MATHEMATICS

In the Department of Mathematics and Statistics

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The Department of Mathematics and Statistics offers courses for students who: (1) plan a career in an area of pure or applied mathematics, including statistics; (2) need mathematics as support for their chosen major; or (3) elect to take mathematics as part of their liberal arts education or to fulfill the area requirement in quantitative reasoning.

The department offers majors in mathematics and in mathematical sciences as well as minors in mathematics and in statistics. The major in mathematical sciences is also offered with a concentration in statistics. Majors in mathematics and in mathematical sciences can be taken with honors.

Colby mathematics majors in recent years have entered graduate school to do advanced work in mathematics, statistics, biostatistics, engineering, economics, computer science, biomathematics, and the sciences. They also have used the major as a solid foundation for careers in teaching, law, medicine, banking, insurance, management, the computer industry, and other areas.

All incoming students who intend to enroll in mathematics courses in the fall semester are required to complete the mathematics placement questionnaire prior to registration.

Requirements for the Major in Mathematics

Completion of each of the following with a grade of C- or higher: one year of calculus; Mathematics 253, 274, 333, 338, and either 434 or 439; four additional courses selected from Mathematics 262 and any three- or four-credit mathematics courses numbered 300 or above (excluding 484). In exceptional cases, with the permission of the department, another 400-level course may be substituted for 434 or 439.

The department recommends that students complete Mathematics 274 or 275 before the end of their sophomore year. Although Mathematics 262 and 352 are not specifically required, the department strongly recommends that mathematics majors complete both courses.

Requirements for the Major in Mathematical Sciences

Completion of each of the following with a grade of C- or higher: one year of calculus; Mathematics 253, 262, 274; Computer Science 151 or 152; one course (to establish an overall theme for the major) selected from Mathematics 311, 332, 372, 381, Computer Science 231; four additional three- or four-credit courses selected from mathematics or statistics courses numbered 200 or above (excluding 484). With written permission of the advisor, one of these courses may be replaced by a course with significant mathematical content from another department.

The department recommends that students complete Mathematics 274 or 275 before the end of their sophomore year.

Requirements for the Major in Mathematical Sciences with a Concentration in Statistics

Completion of each of the following with a grade of C- or higher: one year of calculus; Statistics 212 or 231, and 382; Mathematics 253, 262, 274, 381; Computer Science 151 or 152; one additional statistics course numbered 300 or above; one additional three- or four-credit course selected from mathematics and statistics courses numbered 300 or above (excluding Mathematics 484 and Statistics 484). Students interested in pursuing a graduate degree in statistics are advised to take Mathematics 338.

The department recommends that students complete Mathematics 274 or 275 before the end of their sophomore year.

Requirements for the Honors Program in Mathematics or Mathematical Sciences

An honors program is available for students majoring in mathematics and mathematical sciences who have a grade point average of at least 3.25 in all mathematics and statistics courses numbered 200 or higher and who complete an additional, preapproved program of independent study in the major (Mathematics 484 or Statistics 484) culminating in both a written paper and a colloquium presentation. Students who successfully complete the requirements and who receive recommendation of the department will graduate with “Honors in Mathematics” or with “Honors in Mathematical Sciences.”

Requirements for the Minor in Mathematics

Six three- or four-credit mathematics courses numbered 121 or above, including Mathematics 122 or 162, Mathematics 253, and at least one course at the 300 level or above. Statistics 212 may substitute for one of the elective mathematics courses.
A minor in data science is described in the “Data Science” section of the catalogue.

A minor in statistics is described in the “Statistics” section of the catalogue.

The point scale for retention of the majors and minor applies to all courses in the majors/minor. No requirement for the majors or minor may be taken satisfactory/unsatisfactory.

Course Offerings

[MA101] Calculus with Pre-calculus I Designed for students who enter Colby with insufficient algebra and pre-calculus background for the standard calculus sequence. It is expected that all students who complete Mathematics 101 will enroll in Mathematics 102 in the following January. The combination of 101 and 102 covers the same calculus material as Mathematics 121. Completion of 101 alone does not constitute completion of a College calculus course for any purpose; in particular, it does not qualify a student to take Mathematics 122 nor does it satisfy the quantitative reasoning requirement. Prerequisite: New first-year students must complete the mathematics placement questionnaire found at www.colby.edu/math/newstudent. Three credit hours.

[MA102] Calculus with Pre-calculus II A continuation of Mathematics 101. Successful completion of both Mathematics 101 and 102 is equivalent to completion of Mathematics 121. Prerequisite: Mathematics 101. Three credit hours. Q.

[MA111] Mathematics as a Liberal Art Mathematics is one of humanity's longest-running conversations. Its crucial role in the thought-world of medieval Europe can be seen in the fact that four of the original seven liberal arts were inherently mathematical. Today, mathematics is just as important, permeating our culture. Students will develop awareness of the historical and contemporary roles of mathematics so that they will better understand the nature of mathematics, will know what kinds of things mathematics does well, and will know when to ask for a mathematician's help with their intellectual work. Specific topics discussed will vary. Four credit hours. Q. FACULTY

MA121fs Single-Variable Calculus Calculus is the result of centuries of intellectual effort to understand and quantify change, such as the position of a moving object or the shape of a curve. Competent users of calculus understand its intellectual structure sufficiently to apply its ideas to a variety of intellectual pursuits. Topics include differential and integral calculus of one variable: limits and continuity; differentiation and its applications, antiderivatives, the definite integral and its applications; exponential, logarithmic, and trigonometric functions. Prerequisite: New first-year students must complete the mathematics placement questionnaire found at www.colby.edu/math/newstudent. Four credit hours. Q. FACULTY

MA122fs Series and Multi-variable Calculus A continuation of Mathematics 121. Students will learn how to use infinite series, both to represent and to approximate functions, and will extend all of their skills from single-variable calculus to the multivariable setting. Topics: infinite series; vectors and analytic geometry in two and three dimensions; partial derivatives, differentials and the gradient; integration in two and three variables. Prerequisite: A course in single-variable calculus. New first-year students must complete the mathematics placement questionnaire found at www.colby.edu/math/newstudent. Four credit hours. Q. FACULTY

MA161f Honors Calculus I The first in a two-course sequence that treats the material of Mathematics 121 and 122 with a focus on the intellectual structure behind the methods. Students will acquire a deep understanding of the theory and foundational facts of calculus, will be able to use the techniques in an intelligent manner, will understand and be able to explain the arguments that undergird those techniques, and will be able to construct original arguments of their own. Topics are presented as a deductive mathematical theory, with emphasis on concepts, theorems, and their proofs. May not be taken for credit if the student has earned credit for Mathematics 122. Prerequisite: One year of calculus in high school. New first-year students must complete the mathematics placement questionnaire found at www.colby.edu/math/newstudent. Four credit hours. Q. MATHES

MA162s Honors Calculus II A continuation of Mathematics 161. Topics are essentially the same as for Mathematics 122, but they are presented as a deductive mathematical theory, with emphasis on concepts, theorems, and their proofs. May not be taken for credit if the student has earned credit for Mathematics 122. Prerequisite: Mathematics 161. Four credit hours. MATHES

[MA194] Mathematics Seminar An opportunity to read and discuss audience-appropriate mathematical material in an informal setting with members of the mathematics faculty, away from problem sets and exams. Successful students will show improvement in reading comprehension of mathematical articles, will increase their knowledge and understanding of the scientific community and the specific ways of mathematicians and statisticians, and will become familiar with mathematical issues of the past and present not normally covered in other courses. May be repeated for additional credit. Prerequisite: Mathematics 102, 121, 122, or 161. One credit hour.

MA253fs Linear Algebra Linear algebra is a crossroads where many important areas of mathematics meet, and it is the tool used to analyze the first approximation of complex systems. Students will learn to understand and use the language and theorems in both abstract and applied situations, gain insight into the nature of mathematical inquiry, and learn how to reason carefully and precisely about formally
described situations. Topics include vectors and subspaces in \( \mathbb{R}^n \), linear transformations, and matrices; systems of linear equations; abstract vector spaces and the theory of single linear transformation: change of basis, determinants, eigenvalues and eigenvectors, and diagonalization. **Prerequisite:** Mathematics 122 or 162; or Mathematics 102, 121, or 161 with permission of the instructor. **Four credit hours.** BRETSCHER, GOUVEA, LIVSHITS

**MA262fs** Vector Calculus Develops ideas first seen in Mathematics 122 by applying the notions of derivative and integral to multi-variable vector-valued functions. The goal is to understand the high-dimensional versions of the fundamental theorem of calculus and to use these theorems in specific scientific applications. Topics include parameterized curves and surfaces; gradient, divergence, and curl; change of variables and the Jacobian; line and surface integrals; conservative vector fields; Green's, Stokes's, and Gauss's theorems; applications. **Prerequisite:** Mathematics 122 or 162. **Four credit hours.** BRETSCHER, YOUNGS

**MA274fs** Mathematical Reasoning Proofs are the main method used by mathematicians to develop and communicate their ideas; this course prepares students to read, write, and communicate mathematical arguments. Topics include logic and standard methods of direct and indirect proof; the set-theoretic approach to functions and relations; the theory of infinite sets; elementary algebraic structures; and techniques from discrete mathematics. Credit can be received for only one of Mathematics 274 and 275. **Prerequisite:** Mathematics 102, 121, 122, or 161, and a W1 course. Two semesters of calculus is recommended. **Four credit hours.** TAYLOR, YOUNGS

**MA275** Topics in Abstract Mathematics Some students are sufficiently proficient with proofs and logic that they do not need to take Mathematics 274; this offers an alternative that focuses less on proof techniques and more on the set theory and related topics. The goal is to equip students to continue their study of mathematics. Topics include set-theoretic approach to functions and relations, the theory of infinite sets, elementary algebraic structures, and techniques from discrete mathematics. Credit can be received for only one of Mathematics 274 and 275. Nongraded. **Prerequisite:** Mathematics 161 and 162 and permission of the department. **Two credit hours.**

**MA311fs** Ordinary Differential Equations Differential equations allow us to deduce the long-term behavior of quantities from information about their short-term rates of change; for that reason they are the language of classical science. Students will learn to analyze concrete situations modeled by differential equations and to draw conclusions using equations, graphical techniques, and numerical methods. Topics include theory and solution methods of ordinary differential equations, linear differential equations, first-order linear systems, qualitative behavior of solutions, nonlinear dynamics, existence and uniqueness of solutions, and applications. **Prerequisite:** Mathematics 122 or 162, and 253. **Four credit hours.** HOLLY, RANDLES

**MA313** Differential Geometry The study of curves and surfaces in three-dimensional space, with the primary focus being on the nature of "curvature" and the distinction between intrinsic and extrinsic geometry. Students will improve their spatial intuition and learn to move easily between general theorems and specific examples. Topics include curves: tangent, normal, and binormal vectors; curvature and torsion; the moving frame; surfaces: the first and second fundamental forms, sectional and Gaussian curvature, the Theorema Egregium, geodesics, parallel transport; and selected additional topics. **Prerequisite:** Mathematics 122 or 162, and 253, and 274 or 275. **Four credit hours.**

**MA314** Geometry of Surfaces Explores the notion of "geometry" by studying the most important two-dimensional geometries: Euclidean, spherical, and hyperbolic. We will prove that every compact two-dimensional surface admits a geometric structure modeled on one of these geometries. As time allows we will also study applications of these geometries and their relationship to Teichmüller space, Kleinian groups, and three-dimensional manifolds. Students will engage in significant self-teaching and will communicate mathematical ideas with oral presentations, written proofs, and short essays aimed at a general audience. **Prerequisite:** Mathematics 162 or 262; 253; and 274 or 275. **Four credit hours.**

**MA331** Topology Begins as the abstract mathematical study of the notions of proximity and continuity and then deploys these methods to understand interesting objects and spaces. Students will develop their ability to construct precise arguments and to explore concrete examples as instances of a general theory. Topics are selected at the discretion of the instructor from the areas of point-set, differential, and algebraic topology. **Prerequisite:** Mathematics 274 or 275. **Four credit hours.**

**MA332** Numerical Analysis In practice, a solution to a problem might be impossible to obtain by classical methods of manipulating equations. Nonetheless, solutions can often be obtained by numerical methods, usually with the aid of a computer. Numerical analysis is the study of those numerical algorithms. Students will acquire the ability to use standard methods and mathematical software for solving the most common types of numerical problems and to analyze the speed and accuracy of the solutions. Topics include solution by numerical methods of linear and nonlinear equations, systems of equations, and differential equations; numerical integration; polynomial approximation; matrix inversion; error analysis. **Prerequisite:** Mathematics 122 or 162, and 253; 274 is recommended. **Four credit hours.**

**MA333f** Abstract Algebra Simply called "algebra" by mathematicians, it is the study of abstract sets with operations and is fundamental in expressing and working in theoretical mathematics. An introduction to that language, to the motivating examples, and to some of the fundamental theorems. Students will develop their ability to discover and write formal arguments, explore the relationship between general
theory and specific examples, and learn to recognize algebraic structures where they occur. Topics include groups, rings, and fields: definition, basic theorems, and important examples. **Prerequisite:** Mathematics 253, and 274 or 275. **Four credit hours.**

**MA336f**  **Mathematical Economics**  Listed as Economics 336. **Four credit hours.**

**MA338s**  **Real Analysis**  An exploration of the theory behind calculus, as well as its extension to more general settings. Students will learn to think carefully and clearly about limiting processes such as differentiation, integration, and summation of series and to interpret their knowledge in terms of the topology of metric spaces. They will develop the ability to read and to produce formal mathematical arguments, with particular attention to handling exceptional cases and delicate issues of convergence. Special focus on foundational issues: topology of metric spaces, continuity, differentiation, integration, infinite series. **Prerequisite:** Mathematics 122 or 162, and 274 or 275. **Four credit hours.**

**MA352f**  **Complex Analysis**  An introduction to functions of a complex variable. Topics include the definition and properties of holomorphic and analytic functions, Cauchy’s integral theorem and formula, meromorphic functions, representation by Laurent series, the residue calculus, and the elementary transcendental functions. Offered in alternate years. **Prerequisite:** Mathematics 122 or 162, and 274 or 275. **Four credit hours.**

**[MA353]**  **Matrix Analysis**  The study of real and complex matrices, beyond the material found in a first course in linear algebra, is essential for many areas of modern mathematics and its applications, and commonly involves analytic methods. We will touch upon topics from the following broad areas of interest for general and more specific matrix classes: the study of the canonical forms, decompositions and factorizations, spectral theory, matrix functions and equations, and multilinear algebra. Applications of the theory will also be considered. **Prerequisite:** Mathematics 253, either 274 or 275, and at least one of 162, 338, or 352. **Four credit hours.**

**MA357s**  **Elementary Number Theory**  Number theory deals with questions about numbers, especially those related to prime numbers and factorization. It offers a wide array of problems that are easily stated and understood but that can be difficult to solve. Students will gain an understanding of the beauty that such problems offer as well as the persistence that is often necessary in tackling them, and they will strengthen their problem-solving and proof-writing skills. Topics include prime numbers and unique factorization; congruences, Fermat’s Little Theorem, the Chinese Remainder Theorem, and RSA cryptography; quadratic residues, reciprocity, quadratic forms, and the Pell Equation. **Prerequisite:** Mathematics 102, 121, or 161. Two semesters of calculus or Mathematics 253 is recommended. **Four credit hours.**

**[MA376]**  **History of Mathematics**  The history of mathematics with emphasis on the interaction between mathematics, culture, and society. Writing-intensive and involving careful reading of original historical documents. By studying the mathematics of different times and cultures, students will deepen their own understanding of mathematics and develop a clearer idea of how society and mathematics influence each other. A survey of the history of mathematics is followed by a more careful tracing of the development of one theme or topic. Specific topics vary from year to year but often include the mathematics of non-Western cultures. **Prerequisite:** Mathematics 274 or 275. **Four credit hours.**

**MA378f**  **Introduction to the Theory of Computation**  Listed as Computer Science 378. **Four credit hours.**

**MA381f**  **Mathematical Statistics I: Probability**  A mathematical introduction to probability theory, the foundation for commonly used inferential statistical techniques (covered in Mathematics 382). Students will learn the basic theorems of probability and computational techniques for finding probabilities associated with stochastic processes. Topics include axiomatic foundations, combinatorics, random variables, discrete and continuous probability distributions, special probability distributions, independence, conditional and marginal probability distributions, properties of expectations, moment generating functions, sampling distributions, weak and strong laws of large numbers, and the central limit theorem. **Prerequisite:** Mathematics 122 or 162; 274 is recommended. **Four credit hours.**

**MA382s**  **Mathematical Statistics II: Inference**  Listed as Statistics 382. **Four credit hours.**

**MA391f**  **Graph Theory and Applications**  Graph theory is the mathematical study of networks. Applications of graph theory are ubiquitous in physics, engineering, and computer science. Introduces the basic terminology and results of graph theory; teaches how to construct rigorous arguments and useful examples; and develops the abilities to present mathematics both orally and in writing. Specific topics include euler, hamiltonian circuits, matching, connectivity and network flow, graph coloring, and algorithms. Particular attention is paid to applications. **Prerequisite:** Mathematics 253 (may be taken concurrently), and 274 or 275. **Four credit hours.**

**MA398s**  **Fourier Analysis**  Fourier analysis is a fundamental tool of pure and applied mathematics in which functions are broken down into periodic building blocks. An introduction to the theory and methods of Fourier analysis with applications to differential equations, mathematical modeling, and other areas. Topics include Fourier series; properties, convergence, summation kernels, and applications;
Fourier transforms: properties, Fourier inversion, Plancherel theorem, integral kernels, and applications; additional topics as chosen by the instructor. **Prerequisite:** Mathematics 253 and 274. **Four credit hours.**

**MA398Bs Mathematical Neuroscience** Neuroscience is an expanding and dynamic field seeking to understand the complexities of the brain. Recent advances in technology have improved our ability to record what the brain is doing, and with this comes the need for new and improved models to understand this influx of information. Students will work with theoretical mathematical models of the brain on different scales, from the cellular and single-neuron level up to interactions between brain regions, using both discrete and continuous techniques. **Prerequisite:** Mathematics 253 and 274. **Four credit hours.**

**[MA411] Topics in Differential Equations** A sequel to Mathematics 311, with higher-level content and a more extensive study of differential equations. Students will implement advanced analytical methods, examine theory, and demonstrate an understanding of further applications. Topics will vary from year to year. May be repeated for credit with permission of instructor. **Prerequisite:** Mathematics 253. **Four credit hours.**

**MA434s Topics in Abstract Algebra** One semester's exposure to algebra is not sufficient for further work in mathematics, so this is a continuation of Mathematics 333. Students will further develop their ability to speak the language of and use the methods of algebra through the study of one particular algebraic theory. Improving one's written and oral communication of mathematics is an integral part of the course. Topics will vary from year to year. May be repeated for credit with permission of instructor. **Prerequisite:** Mathematics 253. **Four credit hours.**

**MA439f Topics in Real Analysis** A sequel to Mathematics 338. Students will deepen their understanding of analysis through the exploration of more-advanced topics and will sharpen their ability to read, analyze, construct, and present proofs. Improving one's written and oral communication of mathematics is an integral part of the course. Topics will vary from year to year. May be repeated for credit with permission of instructor. **Prerequisite:** Mathematics 253. **Four credit hours.**

**MA472s Topics in Mathematical Modeling** Mathematical modeling provides a means to explain and predict phenomena. Applications are numerous, especially in the physical and social sciences. Students will learn to correctly interpret existing models and create new ones and will develop an understanding of the purpose and uses of mathematical models. The emphasis will be on analyzing research publications and on producing research-level mathematical models. Writing and discussion will be important components. Computers will be used for analysis and simulation. Topics will vary from year to year. May be repeated for credit with permission of instructor. **Prerequisite:** Mathematics 253. **Four credit hours.**

**MA484s Honors Independent Study** The independent study component of the honors program in mathematics. Cannot be counted toward the major or minor. **Prerequisite:** Permission of the instructor and admission to the honors program. **Three or four credit hours.**

**MA491f, 492s Independent Study** Independent study in an area of mathematics of particular interest to the student. **Prerequisite:** Permission of the instructor. **One to four credit hours.**