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MR2375070 (2009j:46173) 46L99 (47D03 47L30)
Livshits, L. [Livshits, Leo] (1-COLBY);
MacDonald, G. [MacDonald, Gordon Wilson] (3-PRIN-MS);
Marcoux, L. W. (3-WTRL-PM); **Radjavi, H.** (3-WTRL-PM)

A Kadison transitivity theorem for C^*-semigroups. (English summary)

J. Funct. Anal. **254** (2008), *no. 1,* 246–266.

The Kadison transitivity theorem for C^*-algebras asserts that a closed, self-adjoint, topologically transitive algebra of operators on a Hilbert space is actually strictly transitive. The authors show its analog for multiplicative semigroups: if a closed, homogeneous, self-adjoint, topologically transitive semigroup of operators acting on a separable Hilbert space contains a nonzero compact operator, then it is strictly transitive. Examples are also given to demonstrate that the result is the best possible.

Reviewed by *Roman Drnovšek*

References

1. K.R. Davidson, Nest Algebras, Pitman Res. Notes Math., vol. 191, Longman Scientific & Technical, 1988. MR0972978 (90f:47062)
2. R.V. Kadison, Irreducible operator algebras, Proc. Natl. Acad. Sci. USA 43 (1957) 273–276. MR0085484 (19,47e)
3. L. Livshits, G. MacDonald, H. Radjavi, Topologically transitive matrix semigroups, Operators and Matrices 1 (2) (2007) 165–179. MR2327578 (2008e:47098)
4. P.R. Halmos, A Hilbert Space Problem Book, second ed., Grad. Texts in Math., vol. 19, Springer-Verlag, 1982. MR0675952 (84e:47001)

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Citations
From References: 1
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MR2363973 (2008i:20074) 20M20 (15A30 15A42 47D03)
Bernik, Janez (SV-LJUBMP); **Drnovšek, Roman** (SV-LJUBMP);
Košir, Tomaž (SV-LJUBMP); **Livshits, Leo** (1-COLBY); **Mastnak, Mitja** (3-WTRL-PM);
Omladič, Matjaž (SV-LJUBMP); **Radjavi, Heydar** (3-WTRL-PM)

Approximate permutability of traces on semigroups of matrices. (English summary)

Oper. Matrices **1** (2007), *no. 4,* 455–467.

It is known that if trace is permutable on a semigroup $\mathcal{S}$ of complex matrices, i.e., $\operatorname{tr}(ABC) = \operatorname{tr}(BAC)$ for all $A, B, C \in \mathcal{S}$, then $\mathcal{S}$ is triangularizable. The authors study an approximate version of this condition: $|\operatorname{tr}(ABC - BAC)| \le \varepsilon\rho(A)\rho(B)\rho(C)$ for all $A, B, C \in \mathcal{S}$, where ρ is the spectral radius. They show that this condition with $\varepsilon < 3$ yields commutativity for compact groups and triangularizability for certain groups including connected ones. For general semigroups additional assumptions are needed. Moreover, the authors also show that any property on semigroups of matrices that satisfies certain pretriangularizing conditions yields similar conclusions. Infinite-dimensional cases are also discussed.

Reviewed by *Wenxue Huang*

References

1. J. Bernik, R. Guralnick and M. Mastnak, *Reduction theorems for groups of matrices,* Linear Algebra Appl. **383** (2004), 119–126. MR2073898 (2005f:20080)
2. J. Bernik and H. Radjavi, *How small can nonzero commutators be?,* Indiana Univ. Math. J. **54** (2005), 309–320. MR2136811
3. H. Radjavi and P. Rosenthal, *From local to global triangularization,* J. Funct. Anal. **147** (1997), 443–456. MR1454489 (98j:47010)
4. H. Radjavi and P. Rosenthal, *Simultaneous triangularization,* Springer, New York, 2000. MR1736065 (2001e:47001)
5. O. J. Schmidt, *Ueber Gruppen deren sämtliche Teiler spezielle Gruppen sind,* Math. Sbornik **31** (1924) 366–372.
6. Y. V. Turovskiĭ, *Volterra semigroups have invariant subspaces,* J. Funct. Anal. **162** (1999), 313–322. MR1682061 (2000d:47017)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
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Citations
From References: 1
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MR2327578 (2008e:47098) 47D03 (15A30 47A16)

Livshits, Leo (1-COLBY); **MacDonald, Gordon [MacDonald, Gordon Wilson]** (3-PRIN); **Radjavi, Heydar** (3-WTRL-PM)

Topologically transitive matrix semigroups. (English summary)

Oper. Matrices **1** (2007), *no. 2,* 165–179.

The authors of the paper under review study multiplicative semigroups of $n \times n$ matrices (real or complex) and find conditions under which a topologically transitive semigroup must be transitive.

Reviewed by *V. V. Peller*

References

1. R. Drnovsek, L. Livshits, G. MacDonald, B. Mathes, H. Radjavi and P. Semrl, *On transitive linear semigroups,* Linear Algebra Appl., **305,** 2000, 67–86. MR1733794 (2000i:20100)
2. F. Kalscheuer, *Die bestimmung aller stetigen fastkörper über dem körper der rellen zahlen als grundkörper,* Abh. Math. Sem. Hansische Univ. **13,** 1940, 413–435. MR0001968 (1,328a)
3. H. Radjavi and P. Rosenthal, *Simultaneous Triangularization,* Universitext, Springer-Verlag, New York, 2000. MR1736065 (2001e:47001)
4. G. Hardy and E. Wright, *An introduction to the theory of numbers,* Fifth Edition, Oxford at the Clarendon Press, 1979. MR0568909 (81i:10002)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
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Citations
From References: 4
From Reviews: 1

MR2255476 (2007k:15025) 15A30 (47A15 47A16 47L05)

Semitransitivity Working Group at LAW'05

Semitransitive subspaces of matrices. (English summary)

The Working Group was coordinated by Heydar Radjavi and its members were: Janez Bernik, Roman Drnovšek, Don Hadwin, Ali Jafarian, Damjana Kokol Bukovšek, Tomaž Košir, Marjeta Kramar Fijavž, Thomas Laffey, Leo Livshits, Mitja Mastnak, Roy Meshulam, Vladimir Müller, Eric Nordgren, Jan Okniński, Matjaž Omladič, Ahmed Sourour and Richard Timoney.

Electron. J. Linear Algebra **15** (2006), 225–238 (*electronic*).

A set of matrices $\mathcal{S} \subseteq M_n(\mathbb{F})$ is said to be semitransitive if for any two nonzero vectors $x, y \in \mathbb{F}^n$, there exists a matrix $A \in \mathcal{S}$ such that either $Ax = y$ or $Ay = x$. This paper investigates the structure of semitransitive linear subspaces $\mathcal{L}$ of $M_n(\mathbb{F})$. Some results of note: such an $\mathcal{L}$ has a cyclic vector, and such an $\mathcal{L}$ contains an invertible matrix when $|\mathbb{F}| \geq n$. The paper also investigates minimal semitransitive linear subspaces and triangularizable semitransitive linear subspaces. Some results of note in these cases: there exist minimal such $\mathcal{L}$ which are intransitive, and every such triangularizable $\mathcal{L}$ contains a nonzero nilpotent.

Reviewed by *Gordon Wilson MacDonald*

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Citations
From References: 0
From Reviews: 2

MR2110258 (2006b:47122) 47L10 (46J10 46L99)
Livshits, Leo (1-COLBY-CS); **Ong, Sing-Cheong** (1-CMI); **Wang, Sheng-Wang**
Schur algebras over function algebras. (English summary)
Houston J. Math. **30** (2004), *no. 4,* 1195–1217 (*electronic*).

In [L. Livshits, S.-C. Ong and S. W. Wang, Integral Equations Operator Theory **41** (2001), no. 3, 343–359; MR1853675 (2002f:46087)] the authors of this paper extended the classical theorems of duality among the compact operators $\mathcal{K}(H)$, the trace class operators $(\mathcal{T}(H))$ and the bounded operators $\mathcal{B}(H)$ to the setting where $\mathcal{B}(H)$ is replaced by the absolute Schur algebra $\mathcal{A}^r(\mathbb{C})$ of matrices $A=[a_{jk}]$, over the complex field $\mathbb{C}$.

In this paper the authors further extend the above results to matrices over a commutative C^*-algebra $\mathfrak{B}$. The main theorems are the following.

Theorem 3.4: $\mathcal{A}^r$ is a Banach algebra under the Schur product and the norm $\|\cdot\|_r$, where $\mathcal{A}^r$ is the space of matrices $A=[a_{jk}]$ with $a_{jk}\in\mathfrak{B}$ such that the matrix $[|a_{jk}|^r]$ defines a bounded operator on $l^2(\mathfrak{B})$ and the norm is given by $\|A\|_r=\|[|a_{jk}|^r]\|^{\frac{1}{r}}$.

Theorem 4.5: $(\mathcal{K}^r)^\#$—the dual of $\mathcal{K}^r$—is isometrically isomorphic to $\mathcal{M}(\mathcal{A}^r,(\mathcal{AS}))$, where $\mathcal{K}^r$ denotes the closure in $\mathcal{A}^r$ of the set of matrices with finitely many nonzero entries in $\mathfrak{B}$, $(\mathcal{AS})$ denotes the space of matrices over the complex field with absolutely summable entries, and $\mathcal{M}(\mathcal{A}^r,(\mathcal{AS}))$ is the space of matrices $\varphi=[\varphi_{jk}]$ with entries in the dual of $\mathfrak{B}$ such that $[\varphi_{jk}(a_{jk})]\in(\mathcal{AS})$.

Theorem 5.1: $\mathcal{K}^r$ cannot be the dual of a Banach space.

Theorem 5.3: Assume that the maximal ideal space of $\mathfrak{B}$ is hyperstonian (so that $\mathfrak{B}=C(X)$ has a predual). Then $\mathcal{A}^r$ is the dual of $\mathcal{M}(\mathcal{A}^r,(\mathcal{AS}))_\#$, where $\mathcal{M}(\mathcal{A}^r,(\mathcal{AS}))_\#$ is the linear space consisting of the matrices $B=[b_{jk}]$ with entries in $L^1(X,\mathcal{M})$ such that

$$\Psi_B(A)=\left[\int_X a_{jk}(t)b_{jk}(t)d\mu(t)\right]\in(\mathcal{AS}),\ \forall A\in\mathcal{A}^r.$$

Reviewed by *Shijie Lu*

References

1. P. Chaisuriya, S.-C. Ong, Absolute Schur algebras and unbounded matrices, *SIAM J. Matrix Anal. Appl.* 20 (1999), 596–605. MR1685044 (2000g:15023)

2. N. Dunford, J. T. Schwartz, *Linear Operators,: Part I* Interscience Publishers, Inc., New York, 1957. MR0188745 (32 #6181)
3. L. Livshits, S.-C. Ong and S. W. Wang, Banach Space Duality in Absolute Schur Algebras, *Int. Eq. & Op. Th.* 41 (2001) 343–359. MR1853675 (2002f:46087)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
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Citations
From References: 0
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MR2073902 (2005c:20102) 20M20 (15A30)

Livshits, Leo (1-COLBY)

On decomposability of periodic semigroups of non-negative matrices.

Linear Algebra Appl. **383** (2004), 163–174.

In this paper, the author proves the following result: Let r and n be two positive integers. There exists an indecomposable semigroup $\mathcal{S}$ having r as the smallest rank of matrices in $\mathcal{S}$ and satisfying the equation $X^{n+1} = X$ if and only if every prime divisor of r divides n. The author observes that this answers a question posed by H. Radjvi in 1999 as to whether there is a common zero entry in a semigroup of non-negative matrices satisfying the equation $A^{n+1} = A$. The author derives some further combinatorial consequences of his result.

Reviewed by *K. S. S. Nambooripad*

References

1. A. Berman, R.J. Plemmons, Nonnegative Matrices in the Mathematical Sciences, in: Computer Science and Applied Mathematics, Academic Press [Harcourt Brace Jovanovich Publishers], New York, 1979. MR0544666 (82b:15013)
2. J.M. Howie, Fundamentals of Semigroup Theory, The Clarendon Press, Oxford University Press, Oxford Science Publications, New York, 1995. MR1455373 (98e:20059)
3. A. Marwaha, Decomposability and structure of non-negative bands in $Mn(R)$, Linear Algebra Appl. 291 (1999) 63–82. MR1685625 (2000b:47012)
4. H. Radjavi, The Perron–Frobenius theorem revisited, Positivity 3 (4) (1999) 317–331. MR1721557 (2000g:47047)
5. H. Radjavi, P. Rosenthal, Simultaneous Triangularization, Springer-Verlag, New York, 2000 (Universitext). MR1736065 (2001e:47001)
6. H. Wielandt, Finite Permutation Groups, Academic Press, New York, 1964 (Translated from the German by R. Bercov). MR0183775 (32 #1252)

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AMERICAN MATHEMATICAL SOCIETY
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Article

Citations
From References: 1
From Reviews: 0

MR2018595 (2004h:47009) 47A15 (47B65 47D03)
Livshits, L. [Livshits, Leonya] (1-COLBY);
Macdonald, G. [MacDonald, Gordon Wilson] (3-PRIN);
Mathes, B. [Mathes, D. Benjamin] (1-COLBY); **Radjavi, H.** (3-DLHS-MSC)
Operator semigroups for which reducibility implies decomposability. (English summary)
Positivity **7** (2003), *no. 3,* 195–202.

The authors study invariant subspaces for multiplicative semigroups of operators on L^p-spaces ($1 \leq p < \infty$) which are closed under multiplication on the left or right by bounded multiplication operators. They show that such a semigroup has a nontrivial invariant band provided it has a nontrivial invariant closed subspace. As an application they give a simple proof of the known theorem that a semigroup of positive compact quasinilpotent operators on a L^p-space has a nontrivial invariant band. This result was simultaneously given in the monograph of H. Radjavi and P. Rosenthal [*Simultaneous triangularization*, Springer, New York, 2000; MR1736065 (2001e:47001)], and for the case of general Banach lattices in the paper by the reviewer [Integral Equations Operator Theory **39** (2001), no. 3, 253–266; MR1818060 (2001m:47012)].

Reviewed by *Roman Drnovšek*

References

1. Y.A. Abramovich, C.D. Aliprantis and O. Burkinshaw: On the spectral radius of positive operators, *Math. Z.* 211 (1992), 593–607.; Corrigendum: *Math. Z.* 215 (1994), 167–168. MR1191098 (94a:47064)
2. Y.A. Abramovich, C.D. Aliprantis and O. Burkinshaw: Invariant subspaces of operators on ℓ_p spaces, *J. Funct. Anal.* 115 (1993), 418–424. MR1234398 (94h:47009)
3. Y.A. Abramovich, C.D. Aliprantis and O. Burkinshaw: Invariant subspaces for positive operators, *J. Funct. Anal.* 124 (1994), 95–111. MR1284604 (95e:47006)
4. T. Ando: Positive operators in semi-ordered linear spaces, *J. Fac. Sci. Hokkaido Univ., Ser.* 1 13 (1957), 214–228. MR0092127 (19,1067b)
5. R. Drnovšek: Common invariant subspaces for collections of operators, to appear in *Journal of Integral Equations and Operator Theory*. cf. MR 2001m:47012
6. P. Halmos: *A HilbertSpace Problem Book,* Springer, New York, 1982. MR0675952 (84e:47001)
7. H.J. Krieger: Beitrage zur Theorie positiver Operatoren, Schriftenreihe der Institute fur Math. Reihe A, Heft 6; Akad.-Verlag, Berlin, 1969. MR0415403 (54 #3491)

8. G. MacDonald: Invariant subspaces for Bishop-type operators, *J. Funct. Anal.* 91(2) (1990) 287–311. MR1058974 (91f:47049)
9. B. de Pagter: Irreducible compact operators, *Math. Z.* 192 (1986) 149–153. MR0835399 (87d:47052)
10. H. Radjavi and P. Rosenthal: *Simultaneous Triangularization,* Universitext, Springer, New York, 2000. MR1736065 (2001e:47001)
11. H. Schaefer: *Banach lattices and positive operators,* Springer, New York, 1974. MR0423039 (54 #11023)
12. Y.V. Turovskiĭ: Volterra operators have invariant subspaces, *J. Funct. Anal.* 162(2) (1999) 313–322. MR1682061 (2000d:47017)
13. W.A.J. Luxemburg and A.C. Zaanen: *Riesz Spaces I,* North-Holland, Amsterdam, 1971. MR0511676 (58 #23483)
14. A.C. Zaanen: *Riesz Spaces II,* North-Holland, Amsterdam, 1983. MR0704021 (86b:46001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
MathSciNet® Mathematical Reviews on the Web

Article

Citations
From References: 2
From Reviews: 0

MR1987503 (2004c:15028) 15A30 (15A03 47D03)
Livshits, L. [Livshits, Leo] (1-COLBY);
MacDonald, G. [MacDonald, Gordon Wilson] (3-PRIN);
Mathes, B. [Mathes, D. Benjamin] (1-COLBY-CS); **Okniński, J.** (PL-WASW);
Radjavi, H. (3-DLHS-MSC)

Matrix semigroups with commutable rank. (English summary)

Semigroup Forum **67** (2003), *no. 2,* 288–316.

A matrix semigroup S (over the complex numbers) has commutable rank if $\operatorname{rank}(AB) = \operatorname{rank}(BA)$ for any A, B in S. S has permutable rank if $\operatorname{rank}(A_1A_2\cdots A_n) = \operatorname{rank}(A_{i_1}A_{i_2}\cdots A_{i_n})$ for any permutation $i_1, i_2, \ldots, i_n$ of $1, 2, \ldots, n$ and any matrices $A_1, A_2, \ldots, A_n$ in S. Partial results are given for when commutability of rank implies permutability, and an example is given of a commutable rank semigroup which does not have permutable rank. It is also shown that a commutable rank semigroup is a semilattice of component semigroups.

Reviewed by *Walter S. Sizer*

References

1. Anderson, F. W. and K. R. Fuller, "Rings and Categories of Modules", Vol. 13, Graduate Texts in Mathematics, second edition, Springer-Verlag, New York, 1992. MR1245487 (94i:16001)
2. Ando, T., *Totally positive matrices*, Linear Algebra Appl. **90** (1987), 165–219. MR0884118

(88b:15023)
3. Gantmacher, F. R. and M. G. Krein, *Sur les matrices completement non-negatives et oscillatoires*, Compositio Math. (1937), 445–476. MR1556987
4. Horn, R. A. and C. R. Johnson, "Matrix Analysis", Cambridge University Press, Cambridge, 1990. Corrected reprint of the 1985 original. MR1084815 (91i:15001)
5. Levitzki, J., *Uber nilpotente unterrringe*, Math. Ann. **105** (1931), 620–627. MR1512728
6. Livshits, L., G. MacDonald, B. Mathes, and H. Radjavi, *Reducible semigroups of idempotent operators*, J. Operator Theory **40**(1) (1998), 35–69. MR1642522 (99m:47004)
7. Okniński, J., *Semigroups of matrices*, in "Series in Algebra", Vol. 6, World Scientific Publishing Co. Inc., River Edge, NJ, 1998. MR1785162 (2001g:20076)
8. Prasolov, V. V., "Problems and Theorems in Linear Algebra", American Mathematical Society, Providence, RI, 1994. Translated from the Russian manuscript by D. A. Leĭtes. MR1277174 (95h:15002)
9. Radjavi, H. and P. Rosenthal, "Simultaneous Triangularization", Springer-Verlag, New York, 2000. Universitext. MR1736065 (2001e:47001)
10. Whitney, A. M., *A reduction theorem for totally positive matrices*, J. Analyse Math. **2** (1952), 88–92. MR0053173 (14,732c)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
MathSciNet® Mathematical Reviews on the Web

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Citations
From References: 6 From Reviews: 0

MR1885443 (2003j:47051) 47D03 (47A15 47B38 47B65)
Drnovšek, R. (SV-LJUBMP); **Kokol-Bukovšek, D.** (SV-LJUBMP);
Livshits, L. [Livshits, Leonya] (1-COLBY-CS);
MacDonald, G. [MacDonald, Gordon Wilson] (3-PRIN); **Omladič, M.** (SV-LJUBMP);
Radjavi, H. (3-DLHS-MSC)

An irreducible semigroup of non-negative square-zero operators. (English summary)

Integral Equations Operator Theory **42** (2002), *no. 4,* 449–460.

A (closed) subspace M of a Banach space $\mathcal{X}$ is invariant under a multiplicative semigroup $\mathcal{S}$ of bounded linear operators acting on $\mathcal{X}$ if $A(M) \subseteq M$ for every $A \in \mathcal{S}$. An operator semigroup is said to be irreducible if the only subspaces invariant under $\mathcal{S}$ are $M = \{0\}$ and $M = \mathcal{X}$. There are a number of results in the literature devoted to the existence of invariant subspaces for semigroups of operators, especially for semigroups of compact operators acting on Hilbert spaces. A recent exposition of the theory is given in the monograph of H. Radjavi and P. Rosenthal [*Simultaneous triangularization*, Springer, New York, 2000; MR1736065 (2001e:47001)]. The present paper makes important, interesting contributions to this literature in two ways: first by resolving an open question about the existence of irreducible semigroups of nilpotent operators of bounded order of nilpotency, and second by illustrating how some fundamental results about semigroups of compact operators cannot be extended, even under quite strong algebraic assumptions, to noncompact operators.

The authors initially consider nonnegative operators on the Banach space $L^p(X,\mu)$ of p-integrable complex-valued functions on a measure space (X,μ), where $p\in[1,\infty)$. (If one defines $f\geq 0$, for $f\in L^p(X,\mu)$, to mean $f(x)\geq 0$ for almost all $x\in X$, then a bounded linear operator A acting on $L^p(X,\mu)$ is said to be nonnegative if $Af\geq 0$ for every $f\geq 0$.) The main result of the paper, Theorem 2.7, is a construction of a multiplicative semigroup $\mathcal{S}$ of nonnegative operators acting on $L^p(X,\mu)$, where (X,μ) is a separable nonatomic σ-finite measure space, such that (i) $A^2=0$, for every $A\in\mathcal{S}$, and (ii) the linear span of $\{Ag\colon A\in\mathcal{S}\}$ is dense in $L^p(X,\mu)$ for every nonzero $g\in L^p(X,\mu)$. Put in the language of invariant subspaces, Theorem 2.7 establishes the existence of an irreducible semigroup of nonnegative square-zero operators.

A celebrated theorem of Y. V. Turovskiĭ [J. Funct. Anal. **162** (1999), no. 2, 313–322; MR1682061 (2000d:47017)] states that every semigroup of compact quasinilpotent operators on a Banach space has a nontrivial invariant subspace. A stronger algebraic assumption that one might impose on a semigroup $\mathcal{S}$ of quasinilpotent operators is that the operators be in fact nilpotent and, moreover, that there exist a positive integer k such that $A^k=0$ for all $A\in\mathcal{S}$. For such semigroups of nilpotents, it is natural to ask whether Turovskiĭ's theorem might hold without assuming that the operators be compact. The answer, however, is no, as Theorem 2.7 of the present paper demonstrates.

The paper also sheds light on the role of the measure space (X,μ) for questions about invariant subspaces. In the proof of the main result, for instance, the Lebesgue density theorem and a version of the fundamental Theorem of calculus for Lebesgue integrals have a key role. Furthermore, the assumption in Theorem 2.7 that (X,μ) be nonatomic is crucial, for a theorem of M. D. Choi et al. [Indiana Univ. Math. J. **42** (1993), no. 1, 15–25; MR1218704 (94e:47009)] implies that if (X,μ) has an atom and if $\mathcal{S}$ is a multiplicative semigroup of nonnegative quasinilpotent operators acting on $L^p(X,\mu)$, then $\mathcal{S}$ has a nontrivial invariant subspace.

Reviewed by *Douglas R. Farenick*

References

1. M.D. Choi, E.A. Nordgren, H. Radjavi, P. Rosenthal, Y. Zhong, *Triangularizing semigroups of quasinilpotent operators with non-negative entries*, Indiana Univ. Math. J. **42** (1993), 15–25. MR1218704 (94e:47009)
2. R. Drnovšek, *Triangularizing semigroups of positive operators on an atomic normed Riesz space*, Proc. Edinburgh Math. Soc., 43 (2000), 43–55. MR1744698 (2001i:47062)
3. R. Drnovšek, *Common invariant subspaces for collections of operators*, J. Integ. Eq. Operator Theory, to appear. MR1818060 (2001m:47012)
4. D. Hadwin, E. Nordgren, M. Radjabalipour, H. Radjavi and P. Rosenthal, *A nil algebra of bounded operators on Hilbert space with semisimple norm closure*, J. Integ. Eq. Operator Theory **9** (1986), 739–743. MR0860869 (87k:47104)
5. P. R. Halmos, *Measure Theory*, D. Van Nostrand Company, 1950. MR0033869 (11,504d)
6. H. Radjavi, P. Rosenthal, *Simultaneous Triangularization*, Universitext, Springer-Verlag, New

York, 2000. MR1736065 (2001e:47001)
7. W. Rudin, *Real And Complex Analysis*, McGraw-Hill, Singapore, 1986. MR0210528 (35 #1420)
8. Y. Turovskiĭ, *Volterra operators have invariant subspaces*, J. Funct. Anal. **162(2)** (1999), 313–322. MR1682061 (2000d:47017)
9. Y. Zhong, *Irreducible semigroups of functionally positive nilpotent operators*, Trans. Amer. Math. Soc. **347** (1995), 3093–3100. MR1264835 (95j:47008)

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AMERICAN MATHEMATICAL SOCIETY
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Citations
From References: 3
From Reviews: 0

MR1897153 (2003m:47071) 47D03 (15A30 20M20 47A15 47L35)
Livshits, L. [Livshits, Leonya] (1-COLBY);
MacDonald, G. [MacDonald, Gordon Wilson] (3-PRIN);
Mathes, B. [Mathes, D. Benjamin] (1-COLBY); **Radjavi, H.** (3-DLHS-MSC)
On band algebras. (English summary)
J. Operator Theory **46** (2001), *no. 3, suppl.,* 545–560.

The linear span of a band (a multiplicative semigroup of idempotent operators) is called a band algebra. The present paper investigates a variety of properties of band algebras, usually acting on complex Hilbert spaces.

An earlier paper of the authors [J. Operator Theory **40** (1998), no. 1, 35–69; MR1642522 (99m:47004)] demonstrates that every operator in a band algebra is algebraic. The present paper establishes the converse, namely that a Hilbert space operator is algebraic only if it belongs to some band algebra. A similar result holds for linear transformations acting on a vector space V over a field $\mathbb{F}$. Every linear transformation T in a band algebra acting on V is algebraic and the minimal annihilating polynomial of T splits over $\mathbb{F}$; conversely, an algebraic linear transformation T on V whose minimal annihilating polynomial of T splits over $\mathbb{F}$ lies in some band algebra acting on V.

There is no known characterisation of the subspace lattices that corresponds to the invariant-subspace lattices of operator bands. Toward such a description, the authors determine which nests $\mathcal{M}$ of closed subspaces of a Hilbert space do arise as the invariant-subspace lattice of a band algebra. They prove that a nest $\mathcal{M}$ is the invariant-subspace lattice of a band algebra if and only if each of the finite-dimensional atoms of $\mathcal{M}$ has dimension 1.

A multiplicative semigroup $\mathcal{S}$ of operators is reducible if it leaves a nontrivial nonzero closed subspace invariant; $\mathcal{S}$ is triangularisable if $\mathcal{S}$ leaves a maximal chain of closed subspaces invariant. A striking theorem of R. Drnovšek [Studia Math. **125** (1997), no. 1, 97–99; MR1455626

(98e:47011)] shows that on every infinite-dimensional Hilbert space there is an irreducible band. Thus, the authors of the present paper are led to formulate sufficient conditions for reducibility and triangularisability of band algebras. For example, every band $\mathcal{S}$ satisfies $[A,B]^3=0$, for all $A,B\in\mathcal{S}$, where $[X,Y]$ is the commutator $[X,Y]=XY-YX$. The present paper demonstrates that if a band $\mathcal{S}$ contains a nonzero operator A different from the identity and such that $[A,B]^2=0$ for all $B\in\mathcal{S}$, then $\mathcal{S}$ is reducible. Several additional algebraic results bearing on reducibility are also established.

Reviewed by *Douglas R. Farenick*

References

1. A.H. Clifford, G.B. Preston, *The Algebraic Theory of Semigroups*, Math. Surveys Monogr., vol. 7, Amer. Math. Soc., RI 1961. MR0132791 (24 #A2627)
2. R. Drnovšek, An irreducible semigroup of idempotents, *Studia Math*. **125**(1997), 97–99. MR1455626 (98e:47011)
3. P. Fillmore, G. MacDonald, M. Radjabalipour, H. Radjavi, Towards a classification of maximal unicellular bands, *Semigroup Forum* **49**(1994), 195–215. MR1283128 (95f:47059)
4. J.A. Green, D. Rees, On semigroups in which $x^r=x$, *Proc. Cambridge Philos. Soc*. **48**(1952), 35–40. MR0046353 (13,720c)
5. D. Hadwin, Radjavi's trace condition for triangularizability, *J. Algebra* **109**(1987), 184–192. MR0898345 (88g:15010)
6. J.M. Howie, *Fundamentals of Semigroup Theory*, Oxford Sci. Publ., The Clarendon Press, Oxford Univ. Press, New York 1995. MR1455373 (98e:20059)
7. I. Kaplansky, *Fields and Rings*, Univ. Chicago Press, Chicago 1972. MR0349646 (50 #2139)
8. M. Lambrou, W.E. Longstaff, H. Radjavi, Spectral conditions and reducibility of operator semigroups, *Indiana Univ. Math. J*. **41**(1992), 449–464. MR1183353 (94a:47069)
9. L. Livshits, G. MacDonald, B. Mathes, H. Radjavi, Reducible semigroups of idempotent operators, *J. Operator Theory* **40**(1998), 35–69. MR1642522 (99m:47004)
10. W.D. Munn, The Jacobson radical of a band ring, *Math. Proc. Cambridge Philos. Soc*. **105**(1989), 277–283. MR0974983 (89j:16034)
11. J.D. Newburgh, The variation of spectra, *Duke Math J*. **18**(1951), 165–176. MR0051441 (14,481b)
12. H. Radjavi, On the reduction and triangularization of semigroups of operators, *J. Operator Theory* **13**(1985), 63–71. MR0768302 (86c:47056)
13. H. Radjavi, Sublinearity and other spectral conditions on a semigroup, *Canad. J. Math*. **52**(2000), 197–224. MR1745707 (2001d:47057)
14. H. Radjavi, P. Rosenthal, *Simultaneous Triangularization*, Universitext, Springer-Verlag, New York 2000. MR1736065 (2001e:47001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
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Citations
From References: 2
From Reviews: 3

MR1853675 (2002f:46087) 46H35 (46A20 47B10 47L50)
Livshits, Leo (1-COLBY-CS); **Ong, S.-C.** (1-CMI);
Wang, S.-W. [Wang, Sheng Wang] (PRC-NAN)
Banach space duality of absolute Schur algebras. (English summary)
Integral Equations Operator Theory **41** (2001), *no. 3,* 343–359.

The main result of this paper is to show that the standard duality triple $\{\mathcal{K}, \mathcal{C}, \mathcal{B}(l^2)\}$ is an operator analogue of the duality triple $\{c_0, l^1, l^\infty\}$.

Given two matrices $A = [a_{jk}]$ and $B = [b_{jk}]$, the Schur product of A and B is $A \bullet B = [a_{jk}b_{jk}]$. The absolute Schur rth power of A is the matrix $A^{[r]} = [|a_{jk}|^r]$, $\|\cdot\|$ is the operator norm on the Hilbert sequence space l^2 and $\|A\|_r = \|A^{[r]}\|^{1/r}$ is a norm. $\mathcal{A}^r$ denotes the class of matrices A such that $A^{[r]}$ defines a bounded linear operator on l^2; then $\mathcal{A}^r$ is a Banach algebra under the Schur product operation and the norm $\|\cdot\|_r$.

Let $\mathcal{K}$ be the set of all compacts in $\mathcal{B}(l^2)$, $\mathcal{C}^p$ be the Schatten p-classes, $\mathcal{C}^1$ be the trace class, $(\mathcal{A}S)$ be the set of all absolutely summable matrices $A = [a_{jk}]$ such that $\sum_{j,k} |a_{jk}| < \infty$. Let $\mathcal{M}(\mathcal{A}^r, (\mathcal{A}S))$ denote the linear space of all matrices B such that $A \bullet B \in (\mathcal{A}S)$ whenever $\mathcal{A} \in \mathcal{A}^r$.

Define $\|B\|_{\mathcal{M}(\mathcal{A}^r,(\mathcal{A}S))} = \|\Psi_B\|_{\mathcal{B}(\mathcal{A}^\triangledown,(\mathcal{A}S))}$, where $\Psi_B(A) = A \bullet B$ defines a bounded operator from the Banach space $(\mathcal{A}^r, \|\ \|_r)$ to the Banach space $((\mathcal{A}S), \|\ \|_{(\mathcal{A}S)})$.

The authors finally identify the Banach space dual of $\mathcal{K}^r$ and the Banach space predual of $\mathcal{A}^r$ with $\mathcal{M}(\mathcal{A}^r, (\mathcal{A}S))$. Thus the duality progression of the Banach algebra $\{\mathcal{K}^r, \mathcal{M}(\mathcal{A}^r, (\mathcal{A}S)), \mathcal{A}^r\}$ imitates the duality progression $\{\mathcal{K}, \mathcal{C}^1, \mathcal{B}(l^2)\}$, and the classical sequential analogue $\{c_0, l^1, l^\infty\}$.

Reviewed by *Shijie Lu*

References

1. Pachara Chaisuriya and S.-C. Ong, Absolute Schur algebras and unbounded matrices, *SIAM J. Matrix Anal. Appl.* 20 (1999), 596–605. MR1685044 (2000g:15023)
2. I. C. Gohberg and M. G. Krein, *Introduction to the Theory of Linear Nonselfadjoint Operators*, AMS Translations of Mathematical Monographs Vol. 18, 1969. MR0246142 (39 #7447)
3. E. Nordgren, M. Radjabalipour, H. Radjavi and P. Rosenthal, On invariant operator ranges, *Trans. Amer. Math. Soc.* 251 (1979), 389–398. MR0531986 (81c:47010)
4. V.I. Paulsen, S.C. Power, and R. Smith, Schur product and matrix completions, *J. Funct. Anal.* 85 (1989), 151–178. MR1005860 (90j:46051)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
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Citations
From References: 0
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MR1822227 (2002d:20102) 20M20 (15A30 47S10)
Livshits, Leo (1-COLBY); **MacDonald, Gordon [MacDonald, Gordon Wilson]** (3-PRIN)
n-transitivity and the complementation property. (English summary)
Linear Algebra Appl. **329** (2001), *no. 1-3,* 157–169.

The authors study the transitivity of certain semigroups of matrices over arbitrary fields. The rank of a semigroup is defined to be the maximum rank of the elements of the semigroup. A semigroup is said to be n-transitive if every set of n linearly independent vectors is taken everywhere by the semigroup. The minimal semigroups of rank 1 that are transitive (i.e., 1-transitive) were characterized in an earlier paper of the authors and co-authors [R. Drnovšek et al., Linear Algebra Appl. **305** (2000), no. 1-3, 67–86; MR1733794 (2000i:20100)]. In the paper under review, the problem of characterizing the minimal n-transitive semigroups of rank n is reformulated, and a number of partial results are obtained.

Reviewed by *P. Rosenthal*

References

1. R. Drnovšek, L. Livshits, G. MacDonald, B. Mathes, H. Radjavi, P. Šemrl, On transitive linear semigroups, Linear Algebra Appl. 305 (2000) 67–86. MR1733794 (2000i:20100)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
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Citations
From References: 1
From Reviews: 0

MR1784874 (2001e:20062) 20M20
Livshits, L. [Livshits, Leonya] (1-COLBY-CS);
Macdonald, G. [MacDonald, Gordon Wilson] (3-PRIN); **Radjavi, H.** (3-DLHS-MSC)
Cone-transitive matrix semigroups. (English summary)
Linear and Multilinear Algebra **47** (2000), *no. 4,* 313–350.

Summary: “Semigroups of matrices (over an ordered field) with nonnegative entries are considered. A complete characterization is obtained for the semigroups that are minimal transitive on the positive (or nonnegative) cone of the underlying vector space. Consequently, an explicit form is derived for the semigroups that are sharply transitive on the cone.”

References

1. Berman, A. and Plemmons, R. J., *Nonnegative matrices in the mathematical sciences*. Academic Press (Harcourt Brace Jovanovich Publishers), New York, 1979. Computer Science and Applied Mathematics. MR0544666 (82b:15013)
2. Radjavi, H. (1999). The Perron–Frobenius theorem revisited. *Positivity,* **3**(4), 317–331. MR1721557 (2000g:47047)
3. Radjavi, H. and Rosenthal, P., *Simultaneous Triangularization*. Springer-Verlag, New York, 2000, Universitext. MR1736065 (2001e:47001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
MathSciNet® Mathematical Reviews on the Web

Article

Citations
From References: 2
From Reviews: 0

MR1763047 (2001c:47045) 47D03 (47A15)

Drnovšek, Roman (SV-LJUBMP); **Livshits, Leo** (1-COLBY-CS); **MacDonald, Gordon W.** (3-PRIN); **Mathes, Ben [Mathes, D. Benjamin]** (1-COLBY-CS); **Radjavi, Heydar** (3-DLHS-MSC); **Šemrl, Peter** (SV-LJUBMP)

On operator bands. (English summary)

Studia Math. **139** (2000), *no. 1,* 91–100.

Summary: "A multiplicative semigroup of idempotent operators is called an operator band. We prove that for each $K > 1$ there exists an irreducible operator band on the Hilbert space l^2 which is norm-bounded by K. This implies that there exists an irreducible operator band on a Banach space such that each member has operator norm equal to 1. Given a positive integer r, we introduce a notion of weak r-transitivity of a set of bounded operators on a Banach space. We construct an operator band on l^2 that is weakly r-transitive and is not weakly $(r+1)$-transitive. We also study operator bands $\mathcal{S}$ satisfying a polynomial identity $p(A,B) = 0$ for all non-zero $A, B \in \mathcal{S}$, where p is a given polynomial in two noncommuting variables. It turns out that the polynomial $p(A,B) = (AB - BA)^2$ has a special role in these considerations."

References

1. J. B. Conway, *A Course in Functional Analysis*, Springer, 1990. MR1070713 (91e:46001)
2. R. Drnovšek, *An irreducible semigroup of idempotents*, Studia Math. 125 (1997), 97–99. MR1455626 (98e:47011)
3. P. Fillmore, G. W. MacDonald, M. Radjabalipour and H. Radjavi, *Principal-ideal bands*, Semigroup Forum 59 (1999), 362–373. MR1847651 (2002e:20136)
4. J. A. Green and D. Rees, *On semigroups in which $x^r = x$*, Proc. Cambridge Philos. Soc. 48

(1952), 35–40. MR0046353 (13,720c)

5. L. Livshits, G. W. MacDonald, B. Mathes and H. Radjavi, *Reducible semigroups of idempotent operators*, J. Operator Theory 40 (1998), 35–69. MR1642522 (99m:47004)
6. L. Livshits, G. W. MacDonald, B. Mathes, H. Radjavi, *On band algebras*, ibid., to appear. cf. MR 2003m:47071
7. M. Petrich, *Lectures in Semigroups*, Akademie-Verlag, Berlin, and Wiley, London, 1977. MR0447437 (56 #5749)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
MathSciNet® Mathematical Reviews on the Web

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Citations
From References: 4
From Reviews: 1

MR1733794 (2000i:20100) 20M20 (15A04 47A15 47D03)

Drnovšek, R. (SV-LJUBMP); **Livshits, L. [Livshits, Leonya]** (1-COLBY-CS);
MacDonald, G. [MacDonald, Gordon Wilson] (3-PRIN);
Mathes, B. [Mathes, D. Benjamin] (1-COLBY-CS); **Radjavi, H.** (3-DLHS-MSC);
Šemrl, P. (SV-LJUBMP)

On transitive linear semigroups. (English summary)

Linear Algebra Appl. **305** (2000), *no. 1-3,* 67–86.

$\mathbf{M}_n(\mathbf{F})$ denotes the semigroup of all $n \times n$ matrices over a field $\mathbf{F}$. A subsemigroup $\mathcal{S}$ of $\mathbf{M}_n(\mathbf{F})$ is transitive if $\{T(x)\colon T \in \mathcal{S}\} = \mathbf{F}^n$ for all nonzero $x \in \mathbf{F}^n$. A transitive subsemigroup of $\mathbf{M}_n(\mathbf{F})$ is said to be left t-simple if it contains no proper transitive left ideals. In the main result of the paper, the authors characterize the transitive left t-simple subsemigroups of $\mathbf{M}_n(\mathbf{F})$.

Reviewed by *K. D. Magill, Jr.*

References

1. P.A. Fillmore, J.P. Williams. On operator ranges. Adv. in Math. 7 (1971) 254–281. MR0293441 (45 #2518)
2. F. Kalscheuer, Die bestimmung aller stetigen fastkörper über dem körper der reellen zahlen als grundkörper. Abh. Math. Sem. Hansische Univ. 13 (1940) 413–435. MR0001968 (1,328a)
3. H. Radjavi, Invariant subspaces and spectral conditions on operator semigroups, in: Linear Operators (Warsaw, 1994), Polish Acad. Sci., Warsaw, 1997, pp. 287–296. MR1457013 (98h:47010)
4. H. Radjavi, P. Rosenthal, Invariant Subspaces, Springer, New York, 1973. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 77. MR0367682 (51 #3924)
5. H. Zassenhaus, Über endliche fastkörper. Abh. Math. Sem. Hansische Univ. 11 (1936) 187–220.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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AMERICAN MATHEMATICAL SOCIETY
MathSciNet® Mathematical Reviews on the Web
Article

Citations
From References: 0
From Reviews: 0

MR1704128 15A04 (47D03)
Livshits, Leo (1-COLBY)
Transitive linear semigroups. (English, Slovenian summaries)
The Second Meeting on Linear Algebra (Slovenian) (Bled, 1999).
Obzornik Mat. Fiz. **46** (1999), *no. 2,* 53–56.

{This item will not be reviewed individually.}
{For the entire collection see MR1704124 (2000c:47002)}

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MathSciNet® Mathematical Reviews on the Web
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Citations
From References: 9
From Reviews: 1

MR1642522 (99m:47004) 47A15 (20M99 47D03)
Livshits, L. [Livshits, Leonya] (1-COLBY-CS);
MacDonald, G. [MacDonald, Gordon Wilson] (3-PRIN);
Mathes, B. [Mathes, D. Benjamin] (1-COLBY-CS); **Radjavi, H.** (3-DLHS-MSC)
Reducible semigroups of idempotent operators. (English summary)
J. Operator Theory **40** (1998), *no. 1,* 35–69.

This paper is concerned with the existence of common invariant subspaces for semigroups of idempotent operators. In finite dimensions, every such semigroup is simultaneously triangularizable. The question of existence of even one nontrivial common invariant subspace (i.e. reducibility) is still open in infinite dimensions. This research contributes to the larger effort to find convenient necessary and sufficient conditions for reducibility of semigroups within various classes of operators [cf. H. Radjavi, J. Operator Theory **13** (1985), no. 1, 63–71; MR0768302 (86c:47056)].

The approach in this paper is from an algebraic point of view. Connections are exploited between the operator structure and the component structure of equivalence classes of idempotents in the semigroup (for the equivalence relation given by: a is equivalent to b if $aba = a$ and $bab = b$). The paper contains a wealth of results on this aspect of the algebraic structure, in general vector space settings as well as in the Hilbert space setting. One such result is that semigroups (of idempotent operators) with finitely many components are always reducible (with special block-upper triangular structure). A powerful tool used to extend results to the unrestricted number of components case is the existence (in the finitely many components case) of a uniformly bounded non-negative integer valued "faithful" trace which extends linearly to a trace on the algebra generated by the semigroup. One main result is that, if the (non-closed) span of a semigroup of idempotent operators on a Hilbert space contains a compact operator, then the semigroup is reducible.

Reviewed by *Cecelia Laurie*

References

1. B. Aupetit, *Propriétés spectrales des algèbres de Banach*, Lecture Notes in Math., vol. 735, Springer, Berlin 1979. MR0549769 (81i:46055)
2. P. Fillmore, G. MacDonald, M. Radjabalipour, H. Radjavi, Towards a classification of maximal unicellular bands, *Semigroup Forum* **49**(1994), 195–215. MR1283128 (95f:47059)
3. J.A. Green, D. Rees, On semigroups in which $x^r = x$, *Proc. Cambridge Philos. Soc.* **48**(1952), 35–40. MR0046353 (13,720c)
4. A. Katavolos, H. Radjavi, Simultaneous triangularization of operators on a Banach space, *J. London Math. Soc. (2)* **41**(1990), 547–554. MR1072047 (91i:47058)
5. M. Petric, *Lectures in Semigroups*, Academie-Verlag, John Wiley & Sons Ltd., Berlin 1977. MR0447437 (56 #5749)
6. H. Radjavi, On the reduction and triangularization of semigroups of operators, *J. Operator Theory* **13**(1985), 63–71. MR0768302 (86c:47056)
7. H. Radjavi, P. Rosenthal, *Invariant Subspaces*, Springer-Verlag, New York 1973. MR0367682 (51 #3924)
8. H. Radjavi, P. Rosenthal, On transitive and reductive operator algebras, *Math. Ann.* **209**(1974), 43–56. MR0350453 (50 #2945)

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MR1361566 (96k:47061) 47B49 (47A05)

Livshits, Leo (1-COLBY-CS)

Continuity of Schur block-multiplication maps with respect to various topologies. (English summary)

J. Operator Theory **34** (1995), *no. 1,* 17–56.

Write M_∞ for the space of (infinite) complex matrices and BM_∞ for the set of matrices corresponding to bounded linear operators on the Hilbert space l^2. The Schur product is defined on M_∞ by multiplying matrices entrywise. Suppose $A \in M_\infty$ is a Schur multiplier in the sense that left Schur multiplication by A induces a linear transformation ${}_A\Psi$ from BM_∞ into itself. The final corollary of the paper states that ${}_A\Psi$ is strong–strong continuous (i.e. continuous when the initial and final spaces are both equipped with the strong operator topology) if and only if it is strong–weak continuous if and only if it is weak–weak continuous if and only if A has finite rank.

The body of the paper deals with the more general setting of block matrices equipped with Schur-block multiplication. Continuity of ${}_A\Psi$ is characterized in a variety of situations involving the weak operator, strong operator, and norm topologies. It is still true that strong–weak and weak–weak continuity coincide, but their relation to strong–strong continuity is more delicate.

Reviewed by *Edward Azoff*

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Citations
From References: 4
From Reviews: 2

MR1332920 (96d:15040) 15A60

Livshits, Leo (3-BISH)

A note on 0-1 Schur multipliers. (English summary)

Linear Algebra Appl. **222** (1995), 15–22.

Summary: "We answer a question of Q. Stout about the role of the triangular truncation in constructing 0-1 matrices that are not Schur multipliers. We also demonstrate that the triangular truncation on $\mathcal{M}_2$ has the third smallest norm (after 0 and 1) that any map induced by a 0-1 Schur multiplier can have."

Reviewed by *Sing Cheong Ong*

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MR1310539 (96c:47007) 47A30 (15A60 46L05)

Livshits, Leo (3-BISH)

Block-matrix generalizations of infinite-dimensional Schur products and Schur multipliers. (English summary)

Linear and Multilinear Algebra **38** (1994), *no. 1-2,* 59–78.

When a is a natural number or $= \infty$, M_a denotes the space of $a \times a$ complex matrices. For such a, b the space of $a \times a$ matrices $(A[i,j])_{1\leq i,j\leq a}$ with entries $A[i,j]$ in M_b is denoted by $M_a(M_b)$, and its subspace consisting of those elements canonically identified with bounded linear operators on the Hilbert space $H \equiv \bigoplus_{i=1}^{a} l_b^2$ is denoted by $BM_a(M_b)$, which becomes a C^*-algebra.

When $a, b < \infty$, generalizing the classical Schur product, R. A. Horn, R. Mathias and Y. Nakamura [Linear and Multilinear Algebra **30** (1991), no. 4, 303–314; MR1129186 (93b:15020)] introduced a product operation $\Box$ on $M_a(M_b)$ by $(A\Box B)[i,j] = A[i,j] \cdot B[i,j]$, where $\cdot$ is the usual matrix product. In the present paper the author investigates what happens when $a = \infty$ or $b = \infty$. Here is a sample result. Each $A \in M_a(M_b)$ induces the left multiplication map ${}_A\Box$ on $M_a(M_b)$ by ${}_A\Box(T) \equiv A\Box T$ for $T \in M_a(M_b)$ and similarly the right multiplication map $\Box_A$, provided required matrix multiplications in the definitions are well defined. The set of A for which the left multiplication map ${}_A\Box$ leaves $BM_a(M_b)$ invariant is denoted by $LSM_a(M_b)$. Similarly, $RSM_a(M_b)$ is defined by means of the right multiplication. The author proves that those two sets coincide if and only if a or b is finite. But even when $a = b = 2$ the operator norm of ${}_A\Box$ is not necessarily equal to that of $\Box_A$.

Reviewed by *T. Ando*

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MR1208200 (94c:47055) 47B49 (15A09 47A30 47A62 47B15)

Livshits, Leonya (1-CMI); **Ong, Sing-Cheong** (1-CMI)

On the invertibility of the map $T \to STS^{-1} + S^{-1}TS$ and operator-norm inequalities. (English summary)

Linear Algebra Appl. **183** (1993), 117–129.

For S normal, a necessary and sufficient condition for the invertibility of the map of the title is found in terms of the spectrum of S. The same condition is shown to be sufficient for invertibility in the case of any bounded S. In addition, there are several results on operators in the kernel of the map in cases where it is not invertible. The proofs rely on Rosenblum's theorem on operator equations and some interesting computations.

Reviewed by *P. Rosenthal*

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MR1159175 (93k:47054) 47D99 (47A99 47D15)
Livshits, Leonya (1-CMI)
Locally finite-dimensional sets of operators.
Proc. Amer. Math. Soc. **119** (1993), *no. 1,* 165–169.

A set F of bounded operators mapping a Banach space V into a Banach space W is defined to be locally finite-dimensional if the span of Fx is finite-dimensional for each vector x in the domain space V. Obvious examples include: (1) finite-dimensional subspaces of $B(V, W)$, (2) collections of operators whose ranges are contained in a common finite-dimensional subspace of W, and (3) subsets of sums of the preceding two types. The main results of this paper show that these are the only possibilities.

Reviewed by *Edward Azoff*

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