HW for Wed. 3-19-09

(1) Prove that the interval $[0,1]$ does not have dimension 0.
(2) Suppose that Y is a compact metric space and that $X \subset Y$ is compact. Prove that the (topological) dimension of X is less than or equal to the (topological) dimension of Y. (X has the subspace topology.). Hint: Mimic the proof that dimension is a homeomorphism invariant.
(3) Prove that the topological dimension of $[0,1] \times [0,1] \subset \mathbb{R}^2$ is at least 1 and is no more than 2. (Hint: To prove that the dimension is at least 1, use the previous problem. To prove that the topological dimension is no more than 2, mimic the proof that the dimension of $[0,1]$ is no more than 1. Do this by covering $[0,1] \times [0,1]$ with small boxes. This won't quite work, but you might be able to modify the construction slightly so that it does work.)