Complex Waveforms-Euler Identity

We can prove that $e^{ix} = \cos x + i \sin x$ by finding the Taylor expansions for $e^{ix}$, $\sin x$, and $\cos x$.

Remember that: 
\[ e^x = 1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \ldots + \frac{1}{n!}x^n \]
or
\[ e^{ix} = 1 + ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \frac{1}{4!}(ix)^4 + \ldots + \frac{1}{n!}(ix)^n \]

Then expanding the trig functions:
\[ \sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \ldots + (-1)^n \frac{1}{(2n+1)!}x^{2n+1} \]
\[ \cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \ldots + (-1)^n \frac{1}{(2n)!}x^{2n} \]

Multiplying the $\sin x$ expansion by $i$ gives:
\[ i \sin x = ix - \frac{1}{3!}ix^3 + \frac{1}{5!}ix^5 - \ldots + (-1)^n \frac{1}{(2n+1)!}ix^{2n+1} \]

Now noting that $i \cdot i = -1$,
\[ i \sin x = ix + \frac{1}{3!}(ix)^3 + \frac{1}{5!}(ix)^5 + \ldots + \frac{1}{(2n+1)!}(ix)^{2n+1} \]
\[ \cos x = 1 + \frac{1}{2!}(ix)^2 + \frac{1}{4!}(ix)^4 + \ldots + \frac{1}{(2n)!}(ix)^{2n} \]

Now combine the $\cos x$ and $i \sin x$ expansions:
\[ \cos x + i \sin x = 1 + ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \frac{1}{4!}(ix)^4 + \frac{1}{5!}(ix)^5 + \ldots \]

and you see that the result is the same expansion as for $e^{ix}$.

From trigonometric identities remember that $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$. This is handy for finding the real and imaginary parts of wavefunctions. To find the real part of a wavefunction note that you can take for the real part:
\[ \cos x = \frac{1}{2} ( e^{ix} + e^{-ix} ) \]
and the imaginary part
\[ \sin x = \frac{1}{2i} ( e^{ix} - e^{-ix} ) \]