Partial Derivative Conversion

\[
C_v + \frac{\alpha}{\kappa} V = \left( \frac{\partial U}{\partial T} \right)_V + \left( \frac{\partial P}{\partial T} \right)_V V
\]

\[
= \frac{1}{\left( \frac{\partial H}{\partial V} \right)_T} = \frac{1}{C_p}
\]

\[
H = U + PV
\]

definition
invert

misplaced numerator
misplaced numerator

\[
\left( \frac{\partial H}{\partial T} \right)_V \quad \left( \frac{\partial T}{\partial H} \right)_P
\]

misplaced denominator
misplaced constant variable
misplaced constant variable

chain rule

total differential, \( dH \)

\[
\frac{\partial H}{\partial P} \quad \frac{\partial P}{\partial V} \quad \frac{1}{\kappa}
\]

\[
\frac{\partial H}{\partial T} \quad \frac{\partial H}{\partial P} \quad \frac{\partial P}{\partial T}
\]

\[
C_p + \alpha \frac{\partial H}{\partial P}_T \kappa
\]

\[
= C_p + \left( \frac{\partial H}{\partial P}_T \kappa \right)
\]

\[
dH = \frac{\partial H}{\partial P}_T dT + \frac{\partial H}{\partial P}_P dP
\]

\[
dH = \left( \frac{\partial H}{\partial T}_P dT + \frac{\partial H}{\partial P}_T dP \right)
\]

\[
dH = \left( \frac{\partial H}{\partial T}_P \right) dT + \left( \frac{\partial H}{\partial P}_T \right) dP
\]

\[
= 0
\]

\[
\left( \frac{\partial T}{\partial P} \right)_H = \frac{\left( \frac{\partial H}{\partial P} \right)_T}{C_p}
\]