribution with the maximum shear stress, \( \tau_o \), along the right bank, for example), followed by the abandonment of that state for one with a nearly uniform shear stress distribution, and ultimately followed by the development of a maximum variance state on the opposite side of the channel (\( \tau_o \) along the left bank) (Figure 8).

For meandering streams, then, there is no single, stable state. They maintain relative pattern stability by oscillating between two attractors having \( \tau_o \) on opposite sides of the channel which are associated with curvature-induced secondary circulation in opposite directions. In the transition between the two attractors, the stream occupies a metastable state. Thus, meandering can be viewed as a dynamic process, produced by the attraction toward conditionally stable maximum variance conditions on alternate sides of the channel. The oscillation is responsible for the characteristic pattern of alternating pools with asymmetric cross-sectional shear stress distributions having a large variance, separated by riffles where the shear stress distribution is generally more uniform. This can be interpreted as a spatial oscillation along the path of sediment and fluid flux, or alternatively as a temporal oscillation at a fixed point, produced by the downstream migration of the meander form.
Comparisons with Experimental Data

The argument presented above suggests that the feedback mechanism should produce some characteristic features in alluvial systems. Experiments on a stream table where \( Q, Q_b, \) and \( S_v \) are treated as independent variables demonstrate many of these expected features (Eaton and Church, 2004).

Observed slope reduction

Most obviously, Eaton and Church’s (2004) experiments demonstrate that initially straight channels with a mobile bed increase their sinuosity until they reach equilibrium configurations at slopes less than the initial slope. Holding \( Q \) constant, the reported equilibrium slope depends on the sediment supply to the system, and the documented trajectories toward equilibrium show sinuosity increasing monotonically – the so-called slope minimizing behaviour. Eaton and Church (2004) also demonstrate that the response of systems at equilibrium to subsequent changes in the governing conditions is consistent with the hypothesized feedback mechanism, since when the sediment supply was lowered, the channel – initially at equilibrium with the higher feed rate – responded by instigating another phase of bank erosion at the apices, producing an increase in sinuosity until equilibrium was re-established at a lower slope. When the sediment supply to an equilibrium channel was increased, the reverse seemed to occur, though no equilibrium channels were established since the single-thread channels tended to give way to multiple-thread ones.

Time-to-equilibrium

The rate of adjustment for these experiments was also consistent with our model, which predicts that when the shear stress acting on the bed is much larger than \( \tau_c \), the amplification of the transport capacity due to non-uniformity of the transverse shear stress distribution is small, and thus the feedback is relatively weak (Figure 2). As the average shear stress approaches the critical value for entrainment, the strength of the feedback increases rapidly. Figure 9 plots the initial specific discharge \( q \) against the time it took for the stream table experiments to reach equilibrium, which must be related in some general way to the strength of the feedback mechanism. Since the initial channel slope (i.e. \( S_v \)) and the bed material grain size distribution were kept constant, \( q \) is a surrogate for shear stress, with the threshold for bed entrainment from a flat bed estimated to be about 0.6 litres per second per decimeter (L/s/dm). There is a strong positive correlation between the time required to establish equilibrium and the initial \( q \). For the longest running experiment (1–3), increases in channel sinuosity did not occur until around 10 hours, when the sediment transport rate and, by inference, average shear stress were substantially reduced, and the strength of the feedback increased. It took a further 14 hours to reach equilibrium. In contrast, when the initial \( q \) was near threshold, equilibrium configurations were reached after one or two hours.

Another perspective supposes that the higher rates of sediment throughput associated with higher mean shear stress should result in a more rapid approach to equilibrium. But it is clear that the lateral change in channel morphology implicit in our feedback mechanism requires divergence of the sediment flux – deposition must be possible within the channel. When the mean shear stress is high and the mean step length is long, relative to channel width, then there can be relatively little divergence of the sediment flux. As a result, construction of an alluvial morphology by deposition is very limited, even though substantial morphologic modification may occur due to net degradation. It is only when the mean shear stress approaches the value for entrainment that divergence (and deposition) readily occurs. Thus, while higher mean shear stress results in a higher sediment throughput, it produces a lower rate of constructional morphologic change.

Figure 9. Time to equilibrium versus initial specific discharge for stream table experiments.
Another testable prediction of our model pertains to the distribution of local energy slope along the channel. The inferred difference in $\beta$ between bend apices and crossovers implies that a higher energy slope must exist at the crossovers if they are to transport the same sediment load. The water surface slopes for three experiments reported by Eaton and Church (2004) are presented in Figure 10. Water surface elevations are used instead of total hydraulic head.

![Figure 10](image-url)

**Figure 10.** Water surface slopes along the thalweg for three equilibrium channel configurations. The minimum slope in each bend is shown in bold italics, and the maximum is shown in bold. $S_v$ is 1.09 for these experiments. The direction of flow is from right to left.
because there is a greater density of these data since the total hydraulic head can only be estimated at the crossovers and the apices (where there are velocity estimates). However, as mentioned above, the mean velocities at the crossovers and apices for these experiments were not significantly different, suggesting that the approximation of energy gradient using water surface slope is appropriate.

The experiments represent a range of thalweg sinuosities and initial transport intensities. For the higher thalweg sinuosity channels (experiments 1–1 and 1–3, Figure 10), the pattern is relatively clear: the lowest slopes are associated with the thalweg apices and the highest slopes are associated with the entrances to and exits from the bends. This pattern is also evident for a lower sinuosity equilibrium channel (experiment 1–6), but there are notable exceptions to the general rule. This most likely implies failure of the simple reasoning that relates the pattern to position along the thalweg trace. The details associated with the individual bedforms that undoubtedly affect the transverse distribution of shear stress seem to become relatively more important as the sinuosity decreases.

Interestingly, the typical pattern of slope variation exhibited in these experiments is also evident in fixed bank, mobile bed experiments. Hooke (1975) discusses an experiment conducted in a channel bend based on a sine-generated curve with a mobile sand bed. In the pool downstream of the bend apex (where helicoidal flow strength is greatest) the energy slope at all of his experimental discharge values is less than over the riffles upstream and downstream of the pool. The magnitude of difference in surface slopes is similar to the differences between the maximum and minimum slopes shown in Figure 10 for our experiments.

Hooke (1975) presents maps of shear stress variance which, for a discharge of 50 L/s, show that over the pool and adjacent bar, 33 per cent of the bed experiences flow less than 50 per cent of the average shear stress ($\bar{\tau}$), while 29 per cent of the bed experiences shear stresses greater than 150 per cent of the average. In contrast, only 34 per cent of the bed at the riffles experience shear stresses outside the range 0.5($\bar{\tau}$) $\leq$ $\tau$ $\leq$ 1.5($\bar{\tau}$), demonstrating that there is indeed a systematic difference in the shear stress variance with channel morphology. This pattern holds over a range of discharge values: for a discharge of 20 L/s, Hooke’s maps indicate that only 40 per cent of the riffle experienced shear stresses outside the range defined above, while 51 per cent of the pool–bar had shear stresses outside the range.

**Discussion and Conclusions**

By changing the scale of inquiry from the reach scale to the scale of individual morphologic units, one can gain an understanding of the optimality criterion necessary to complete the regime formulation presented by Eaton *et al.* (2004), and evident in the graded channel response documented by Eaton and Church (2004). Eaton *et al.* (2004) interpreted the optimality criterion as a maximization of the system-scale flow resistance, which is conceptually equivalent to a potential energy well (cf. Huang *et al.*, 2004). When the reach-scale resistance component is dominant, system-scale resistance is maximized by a feedback between the transport capacity for a section and the variance of the transverse shear stress distribution that reduces the channel slope. Slight initial variations in the shear stress distribution result in local net scour, typically along the banks, which increases the local transport capacity by increasing the variance of the shear stress distribution. This process leads to further local net scour in a positive feedback (Figure 4). The local net scour may be primarily vertical until the channel banks begin to fail, at which point bank erosion initiates a negative feedback that increases channel sinuosity (Figure 4). If the reach-average sediment throughput is to remain constant (that is, if equilibrium is to be regained), then the local channel slope at the locus of most intense feedback (the attractor) must decrease since more material is leaving the area than is entering it.

Once one initial bend develops, others necessarily follow, because of the non-uniform cross-sectional distribution of sediment transport. That is, the local increase in sediment transport resulting from the net scour at an asymmetric cross-section will induce significant divergence in the sediment flux field, expressed as periodic deposition downstream on alternate sides of the channel. Order is generated in the system by alternating loci of erosion and deposition on either side of the channel. The longitudinal scale for this cycle can be approximated in a physically meaningful way as the length scale for the diffusion of a sediment wave across the channel, moving laterally at a speed proportional to that for a gravity wave. The initiation of meandering, then, is proposed to be related to the characteristics of the sediment transport field. The secondary flow circulation that is closely related to the meandering pattern is thought to be a trailing phenomenon that develops as a consequence of the sediment transport field dynamics, but which ultimately influences the equilibrium channel form (after Parker, 1976).

Based on the feedback process, we can identify two features of an equilibrium produced this way, beyond the definitive equality of local transport capacity and sediment supply. First, the cut-bank at the bend apex must be critically stable at formative discharge. In fact, this may be an appropriate way of defining the formative discharge, at
least with respect to the channel planform. A reasonable (and testable) hypothesis claims that cut-banks are near the threshold of motion for bankfull flows in alluvial channels, which may be the most effective means of estimating bank strength for structurally complex stream banks. In any case, we argue that bank stability is a key feature of equilibrium, as have many previous researchers (e.g. Millar and Quick, 1993; Griffiths and Carson, 2000).

In comparison, the discharge responsible for bed surface texture or in-channel bedforms is probably different from that controlling the channel pattern. That is, channel morphology is best thought of as the result of a range of flows, each effecting different adjustments: we have been concerned herein only with the flows that initiate and maintain the channel planform.

The second point relates to the shear stress distribution. A channel formed in erodible material has some maximum limit to the variance in the shear stress that can be sustained without the bed failing, just as banks can withstand only a certain shear stress before bank retreat occurs. That is, there is some configuration at which the variance is maximized and beyond which the boundary will fail, thereby reducing the variance. In our framework, this is equivalent to maximizing $\beta$ by maximizing $Y_\alpha$, subject to the strength of the channel boundary. However, since $\beta$ for a specified slope – an increase in variance will produce an increase in $TC$, the system will tend to exhibit a maximum variance consistent with cross-sectional stability. If the variance is not maximized, then there is still scope for the positive feedback shown in Figure 4 to act on the system, causing further deformation and increasing $\beta$. Therefore the equilibrium channel configuration will be characterized by cut-banks at the apices that are at the threshold of movement and by a cross-sectional shape that exhibits the maximum variance that can be supported by the bed material.

However, since crossovers between two apices will have a lower $\beta$, the feedback mechanism described above implies that water surface slope must vary systematically downstream in order to maintain a steady-state mass flux throughout the system. Experimental observations from stream table experiments and from fixed bank physical models support this deduction.

The theory also predicts that the strength of the feedback mechanism falls off rapidly as excess shear stress increases. This is consistent with the observed time-to-equilibrium for laboratory experiments for a range of initial shear stresses, which proceed more quickly at lower shear stresses. The relation between relative variance, transport capacity and shear stress also has implications for the magnitude of the variation in the energy gradient for different systems. In streams that are always near threshold, which is true of most gravel-bed streams, the effect of variance on the transport capacity is relatively large, and there ought to be relatively large differences in the energy gradient over the pools and the riffles. As discussed above, we certainly see such differences in laboratory models of this kind of stream. However, in stream channels with very high dimensionless shear stress, such as sand-bed channels with cohesive banks, the relative effect of shear stress variance on the transport capacity will be much smaller and, presumably, so too will be the relative change in energy slope along the stream channel.

In all of the experiments reported by Eaton and Church (2004), the equilibrium channel form was the result of a reduction of the channel slope following the development of alternate bars. The equilibrium slope achieved is functionally determined by the sediment concentration, while other possible adjustments, such as changes in the resistance coefficients (e.g. Manning’s $n$), could not be identified. This slope minimizing behaviour is the product of the initiation and development of pools on alternating sides of the channel. In this way, a three-dimensional interaction amongst channel morphology, local hydraulics and sediment transport responsible for pool growth informs the one-dimensional properties of the fluvial system, and produces the observed SMB. SMB is consistent with the cross-scale linkage described by the feedback mechanism that occurs between the width-scale variables $Y_\alpha$ and $\beta$ and the length-scale variables $L^*$ and $S$.

Since SMB is similar to the behaviour predicted by the minimum slope hypothesis (and by extension, all analogous optimality criteria), the process–form interactions embodied in the feedback mechanisms suggest a physically based, rational interpretation of what such hypotheses actually represent. Regime models are one-dimensional, and since they cannot explicitly consider the transverse distribution of shear stress, they cannot distinguish which solution among the many possible solutions admitted by the model is actually stable in a three-dimensional fluvial system. That is, they cannot explicitly describe the self-forming action of the fluvial system. The selection of a single, stable solution is accomplished by application of an optimality criterion that facilitates the description of a three-dimensional reality by a one-dimensional model.

Acknowledgements

This work has benefited from comments made by Trevor Hoey, Gerald Nanson, Jim Pizzuto, Peter Wilcock and, especially, Rob Ferguson. We thankfully acknowledge funding provided by the Natural Sciences and Engineering Research Council of Canada through M. Church’s discovery grant and B. Eaton’s post-graduate scholarship.
References


