

Integration CompuTest with Solutions

In each case, compute the integral. Some of these are definite integrals, others just ask for anti-derivatives. There are seven questions, and each is worth 15 points. The test will be graded out of 100 points, so that there is a built-in 5-point bonus for everyone.

In these solutions, I've done substitutions even in cases where you could plausibly just guessed, or done the substitution in your head.

$$1. \int_0^{\pi/2} \sin(2\theta) d\theta = \left[\begin{array}{l} u = 2\theta \\ du = 2d\theta \end{array} \right] = \frac{1}{2} \int_0^{\pi} \sin(u) du = -\frac{1}{2} \left[\cos(u) \right]_0^{\pi} = -\frac{1}{2}((-1) - 1) = 1$$

$$2. \int e^{5t} dt = \left[\begin{array}{l} u = 5t \\ du = 5dt \end{array} \right] = \frac{1}{5} \int e^u du = \frac{1}{5} e^u = \frac{1}{5} e^{5t}$$

$$3. \int \frac{3x}{\sqrt{1+x^2}} dx = \left[\begin{array}{l} u = 1+x^2 \\ du = 2xdx \end{array} \right] = 3 \int \frac{1}{2\sqrt{u}} du = 3\sqrt{u} = 3\sqrt{1+x^2}$$

$$4. \int_0^1 (2x^5 - 3x^3 + 5) dx = \left[\frac{2x^6}{6} - \frac{3x^4}{4} + 5x \right]_0^1 = \frac{1}{3} - \frac{3}{4} + 5 = \frac{55}{12}$$

$$5. \int_0^3 \frac{2x}{1+x^2} dx = \left[\begin{array}{l} u = 1+x^2 \\ du = 2xdx \end{array} \right] = \int_1^{10} \frac{1}{u} du = \ln(10) - \ln(1) = \ln(10)$$

$$6. \int_0^a e^{-x} dx = \left[-e^{-x} \right]_0^a = 1 - e^{-a}$$

$$7. \int_0^{\pi} x \sin(x) dx = \left[-x \cos(x) \right]_0^{\pi} + \int_0^{\pi} \cos(x) dx = \left[-x \cos(x) \right]_0^{\pi} + \left[\sin(x) \right]_0^{\pi} \\ = (-\pi \cos(\pi) + 0 \cos(0)) + (\sin(\pi) - \sin(0)) = \pi$$