How to Calculate Determinants

2X2

\[\begin{align*}
ax + by &= 0 \\
cx + dy &= 0
\end{align*}\]

\[\begin{vmatrix}
a & b \\
c & d
\end{vmatrix} = ad - bc \quad (1)\]

3X3

\[|A| = \begin{vmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{vmatrix} \quad (2)\]

Expand in terms of minors of any row or column (pick the row or column with the most zeros). For example, choosing the first column:

\[|A| = a \begin{vmatrix}
e & f \\
h & i
\end{vmatrix} - d \begin{vmatrix}
 b & c \\
h & i
\end{vmatrix} + g \begin{vmatrix}
 b & c \\
e & f
\end{vmatrix} \quad (3)\]

where you find the appropriate determinant in the expansion by striking out the row and column of the chosen coefficient. These smaller determinants are called minors. For example:

\[|A| = a \begin{vmatrix}
a & -b & -c \\
& e & f \\
g & h & i
\end{vmatrix} - d \begin{vmatrix}
 b & c \\
& e & f \\
g & h & i
\end{vmatrix} + g \begin{vmatrix}
 b & c \\
& e & f \\
& h & i
\end{vmatrix} \quad (4)\]

Minors have an associated sign, alternating through the matrix:

\[+ - + -
- + - +
+ - + -
- + - +\] \quad (5)

This is why the second term in equation 3 is negative.

We could also have expanded in terms of a row. For example, choosing the second row:

\[|A| = -d \begin{vmatrix}
b & c \\
h & i
\end{vmatrix} + e \begin{vmatrix}
a & c \\
g & i
\end{vmatrix} - f \begin{vmatrix}
a & b \\
g & h
\end{vmatrix} \quad (6)\]

For Larger Matrices: Do the determinant in steps. For example, a 4x4 is expanded in terms of 3x3 determinants and then the 3x3 determinants are expanded in terms of 2x2 determinants.