MA121, Spring 2008 — Quiz The Second with Solutions

1. If you know that the graph of a function f is a line that has slope 2 and goes through the point $(1, 1)$, what is $f(x)$ equal to?

The standard solution was to set $f(x) = 2x + b$, plug in $f(1) = 1$, and solve for b. Alternatively, one could use the point-slope form of the equation of a line and write $y - 1 = 2(x - 1)$. Either way, the line is $f(x) = 2x - 1$.

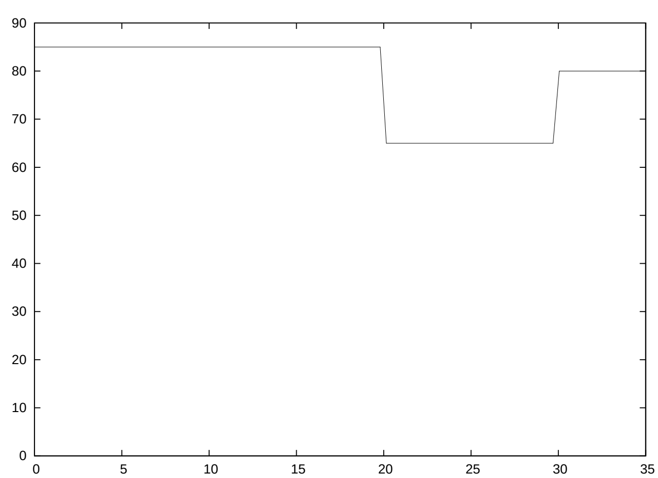
2. *Car Talk* class valedictorian Sidney Furst Rowe was driving from Podunk to Nowheresville. When he was ten miles out, he set the cruise control to 85 miles per hour. He zoomed along happily until, 20 minutes later, he saw a squad car parked by the side of the road. Hitting the brakes at once, he reset the cruise control for 65 miles per hour, and continued at that speed for 10 minutes. Then, feeling safe again, he hit the gas and set the cruise control at 80 miles per hour. Five minutes later, he was stopped by the police.

a. Graph Sidney's speed from the time he set his cruise control until the time he was stopped.

I assumed all the transitions were instantaneous. Then the speed function is this:

$$v(t) = \begin{cases} 85 & \text{if } 0 < t < 20 \\ 65 & \text{if } 20 < t < 30 \\ 80 & \text{if } 30 < t < 35 \end{cases}$$

The graph is a step function. I drew it with *gnuplot*, which doesn't like step functions, so I ended up with almost-vertical lines.



b. Let $P(t)$ denote Sidney's position t minutes after he set his cruise control. Sketch the graph of $y = P(t)$ for $0 \leqslant t \leqslant 35$.

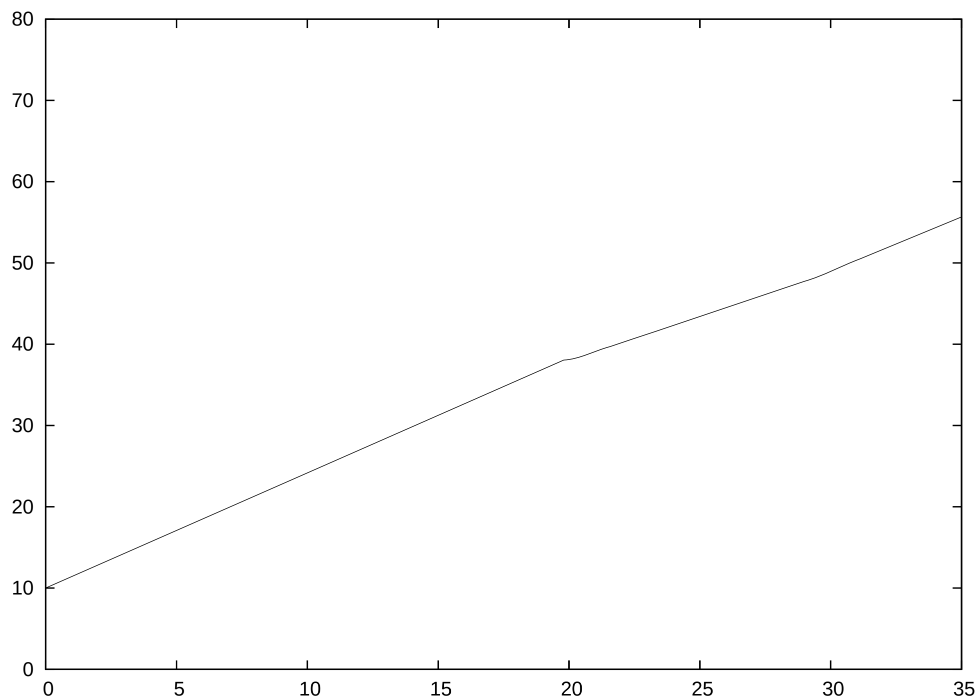
I just wanted to see a piecewise-linear graph with slopes going big-smaller-big. I'm going to do it in ridiculous detail here, though.

Notice that $P(0) = 10$, because we start counting time at that point. Also, since the t variable is in minutes, we should convert the data accordingly. So the speeds are $\frac{17}{12}$ miles per minute for the first stretch, $\frac{13}{12}$ miles per minute for the second, and $\frac{4}{3}$ miles per minute for the third.

The first line goes through $(0, 10)$ with slope $\frac{17}{12}$, so the equation for that stretch is $10 + \frac{17}{12}t$. When $t = 20$, that gives $P(20) = 115/3 = 38.33$ miles.

The second line starts at $(20, 115/3)$ and has slope $\frac{13}{12}$, so its equation is $\frac{115}{3} + \frac{13}{12}(t - 20)$. The final position is $P(30) = 295/6 = 49.17$.

The third line starts at $(30, 295/6)$ and has slope $\frac{4}{3}$, so the equation is $\frac{295}{6} + \frac{4}{3}(t - 30)$. Here's the graph:



As you can see, that is deeply uninformative! The slopes are just too close to one another. In a graph, it's often better to go for *insight* instead of precision. In other words, it would've been nicer, if less precise, to draw the thing by hand, exaggerating the differences between the slopes.

c. What was Sydney's average speed over the whole period?

To compute average speed, you take the total distance and divide by the total time. Here the total distance is 45.833 miles, and it took 35 minutes in all. That's 1.309 miles per minute, or 78.57 miles per hour.

Just taking the average of the three numbers won't work because it ignores the fact that we spent different amounts of time at each speed!

3. (Extract of a letter from Gustave Flaubert.) Since you are now studying geometry and trigonometry, I will give you a problem. A ship sails the ocean. It left Boston with a cargo of wool. It grosses 200 tons. It is bound for Le Havre. The mainmast is broken, the cabin boy is on deck, there are 12 passengers aboard, the wind is blowing East-North-East, the clock points to a quarter past three in the afternoon. It is the month of May. How old is the captain?

A few years older than his oldest teeth.