MA121, Spring 2008 — Problem Set Last

This is it, boys and girls: the last of the problem sets. There's some optimization, a little bit of differential equations, and various forms of review. This problem set is due on **Wednesday, May 7**.

1. Problems from the textbook:

a. Section 4.5: 16, 24, 36.

b. Read section 8.1, then do problems 2, 4, 8, 12, except that you may ignore the instruction to "write a Riemann sum."

c. Section 11.4, problems 4, 12, 20, 26.

d. Section 11.6, problems 22 and 23.

2. A hallway of width a meters makes a right angle turn into a hallway of width b meters. You want to carry a ladder horizontally around the corner. What is the largest ladder that you can carry?

3. The function $\arctan(x)$ is the inverse function of the tangent function, arranged so that the value of $\arctan(x)$ is always between $-\pi/2$ and $\pi/2$. As θ runs from $\pi/2$ to $\pi/2$, $\tan(\theta)$ takes all possible real values, so the domain of $\arctan(x)$ is all the real line. Use the equation

$$\tan(\arctan(x)) = x$$

to find the derivative of $\arctan(x)$. (You'll need to remember some trigonometric identities in order to simplify.)

Reading the result backwards, you'll get one more antiderivative to add to your "internal table."

4. A motorist who had been charged with speeding on the basis of evidence from a police radar trap claimed that she should be acquitted. She argued as follows. The police car with the radar unit was parked 50 ft off to the side of the road. The radar measured not the speed with which the car was traveling along the road, but the rate of change of the distance from the car to the radar. When the car was 200 ft up the road from the point on the road that is closest to the radar, the radar registered that this rate of change was 80 mph. Since the law says that it is illegal to travel at 80 mph along the road but says nothing about the rate of change of the distance to a point 50 ft to one side, the motorist argued that the charge ought to be dismissed. The judge had never studied calculus, and let the motorist off with a warning. You *have* studied calculus: what should the judge have done? (Assume the road was perfectly straight, and be careful about units.)

5. (One of the much-too-hard problems in last year's final exam...) We are trying to fill a large leaky bucket. Each minute, we pour in 2 liters of water, but every minute one quarter of the volume that is in the bucket leaks out. What will happen in the long run?