MATHEMATICS

In the Department of Mathematics and Statistics

Chair, *Associate Professor* Scott Taylor
Associate Chair, Associate Professor James Scott
Professors Fernando Gouvêa, Jan Holly, Leo Livshits, Benjamin Mathes, and Liam O'Brien; Associate Professors James Scott, Scott Taylor, and George Welch; Assistant Professors Tamar Friedmann, Evan Randles, Jerzy Wieczorek, Nora Youngs, and Bret Zeldow; Visiting Assistant Professor Costel-Gabriel Bontea

The Department of Mathematics and Statistics offers courses for students who: (1) plan a career in an area of pure or applied mathematics, including statistics; (2) need mathematics as support for their chosen major; or (3) elect to take mathematics as part of their liberal arts education or to fulfill the area requirement in quantitative reasoning.

The department offers majors in mathematics and in mathematical sciences as well as minors in mathematics and in statistics. The major in mathematical sciences is also offered with a concentration in statistics. Majors in mathematics and in mathematical sciences can be taken with honors.

Colby mathematics majors in recent years have entered graduate school to do advanced work in mathematics, statistics, biostatistics, engineering, economics, computer science, biomathematics, and the sciences. They also have used the major as a solid foundation for careers in teaching, law, medicine, banking, insurance, management, the computer industry, and other areas.

All incoming students who intend to enroll in mathematics courses in the fall semester are required to complete the mathematics placement questionnaire prior to registration.

Requirements for the Major in Mathematics

Completion of each of the following with a grade of C- or higher: one year of calculus; Mathematics 253, 274, 333, 338, and either 434 or 439; four additional courses selected from Mathematics 262 and any three- or four-credit mathematics courses numbered 300 or above (excluding 484). In exceptional cases, with the permission of the department, another 400-level course may be substituted for 434 or 439.

The department recommends that students complete Mathematics 274 or 275 before the end of their sophomore year. Although Mathematics 262 and 352 are not specifically required, the department strongly recommends that mathematics majors complete both courses.

Requirements for the Major in Mathematical Sciences

Completion of each of the following with a grade of C- or better: Mathematics 122 or 162, 253, 274, 311; one three- or four-credit course from Computer Science; four additional three- or four-credit courses selected from mathematics or statistics courses numbered 200 or above; one "Topics" course in Mathematics or Statistics numbered 400 or above (excluding 484). The department recommends that Mathematics 274 be completed before the end of the sophomore year.

Requirements for the Major in Mathematical Sciences with a Concentration in Statistics

Completion of each of the following with a grade of C- or better: Mathematics 122 or 162, 253, 274, 311, 381; one three- or four-credit course from Computer Science; Statistics 212, 321, one additional Statistics course numbered 300 or above (excluding 484), and Statistics 482. The department recommends that Mathematics 274 be completed before the end of the sophomore year. Students considering graduate school in statistics are strongly encouraged to take Mathematics 338.

Requirements for the Honors Program in Mathematics or Mathematical Sciences

An honors program is available for students majoring in mathematics and mathematical sciences who have a grade point average of at least 3.25 in all mathematics and statistics courses numbered 200 or higher and who complete an additional, preapproved program of independent study in the major (Mathematics 484 or Statistics 484) culminating in both a written paper and a colloquium presentation. Students who successfully complete the requirements and who receive recommendation of the department will graduate with "Honors in Mathematics" or with "Honors in Mathematical Sciences."

Requirements for the Minor in Mathematics

Six three- or four-credit mathematics courses numbered 121 or above, including Mathematics 122 or 162, Mathematics 253, and at least one course at the 300 level or above. Statistics 212 may substitute for one of the elective mathematics courses.

A minor in data science is described in the "Data Science" section of the catalogue.

A minor in statistics is described in the "Statistics" section of the catalogue.

The point scale for retention of the majors and minor applies to all courses in the majors/minor. No requirement for the majors or minor may be taken satisfactory/unsatisfactory.

Course Offerings

MA101f Calculus with Pre-calculus I Designed for students who enter Colby with insufficient algebra and pre-calculus background for the standard calculus sequence. It is expected that all students who complete Mathematics 101 will enroll in Mathematics 102 in the following January. The combination of 101 and 102 covers the same calculus material as Mathematics 121. Completion of 101 alone does not constitute completion of a College calculus course for any purpose; in particular, it does not qualify a student to take Mathematics 122 nor does it satisfy the quantitative reasoning requirement. *Three credit hours.* MOURNING-STAR

MA102j Calculus with Pre-Calculus II A continuation of Mathematics 101. Successful completion of both Mathematics 101 and 102 is equivalent to completion of Mathematics 121. *Prerequisite:* Mathematics 101. *Three credit hours.* **Q.** GOUVEA, TAYLOR

[MA111] Mathematics as a Liberal Art *Four credit hours.* **Q.**

MA121fs Single-Variable Calculus Calculus is the result of centuries of intellectual effort to understand and quantify change, such as the position of a moving object or the shape of a curve. Competent users of calculus understand its intellectual structure sufficiently to apply its ideas to a variety of intellectual pursuits. Topics include differential and integral calculus of one variable: limits and continuity; differentiation and its applications, antiderivatives, the definite integral and its applications; exponential, logarithmic, and trigonometric functions. *Prerequisite:* New first-year students must complete the mathematics placement questionnaire found at www.colby.edu/math/newstudent. *Four credit hours.* **Q.** FACULTY

MA122fs Series and Multi-variable Calculus A continuation of Mathematics 121. Students will learn how to use infinite series, both to represent and to approximate functions, and will extend all of their skills from single-variable calculus to the multivariable setting. Topics: infinite series; vectors and analytic geometry in two and three dimensions; partial derivatives, differentials and the gradient; integration in two and three variables. *Prerequisite:* A course in single-variable calculus. New first-year students must complete the mathematics placement questionnaire found at www.colby.edu/math/newstudent. *Four credit hours.* **Q.** ABDELGHANY, GOUVEA, RANDLES, YOUNGS

MA161f Honors Calculus I The first in a two-course sequence that treats the material of Mathematics 121 and 122 with a focus on the intellectual structure behind the methods. Students will acquire a deep understanding of the theory and foundational facts of calculus, will be able to use the techniques in an intelligent manner, will understand and be able to explain the arguments that undergird those techniques, and will be able to construct original arguments of their own. Topics are presented as a deductive mathematical theory, with emphasis on concepts, theorems, and their proofs. May not be taken for credit if the student has earned credit for Mathematics 122. *Prerequisite:* One year of calculus in high school. New first-year students must complete the mathematics placement questionnaire found at www.colby.edu/math/newstudent. *Four credit hours.* **Q.** LIVSHITS

MA162s Honors Calculus II A continuation of Mathematics 161. Topics are essentially the same as for Mathematics 122, but they are presented as a deductive mathematical theory, with emphasis on concepts, theorems, and their proofs. Student who receive an A- or above will receive an exemption from taking MA274. May not be taken for credit if the student has earned credit for Mathematics 122. *Prerequisite:* Mathematics 161. *Four credit hours.* LIVSHITS

MA253fs Linear Algebra Linear algebra is a crossroads where many important areas of mathematics meet, and it is the tool used to analyze the first approximation of complex systems. Students will learn to understand and use the language and theorems in both abstract and applied situations, gain insight into the nature of mathematical inquiry, and learn how to reason carefully and precisely about formally described situations. Topics include vectors and subspaces in $\mathbf{R}^n$, linear transformations, and matrices; systems of linear equations; abstract vector spaces and the theory of single linear transformation: change of basis, determinants, eigenvalues and eigenvectors, and diagonalization. *Prerequisite:* Mathematics 122 or 162; or Mathematics 102, 121, or 161 with permission of the instructor. *Four credit hours.* FRIEDMANN, GOUVEA, LIVSHITS

MA262s Vector Calculus Develops ideas first seen in Mathematics 122 by applying the notions of derivative and integral to multi-variable vector-valued functions. The goal is to understand the high-dimensional versions of the fundamental theorem of calculus and to use these theorems in specific scientific applications. Topics include parameterized curves and surfaces; gradient, divergence, and curl; change of variables and the Jacobian; line and surface integrals; conservative vector fields; Green's, Stokes's, and Gauss's theorems; applications. *Prerequisite:* Mathematics 122 or 162. *Four credit hours.* TAYLOR

MA274fs Mathematical Reasoning Proofs are the main method used by mathematicians to develop and communicate their ideas; this

course prepares students to read, create, write, and communicate mathematical arguments. Topics include logic and standard methods of direct and indirect proof; the set-theoretic approach to functions and relations; the theory of infinite sets; elementary algebraic structures; and techniques from discrete mathematics. Credit can be received for only one of Mathematics 274 and 275. *Prerequisite:* Mathematics 102, 121, 122, or 161, and a W1 course. Two semesters of calculus is recommended. *Four credit hours.* **W2.** LIVSHITS, MATHES, YOUNGS

MA311fs Ordinary Differential Equations Differential equations allow us to deduce the long-term behavior of quantities from information about their short-term rates of change; for that reason they are the language of classical science. Students will learn to analyze concrete situations modeled by differential equations and to draw conclusions using equations, graphical techniques, and numerical methods. Topics include theory and solution methods of ordinary differential equations, linear differential equations, first-order linear systems, qualitative behavior of solutions, nonlinear dynamics, existence and uniqueness of solutions, and applications. *Prerequisite:* Mathematics 122 or 162, and 253. *Four credit hours.* RANDLES

[MA314] Geometry of Surfaces Explores the notion of "geometry" by studying the most important two-dimensional geometries: Euclidean, spherical, and hyperbolic. We will prove that every compact two-dimensional surface admits a geometric structure modeled on one of these geometries. As time allows we will also study applications of these geometries and their relationship to Teichmüller space, Kleinian groups, and three-dimensional manifolds. Students will engage in significant self-teaching and will communicate mathematical ideas with oral presentations, written proofs, and short essays aimed at a general audience. *Prerequisite:* Mathematics 162 or 262; 253; and 274 or 275. *Four credit hours.*

[MA331] Topology Begins as the abstract mathematical study of the notions of proximity and continuity and then deploys these methods to understand interesting objects and spaces. Students will develop their ability to construct precise arguments and to explore concrete examples as instances of a general theory. Topics are selected at the discretion of the instructor from the areas of point-set, differential, and algebraic topology. *Prerequisite:* Mathematics 274 or 275. *Four credit hours.*

[MA332] Numerical Analysis In practice, a solution to a problem might be impossible to obtain by classical methods of manipulating equations. Nonetheless, solutions can often be obtained by numerical methods, usually with the aid of a computer. Numerical analysis is the study of those numerical algorithms. Students will acquire the ability to use standard methods and mathematical software for solving the most common types of numerical problems and to analyze the speed and accuracy of the solutions. Topics include solution by numerical methods of linear and nonlinear equations, systems of equations, and differential equations; numerical integration; polynomial approximation; matrix inversion; error analysis. *Prerequisite:* Mathematics 122 or 162, and 253; 274 is recommended. *Four credit hours.*

MA333f Abstract Algebra Simply called "algebra" by mathematicians, it is the study of abstract sets with operations and is fundamental in expressing and working in theoretical mathematics. An introduction to that language, to the motivating examples, and to some of the fundamental theorems. Students will develop their ability to discover and write formal arguments, explore the relationship between general theory and specific examples, and learn to recognize algebraic structures where they occur. Topics include groups, rings, and fields: definition, basic theorems, and important examples. *Prerequisite:* Mathematics 253, and 274 or 275. *Four credit hours.* FRIEDMANN

MA335f Mathematical Neuroscience Neuroscience is an expanding and dynamic field, seeking to understand the complexities of the brain. Recent advances in technology have improved our ability to record brain activity, and with this comes the need for new and improved models to understand this influx of information. In this course, students will work with theoretical mathematical models of the brain on different scales, from the cellular and single-neuron level up to interactions between brain regions, using both discrete and continuous techniques. *Prerequisite:* Mathematics 122 or 162, and 253. *Four credit hours.* YOUNGS

[MA336] Mathematical Economics Listed as Economics 336. *Four credit hours.*

MA338s Real Analysis An exploration of the theory behind calculus, as well as its extension to more general settings. Students will learn to think carefully and clearly about limiting processes such as differentiation, integration, and summation of series and to interpret their knowledge in terms of the topology of metric spaces. They will develop the ability to read and to produce formal mathematical arguments, with particular attention to handling exceptional cases and delicate issues of convergence. Special focus on foundational issues: topology of metric spaces, continuity, differentiation, integration, infinite series. *Prerequisite:* Mathematics 122 or 162, and 274 or 275. *Four credit hours.* MATHES

[MA352] Complex Analysis An introduction to functions of a complex variable. Topics include the definition and properties of holomorphic and analytic functions, Cauchy's integral theorem and formula, meromorphic functions, representation by Laurent series, the residue calculus, and the elementary transcendental functions. Offered in alternate years. *Prerequisite:* Mathematics 122 or 162, and 274 or 275. *Four credit hours.*

MA353s Advanced Linear Algebra Continues the exploration of linear algebra initiated in Mathematics 253. The emphasis is on the theory of matrices, linear spaces, and linear transformations, investigating them more deeply. Topics will come from the following list: canonical forms, factorizations, spectral theory, matrix functions and equations, and multilinear algebra. Applications of the theory will also be considered. *Prerequisite:* Mathematics 253 and 274, or equivalent. *Four credit hours.* LIVSHITS

MA355s Combinatorics Combinatorics can be thought of as the study of counting things. Topics to be covered in this course include basic counting principles, recurrence relations, graphs and trees, distributions and partitions, generating functions, inclusion/exclusion, and permutations. *Prerequisite:* Mathematics 274. *Four credit hours.* FRIEDMANN

[MA357] Elementary Number Theory Number theory deals with questions about numbers, especially those related to prime numbers and integral and rational solutions of equations. The subject offers a wide array of problems that are easily stated and understood but that can be difficult to solve. Students will gain an understanding of the beauty that such problems offer as well as the persistence that is often necessary in tackling them, and they will strengthen their program-solving and proof writing skills. Topics may include prime numbers and unique factorization; Diophantine equations; congruences; Fermat's Little Theorem, the Chinese Remainder Theorem, and RSA cryptography; quadratic residues, reciprocity. *Prerequisite:* Mathematics 274. *Four credit hours.*

[MA359] Finite Fields and Error Correcting Codes How can data be transmitted effectively over a wired or wireless connection without constant errors? The key is the use of error-correcting codes. This course is an introduction to the mathematics behind coding, including error detection and error correction. Students will be introduced to finite fields and use them to create codes and to investigate their properties. A small amount of information theory will be included. *Prerequisite:* Mathematics 122 or 162 and 253. *Four credit hours.*

MA376f History of Mathematics The history of mathematics with emphasis on the interaction between mathematics, culture, and society. Writing-intensive and involving careful reading of original historical documents. By studying the mathematics of different times and cultures, students will deepen their own understanding of mathematics and develop a clearer idea of how society and mathematics influence each other. A survey of the history of mathematics is followed by a more careful tracing of the development of one theme or topic. Specific topics vary from year to year but often include the mathematics of non-Western cultures. *Prerequisite:* Mathematics 274 or 275. *Four credit hours.* **H.** GOUVEA

MA378s Introduction to the Theory of Computation Listed as Computer Science 378. *Four credit hours.* AARON

MA381fs Probability A mathematical introduction to probability theory, the foundation for commonly used inferential statistical techniques (covered in Statistics 482). Students will learn the basic theorems of probability and computational techniques for finding probabilities associated with stochastic processes. Topics include axiomatic foundations, combinatorics, random variables, discrete and continuous probability distributions, special probability distributions, independence, conditional and marginal probability distributions, properties of expectations, moment generating functions, sampling distributions, weak and strong laws of large numbers, and the central limit theorem. *Prerequisite:* Mathematics 122 or 162; 274 is recommended. *Four credit hours.* BONTEA, O'BRIEN

[MA382] Mathematical Statistics II: Inference Listed as Statistics 382. *Four credit hours.*

MA397f Geometry and Topology of Knots The mathematics of knot theory is strongly connected to many different mathematical and scientific disciplines. This course provides an introduction to knot theory, emphasizing its combinatorial, topological, and geometric aspects. Along the way, weùll learn foundational concepts from geometry, topology, and algebra and see the relevance to certain problems in biology, chemistry, and physics. The course will emphasize the process of posing creative questions about mathematical objects and turning intuitive ideas into precise, mathematical ones. Coursework will involve reading carefully selected journal articles, solving mathematics problems, writing solutions, and pursuing and presenting targeted research projects. *Prerequisite:* Mathematics 274. *Four credit hours.* TAYLOR

MA411s Topics in Differential Equations A sequel to Mathematics 311, with higher-level content and a more extensive study of differential equations. Students will implement advanced analytical methods, examine theory, and demonstrate an understanding of further applications. Topics will vary from year to year. May be repeated for credit with permission of instructor. *Prerequisite:* Mathematics 122 or 162, and 253, and 311. *Four credit hours.* RANDLES

MA434s Topics in Abstract Algebra One semester's exposure to algebra is not sufficient for further work in mathematics, so this is a continuation of Mathematics 333. Students will further develop their ability to speak the language of and use the methods of algebra through the study of one particular algebraic theory. Improving one's written and oral communication of mathematics is an integral part of the course. Topics will vary from year to year. May be repeated for credit with permission of instructor. *Prerequisite:* Mathematics 333. *Four credit hours.* YOUNGS

MA439f Topics in Real Analysis A sequel to Mathematics 338. Students will deepen their understanding of analysis through the exploration of more-advanced topics and will sharpen their ability to read, analyze, construct, and present proofs. Improving one's written and oral communication of mathematics is an integral part of the course. Topics will vary from year to year. May be repeated for credit with permission of instructor. *Prerequisite:* Mathematics 338. *Four credit hours.* MATHES

[MA472] Topics in Mathematical Modeling Mathematical modeling provides a means to explain and predict phenomena. Applications are numerous, especially in the physical and social sciences. Students will learn to correctly interpret existing models and create new ones and will develop an understanding of the purpose and uses of mathematical models. The emphasis will be on analyzing research publications and on producing research-level mathematical models. Writing and discussion will be important components. Computers will be used for analysis and simulation. Topics will vary from year to year. May be repeated for credit with permission of instructor. *Prerequisite:* Mathematics 122 or 162, and 253, and 311. *Four credit hours.*

MA482s Topics in Statistical Inference Listed as Statistics 482. *Four credit hours.* ZELDOW

MA484s Honors Project The independent study component of the honors program in mathematics. Cannot be counted toward the major or minor. *Prerequisite:* Permission of the instructor and admission to the honors program. *Three or four credit hours.* FACULTY

MA484s Honors Project The independent study component of the honors program in mathematics. Cannot be counted toward the major or minor. *Prerequisite:* Permission of the instructor and admission to the honors program. *Three or four credit hours.* FACULTY

MA491f, 492s Independent Study Independent study in an area of mathematics of particular interest to the student. *Prerequisite:* Permission of the instructor. *One to four credit hours.* FACULTY