MA121, Spring 2008 — Problem Set 7

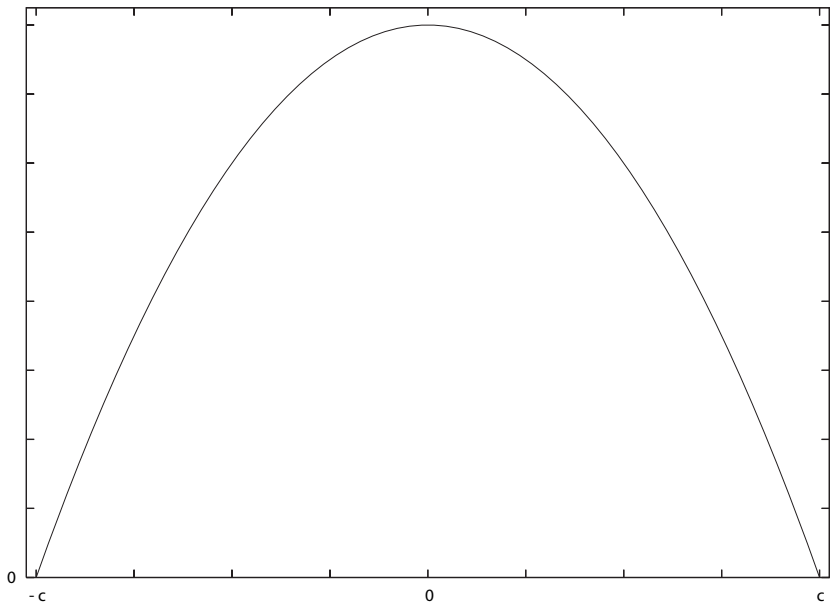
Integrals, integrals, integrals, plus one really hard problem on derivatives. This problem set is a little shorter than usual, since you need to prepare for the midterm. Because this problem set is intended to help you prepare, it is due on **Tuesday, April 22**.

1. Problems from the textbook:

a. Section 6,2, problems 44, 48, 54, 58, 66, 68.

b. Section 6.4, problems 18, 21, 24, 33–36.

2. The area between the graph of the function $f(x) = c^2 - x^2$ and the x-axis is 36. Find the value of c.



3. Let R be the region under the graph of the function $f(x) = (x-2)^2$ for $-1 \leqslant x \leqslant 1$. Find a number c such that the line $x = c$ splits the region in a 2 : 1 ratio.

4. Let R_a the region enclosed by the graph of the parabola

$$y = \frac{2}{a^2}x - \frac{1}{a^3}x^2$$

(for some constant $a > 0$) and the x-axis. Show that the area of R_a does not depend on a. How large is the area? What curve is determined by the vertices of all these parabolas?

5. (This one is quite hard!) What is the area of the largest triangle that can be formed in the first quadrant by the x-axis, the y-axis, and a line tangent to the graph of $y = e^{-x}$?

(Hints: The figure below shows one such triangle, using the tangent line through $(2, e^{-2})$. You need to find the value a so that taking the tangent line at (a, e^{-a}) gives the smallest triangle. So the problem has two steps: compute the area as a function of a, then find the maximum value of that function.)