

MA121, Spring 2008 — Midterm 2

Name: _____ Section: _____

This test consists of six problems. The number of points for each problem is indicated; the total is 100 points. There is also a bonus problem, which you do not need to do. It is worth ten extra points.

Do look over the test before you start. The problems are not necessarily organized in order of difficulty!

As you work on the test, you may use your calculator, and you may also use one 8.5×11 sheet of notes. (But consulting notes takes time — watch the clock!) Remember to *write up* your answers, that is, to *explain your reasoning and your computations in good prose*. In particular, try to make sure your “mathematical sentences” make sense. For example, don’t use “=” unless you actually mean to say two things are equal.

The problems begin on the next page. Good luck!

I’m very good at integral and differential calculus,
I know the scientific names of beings animalculous;
In short, in matters vegetable, animal, and mineral,
I am the very model of a modern Major-General.

– Gilbert & Sullivan, *Pirates of Penzance*

1. [15 points] For all $x \geq 2$, we define

$$\text{Li}(x) = \int_2^x \frac{1}{\ln(x)} dx.$$

Sketch the graph of $\text{Li}(x)$.

(Hint: start by determining whether $\text{Li}(x)$ is increasing or decreasing, and whether the graph is concave up or down.)

2. [15 points] Compute the area of the region under one “bump” of the graph of the sine function.

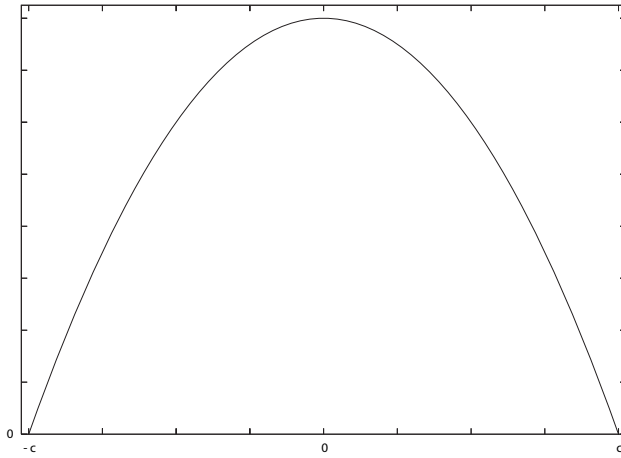
3. [15 points] Given that

$$\int_0^1 f(x) \, dx = 6, \quad \int_1^2 f(x) \, dx = 10, \quad \text{and} \quad \int_2^3 f(x) \, dx = 3,$$

find the value of

$$\int_0^3 (3f(x) + 2x) \, dx.$$

4. [15 points] The area between the graph of the function $f(x) = c^2 - x^2$ and the x -axis is 20 square units. Find the value of c . (The plot below shows the function for one value of c , but not necessarily the one you are looking for.)



5. [20 points] Let $f(x) = xe^{-x^2}$.

a. For which values of x is $f(x)$ decreasing? For which is it increasing?

b. For which values of x is the graph of $f(x)$ concave up? For which is it concave down?

6. [20 points] At time $t = 0$ I set off along the highway, and for the first 5 minutes my speed increases at a constant rate of 20 km/h every minute. After six minutes, I'm going at 120 km/h, which is the speed limit, so I stay at that speed for the rest of the trip.

- a. How far will I travel in the first 30 minutes?
- b. How long will it take me to go 50 km?

Bonus Problem [10 points] What is so fundamental about the Fundamental Theorem of Calculus?