MA121, Spring 2008 — Problem Set 5

Welcome back, everyone. This assignment tries to do two things. First, it has several problems intended to remind you of what you know (still, I hope) about derivatives and what can be done with them. Most of them come from sections 4.1 and 4.2 in the book, which you should read. Second, it has a few problems about the concept of the integral, which will occupy us beginning today. This problem set is due on **Wednesday, April 9**.

1. Problems from the textbook:

a. Section 4.1: 21, 22, 28

b. Section 4.2: 16, 18, 30

c. Section 5.2: 2, 6, 10.

2. Bernhard Riemann V is planning to take a Sunday afternoon trip in his 1976 Mercedes Benz whose odometer broke when it turned over 200,000 miles. He decides to estimate the miles driven by recording his speed at various times during the trip. Because this is to be a restful trip and not a military drill, he does not plan to take readings at specified intervals of time, but only when he remembers to do so. He takes a stopwatch to record the time intervals (in minutes) to make the calculations easier for himself (and consequently for you). Table 1 contains the data he collects during his trip. Each speedometer reading (in miles per hour) was made during the time interval recorded. Approximately how far does he travel?

After completing the arithmetic to find the distance he has driven, Bernhard wonders about his average speed. Can you tell him what it is?

length of time interval	15	25	30	15	20	35	40	20	10
speed measured	30	45	50	25	45	55	50	35	25

Table 1: Bernhard's measurements

3. If you were asked to find

$$\int_1^2 x^2 e^x \, dx,$$

you would not be able to do it (yet). However, you should be able to estimate the size of the answer. Is it less than zero, between 0 and 10, between 10 and 20, or what? Can you approximate it so that the error is less than one?

4. Here's an example of using our theory of maximum and minimum values to find inequalities between numbers.

a. Find where the function $f(x) = (1000-x)^2+x^2$ is increasing and where it is decreasing.

b. Use part (a) to decide whether 1000^2 is bigger or smaller than 998^2+2^2. Check your answer by using a calculator.

c. Generalize what you have discovered in part a to the function $f(x) = (c-x)^n+x^n$, where c is some positive constant and n is any even positive integer. Use this to decide whether 10000^{100} is bigger or smaller than $9000^{100} + 1000^{100}$. (Note that a calculator won't be of much use with this one!)

5. What is the area of the largest rectangle that can be inscribed in a semicircle of radius R so that one of the sides of the rectangle lies on the diameter of the semicircle? (Hint: the hardest thing here is to choose the best variable. The angle ϑ in the diagram is a good bet.)