MA121, Spring 2008 — Problem Set 6

This one is mostly integrals, focusing primarily on properties and a little on the Fundamental Theorem. This problem set is due on **Wednesday, April 16.**

1. Problems from the textbook:

a. Section 5.2: 20, 26, 27.

b. Section 5.3: 2, 4, 16, 20.

c. Section 5.4: 17, 18, 26, 28.

2. Suppose you know that

$$\int_a^b f(x)\,dx = 18, \qquad \int_a^b g(x)\,dx = 5, \qquad \text{and} \qquad \int_a^b h(x)\,dx = -11.$$

Evaluate as many of the following as you can by using the properties of integrals. (Some of them may not be possible to determine with the data you have been given!)

a. $\displaystyle\int_a^b (f(x) + g(x))\,dx$

b. $\displaystyle\int_a^b (f(x) - g(x))\,dx$

c. $\displaystyle\int_a^b f(x)g(x)\,dx$

d. $\displaystyle\int_a^b (g(x) + h(x))\,dx$

e. $\displaystyle\int_a^b \frac{g(x)}{h(x)}dx$

f. $\displaystyle\int_a^b (f(x) + g(x) + h(x))\,dx$

3. Given that

$$\int_0^1 f(x)\,dx = \frac{4}{3}, \qquad \int_1^2 f(x)\,dx = \frac{8}{3}, \qquad \text{and} \qquad \int_0^3 f(x)\,dx = \frac{11}{3},$$

find the values of

$$\text{a.} \quad \int_0^2 f(x)\,dx \qquad \text{b.} \quad \int_1^3 f(x)\,dx \qquad \text{c.} \quad \int_2^3 f(x)\,dx$$

4. What is $\displaystyle\int_{-3}^{3} (x+5)\sqrt{9-x^2}\,dx$ equal to?

(Hints: First, use linearity to write this as the sum of two integrals. Second, remember that the integral can be interpreted as an area under a curve; what do the graphs of $y = x\sqrt{9-x^2}$ and $y = \sqrt{9-x^2}$ look like?)

5. Four calculus students disagree as to the value of the integral

$$\int_0^{\pi} \sin^8(x)\,dx.$$

Jack says that it is equal to π, Joan says that it is equal to $\frac{35\pi}{128}$. Ed claims it is equal to $\frac{3\pi}{90} - 1$, while Lesley says it is equal to $\frac{\pi}{2}$. One of them is right; which one is it?

(Hint: do *not* try to evaluate the integral! Instead, try to eliminate the three wrong answers.)

6. Use the geometric interpretation of the integral as an area to explain why it is true that

$$\int_0^1 x^n\,dx + \int_0^1 x^{1/n}\,dx = 1$$

for any positive integer n.