MA121 — Single-Variable Calculus

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The calculus is the result of many centuries of intellectual work. What was once front-line mathematics, very difficult and hard to use, has now become the standard first college course in mathematics. My goals in this course are

- to pass on to you some of the intellectual excitement of the calculus;
- to help you begin to understand this tool, so that you may use it intelligently in the future;
- to help you understand what scientists are talking about when they write down differential equations;
- to give you some proficiency in the actual use of the calculus.

To that end, I will put the emphasis on *understanding* rather than on the mechanical manipulation of symbols. We will manipulate enough to get to the point where we understand; anything harder we can leave to the computer.

This is not a high school mathematics course! So please note certain things:

- *It goes very quickly.* Indeed, when done as a high school course, calculus usually is taught over a full school year, an hour a day. That's 180 hours of class time. We'll do it in 12 weeks, four times a week, so 48 hours of class time. That's four times as fast!
- *It is not plug-and-chug.* Many high-school math courses are designed around "typical problems." You learn to do those, do identical ones in the homework, then again in the test. Forget it. For one thing, the theory is much more interesting than the applications. Anyway, I'm the wrong person to teach the applications. Instead, my focus is on an important mental skill: the ability to move from theory to applications.
- *I will not waste your time.* Nothing in this course is "filler." If I talk about it, I mean you to learn it. Yes, it will be on the test. There, I've answered the question in advance.
- *You should take charge of your own education.* You're adults now. If something isn't working, do something about it. Ask questions, email me, send anonymous feedback, make a fuss, come to my office, talk to the department chair. After all, there's no reason to expect others to care more about your education than you do.

- *But I do care.* I want you to succeed, both because it's more fun that way and because there's nothing as exciting really understanding a bit of mathematics. It's your life, but I'm here to help. And so is *Calculus After Hours*, for which see below.

I should point out that this course is far from being enough mathematics for you to actually *use* in science, economics, and other areas. First of all, there is a lot more to mathematics than just calculus. The American Mathematical Society classifies mathematical books and articles into some 65 broad subject areas, each of which has many sub-areas. All of calculus goes in just *one* of those areas.

In addition, this course won't come even close to covering all of "usable" mathematics. Recently, the Mathematical Association of America organized a series of conversations between mathematicians and folks from other disciplines about what students should know. The question was "what mathematics would you like your students to know by the end of their second year?" The biologists, for example, wanted their students to know about derivatives (single and multi-variable), differential equations, systems of linear equations and other basic linear algebra, and a hefty amount of statistics. There's no way all that can be packed into one course. My goal is to get started on that list, and to try not to discourage you from going further.

General Information

Office hours: I will be in my office and available for questions, discussion, and general conversation on Mondays, Wednesdays, and Fridays from 1:00 to 3:00. If you can't come during any of these times, please call and make an appointment.

Please do not hesitate to come see me — in fact, I strongly encourage you to come. It is part of your education, and one of your privileges as a Colby student.

Where to find me: The mathematics department no longer fits in the fourth floor of the Mudd building, which is its traditional home. The college has promised us space in a new academic building, but that's still in the future. Meanwhile, someone had to be exiled, and I volunteered. So my office is in the Olin building, where all the biologists live. I get lonely there... do come visit!

office: Olin 342
phone: 5836
email: fqgouvea

If you need to reach me when I'm not in my office, email is the best method.

Web site : Our course web site is at

`http://www.colby.edu/personal/fqgouvea/121`

Go there. I'll try to post all the course handouts and other interesting stuff. This includes assignments and, with luck, solutions.

Anonymous Feedback: I have set up a web page from which you can send me feedback about the course. It protects your identity automatically, so this is the way to send me comments if you want to do it anonymously. All I ask is for you to tell me which class you are in. Go to:

`http://www.colby.edu/personal/fqgouvea/feedback/`

This page is set up so that it can only be accessed from a computer on the Colby network.

How the class will be organized: I will be trying to run the course using a mixture of lectures and discussion and in-class activity. You will often be asked to work in class in a group with others. At any time (and especially in classes where I'm doing most of the talking) you should feel free to interrupt with questions and comments. I will try to prompt this participation by asking *you* questions too.

Textbook: Our basic text will be *Calculus*, by Hughes-Hallett, Gleason, et. al. (third edition). We will be covering the first eight chapters. This is designed to be *read*; it puts the emphasis on understanding rather than on mechanical proficiency. I will be asking you to read sections of the book as we go along, and I will expect you to gain some understanding from this reading. But don't stop there. Colby has a library, and it contains a great many books about calculus. Some of them will be textbooks, and others are supplementary books of various kinds. Do check them out!

Some students find that supplementary materials are helpful. These do exist for our textbook. The *Student Solution Guide*, for example, includes full solutions (not just answers) for the odd-numbered problems in the book. If you think something like this would help, check with the bookstore.

Finally, there are plenty of books designed to help you learn calculus. I'll list only a few that I like, and urge you to look for them if you think they might help.

- *The Calculus Lifesaver: All the Tools You Need to Excel at Calculus*, by Adrian Banner, is both more serious and hefty. I have asked the bookstore to get this is as an optional textbook.

- *Calculus for the Forgetful*, by Wojciech K. Kosek, argues that it is easier to *understand* than it is to *remember*. I agree!

- *How to Ace Calculus*, by Adams, Hass, and Thompson, is an informal and irreverent guide to single-variable calculus.

- *A Hitchhiker's Guide to the Calculus*, by Michael Spivak, is a very small book that tries to emphasize what is really important.

- *A Tour of the Calculus*, by David Berlinski, is a "trade book," that is, a book for reading rather than a textbook. It is incredibly over-written and over-dramatized, but it's a great introduction to the *ideas* of the calculus.

Technology: Calculators and computers are becoming ever more useful to people who need to use mathematics, and it is important to learn how to use these tools. For this class, I will often be using a calculator and will also often ask you to do computations and/or to graph functions on a calculator. If you do not own a graphing calculator, you might want to consider getting one. If you do own one, learn to use it!

On the other hand, a calculator can become a crutch. For some assignments and tests, I will be asking you *not* to use your calculator. Just as you don't need your calculator to compute 2×3, you shouldn't need your calculator to integrate $2x$.

In addition to using a graphing calculator, I will occasionally make use of a very powerful computer program called *Mathematica*. This is rather hard to use, but if you are interested in computing and are willing to put in the effort you might want to investigate this program. People that use mathematics in their work in the sciences or in economics are more likely to use computer tools than calculators, so it makes sense to begin to learn how to use these tools.

email: Email has become a fundamental communication tool, and we will be using it in our course. Your first assignment will involve sending me an email message, and I will occasionally use email to communicate with you either individually or as a class.

Prerequisites: This course is an introduction to the differential and integral calculus. My assumption is that you have not yet had a calculus course. We will rely on high school algebra, on understanding functions and their graphs, and other elementary ideas. If I ever use something you don't know, however, do feel free to ask me what's going on. I can always either do a quick review in class or give you some help in my office.

Note on algebra: There's no way around this. Trying to learn calculus without being fairly good at algebra is like trying to read Shakespeare without knowing much English. It *can* be done, but it is *very, very slow*. And difficult. So brush up your algebra!

Note to those who already know some calculus: There are always students in this class who actually *have* taken calculus before. Perhaps you don't feel the course worked very well, or you're not confident you learned much, or whatever. That's fine, but keep in mind that *you are not my primary target audience*. Don't show off your knowledge, don't use it to intimidate others, don't push me to do it the way your teacher did. This is really a matter of respect for me and for the other students. If you decided you needed to do this again, then put your whole heart in it and try to enjoy the ride.

Outside of class: You should expect to have to study two to three hours outside of class for every hour you spend in class. The best use of this time is to spend most

of it working with a group of friends. This will add a social dimension to your study, and will also help you resolve difficulties by using the differing strengths of people in your group. The ideal sequence seems to be to work the problems on your own first, then to work in a group to resolve any difficulties and reconcile any differences (when two people in the group get different answers, it can be a great learning opportunity).

An important resource to help you with studying and homework is *Calculus After Hours*, a Calculus lab where you can ask questions, work on homework, and generally learn with others. Watch for the handout with more information about this program.

While you are free to work on homework with a group, *you should write your final draft by yourself*, so that it reflects your state of understanding of the material. In other words, getting help from others to understand, then writing a solution that reflects that understanding is completely acceptable, and even desirable. Copying someone else's solution without understanding is *not* acceptable, and will be treated as a form of academic dishonesty. When you write up your solutions, be careful: use good grammar and full sentences, explain your reasoning, justify your approach, make it readable. Writing well requires real understanding; by requiring yourself to write well you will make sure that you really do understand what is going on. If you have questions about what kind of writing I'll be looking for, please feel free to ask.

Cheating: While you are encouraged to cooperate with others in class and in your homework assignments, your other work is expected to be your own. In particular, you are forbidden to get or give help during quizzes and tests. Please refer to the Colby policy on academic honesty as stated in the College Catalogue.

Attendance: Class attendance is required. Should you need to miss a class, please talk to me in advance to see if your absence can be excused. Missing too many classes will result in academic warnings, grade penalties, and eventually dismissal from the class. **Please note:** If you miss a class during which a quiz was given, you will *not* be offered an opportunity to make up that grade.

Assignments

Your grade will be computed as follows:

quizzes	10%
writing assignments	15%
homework	20%
four midterm tests	30%
final exam	25%

The following paragraphs explain what all of those things will entail.

Quizzes: There will be frequent short in-class quizzes, probably about once a week. These are intended simply to give you (and me) a reference point about how well you are absorbing the material. These will *not* be announced, nor can they be made up if you miss one. Since these are partially a diagnostic tool, I will discard the worst of your quiz scores when I work out your grade. The first quiz will be an algebra diagnostic, asking you to do simple computations.

Writing Assignments: There will be at least two writing assignments during the semester. These will involve doing some research, thinking about what you found out, then writing up the results. I will provide more information on these soon.

Homework: One can't learn mathematics without doing mathematics, and doing mathematics, in the context of the calculus, consists in solving non-trivial problems. On the other hand, learning a new subject often requires doing a number of "five finger exercises": relatively simple problems that basically drill you on what you have just learned. The odd-numbered exercises in the book, which have answers in the back, are a good way to provide yourself with this kind of practice. The Student Solutions Guide offers full solutions for odd-numbered problems.

You will receive a homework assignment every Wednesday, and it will be due the following Wednesday. These assignments will often contain problems that require a little creativity to solve. It is by solving these types of problems that you will really begin to understand what the mathematics is about, and it is also this kind of problem that you will meet in the "real world" when it comes time to use your knowledge. So, while these problems will be difficult, solving them will be worth the effort.

Since these assignments are difficult, I will separate some class time every Monday to discuss homework problems. This means you should make sure to take a first stab at the problems before that class so that you know what questions you need to ask.

My assignments are *long*. Please don't leave your assignments for the last day, because you will probably not be able to do them in one day. The problems I'll assign will require time for their solution, and you should plan to put in that time. (*Read this paragraph again. You have been warned!*)

Since homework is so important, it will have a relatively heavy weight in your grade. On the other hand, since homework is also the place to make mistakes and learn from them, I will discard the worst homework score from each half of the course.

Midterm tests: We will have four midterm tests: two in-class tests and two evening tests. The in-class tests will focus entirely on *computation*. The first will be entirely about computing derivatives, the second about computing integrals. For these tests, you will not be allowed to use a calculator. I will announce these about a week in advance.

The evening tests will be on **March 18** and **April 22**. They will be held in the evening, from 6:30 to 8:00 PM, in order to allow a little more time for thought.

Please mark your calendars. These tests will consist of problems similar in style (but not in difficulty!) to the problems assigned for homework. The emphasis here will be on *understanding*, not computation; you will be allowed to use your calculator and a sheet of notes.

Final exam: The final exam will blend both things, of course, but will tend more towards the style of the evening tests. The exam number for this course is 10, so that the exam will be in the morning on Saturday, May 17.

Outline

While I do make a day-by-day plan for what I want to do, it never works out that way. I always end up adjusting my plan to account for what actually happens in class. In particular, there's not much point in moving on when my students haven't yet understood. I don't like talking to myself.

So instead of a week-by-week schedule, here's an outline of what I'm planning to do, and in what order.

- Lines, linear functions of one or more variables, rates of change, modeling the world with linear functions.
- Functions in general, modeling with general functions, rates of change in general.
- Derivatives: what they are, what they mean, how they are used.
- Computing derivatives: by hand, by calculator, by computer.
- Accumulation functions, area, average value.
- Integrals: what they are, what they mean, how they are used.
- The relationship between derivatives and integrals.
- Computing integrals: by hand, by calculator, by computer.
- Calculus in science: differential equations.
- Good functions and bad: continuity, differentiability, integrability, and so on.
- Optimization and other applications.

That's a somewhat unusual sequence. I will point you to the correct parts of the book as we go on.

About Me

Students often wonder about their professors. You can find out more about me by looking at my home page, at

`http://www.colby.edu/~fqgouvea`

But, just for fun, here are some factoids:

- I was born in Brazil.
- I have been at Colby since 1991.
- I have two sons. They are currently both graduate students, one in Classics and the other in Political Science.
- My main research interests are in number theory and in the history of mathematics.
- I'm also interested in lots of other things. (The web site has a partial list.)
- I'm a Christian; currently I am a member of the Lutheran Church of the Resurrection in Waterville.
- I sing, but only in church.
- According to philosopher Richard Rorty, I am "frightening, dangerous, vicious..."
- I have had a beard since 1980, but it has only been gray for the last few years.
- I vote.
- On my bookshelves, you'll find books by Gene Wolfe, Dante, D. J. Enright, Dorothy L. Sayers, Jaroslav Pelikan, Reviel Netz, Barry Mazur, Gordon Fee, Terry Pratchett, G. K. Chesterton, Walt Kelly, Charles Williams, David B. Hart, Thomas Browne, W. H. Auden, Nicole Oresme, Monteiro Lobato, Ricardo Q. Gouvêa. And lots more.