MA121, Spring 2008 — Problem Set 2

This assignment is due on **Wednesday, February 27**. We will spend about twenty minutes of our class on Monday, February 25, discussing these problems. Make sure you have looked over the problems so that you know which ones you need to ask about.

Some of the problems in this set are still "review" of material about functions that is found in chapter 1, this time focusing on the most important "transcendental" functions: exponential and logarithm, sine and cosine. The rest relate to the ideas about "local slope" and "local rate of change," which is, of course, the same thing as "the derivative."

1. Problems from the textbook:

a. Section 1.2, problems 12, 14, 34

b. Section 1.4, problems 32, 38, 40, 46.

c. Section 1.5, problems 34, 38, 42.

2. Explain why a persnickety person might claim that the equation

$$\frac{x^2 - 1}{x - 1} = x + 1$$

is incorrect as stated.

3. Phidias takes a trip from St. Louis to Chicago. He leaves at 9 AM on Monday and arrives at 7 PM that day. He returns on Tuesday, leaving at 9 AM and arriving back in St. Louis at 2 PM, retracing exactly the same route. Explain why there must be a point on the road through which he passes at the same time in both days.

(Hint: graph Phidias' position versus time on each day on the same set of axes.)

4. Penthesilea takes a trip from Chicago to Milwaukee. Due to road construction, she drives the first 10 miles at a constant speed of 20 mph. For the next 30 miles she maintains a constant speed of 60 mph and then stops at McDonald's for a 10 minute snack. She drives the next 45 miles at a constant speed of 45 mph.

a. Draw a graph which shows her distance from her starting point as a function of time.

b. Draw a graph which shows her velocity as a function of time.

c. What is her average speed for the trip (including the stop at McDonald's)?

5. Each of the sentences below can be interpreted as an assertion about the derivative of a function. In each case, give such an interpretation. You need to specify the function and its independent variable, and then translate the statement into the language of derivatives. Finally, sketch the graph of a function which satisfies the properties indicated by each sentence.

a. The price of a product decreases as more of it is produced.

b. The child's temperature has been rising over the last three hours, but not as rapidly since we gave him the antibiotic an hour ago.

c. The cost of health insurance is rising at an ever-increasing rate.

d. The car is gradually slowing to a stop.

6. The weight $w(t)$ (in grams) of a tumor, t weeks after it first is formed, is given by $w(t) = t^2/18$. Find the average rate at which the tumor is growing during the fifth week after it was formed. Find the instantaneous rate of growth at $t = 5$. Compare the two rates. If they are different, explain what the difference means.

7. Water is flowing into a large spherical tank at constant rate. Let $V(t)$ be the volume of water in the tank at time t, and let $H(t)$ be the height of the water level at time t.

a. Give a physical interpretation of the derivatives $\dfrac{dH}{dt}$ and $\dfrac{dV}{dt}$.

b. Is $\dfrac{dV}{dt}$ positive, negative, or zero when the tank is one quarter full? Explain your answer.

c. Is $\dfrac{dH}{dt}$ positive, negative, or zero when the tank is one quarter full? Explain your answer.

d. Which of $\dfrac{dH}{dt}$ and $\dfrac{dV}{dt}$ is constant? Explain your answer.

8. Give a reason why the cosine function is not a polynomial.

(This is not a "duh!" question... After all, the cosine function is just a button in your calculator. Maybe all the calculator does when you press it to compute $\cos x$ is to compute something like

$$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8 - \frac{1}{3628800}x^{10}$$

and return the answer! If you graph this monster you'll see it looks a lot like the cosine. But it *isn't* the cosine, and you know that from the git-go. Why?)