# Error Analysis Example

Error analysis is always a difficult area for students. However, the careful consideration of experimental error is one of the important skills that we need to learn to be effective scientists. In the following discussion, the errors in a titration experiment are considered. The first section is a detailed look at how to determine the most important errors. The second section is an example of the corresponding text that would be written in a lab report for CH141.

## Determining the Important Errors

• The **purpose** of the error analysis section of the lab report is to determine the most important errors and the effect that those errors have on the final result.

• **Random Errors**: Random errors cause positive and negative deviations from the average value of a measurement. Random errors cancel by averaging, if the experiment is repeated many times. Upon averaging many trials, random errors have an effect only on the precision of a measurement. For a single determination, a random error has the same numerical effect on the precision and accuracy. For a small number of trials, random errors have a small effect on the accuracy, because the positive and negative fluctuations do not completely cancel. In a quick review of the errors in an experiment, it is sufficient to consider that the effect of random errors is primarily on the precision. Every non-integer experimental measurement is a source of random error. The size of the random error is estimated from the readability of the device.

• **Systematic Errors**: Without any changes in the procedure, systematic errors are repeated if the experiment is repeated. Systematic errors have a biased effect on the final results; systematic errors make the final result high or low, but not both. Instrument calibration errors are examples of systematic errors. An example of a systematic error from the $CaCO_3$ precipitation experiment is that small particles pass through the glass frits in a Gooch crucible, making the final precipitate mass too small. Systematic errors affect the accuracy of the final results. However, systematic errors also have a random component. For example, a miscalibrated balance is a source of random and systematic error. Systematic errors are often corrected by completing a determination using a different method or by comparing results among different laboratories.

• **Student Mistakes**: Student mistakes are just student mistakes; they are neither random nor systematic errors. Examples in this category are spills, misreading a device such as a burette, misinterpretation of the procedure, incorrect handling of a micro-pipettor, and forgetting to rinse out a beaker when doing a quantitative transfer. These errors are easily preventable, if the experiment is repeated. Mistakes should be noted in the Results section of your report.

***Example***: Titration of an Unknown Acid:
A 25.00 mL sample of an unknown acid is titrated with 15.67 mL of 0.1042 M NaOH. The volume of the acid is determined using a volumetric pipette and the burette used in the experiment has scale divisions every 0.1 mL. The standard base was made using an analytical balance and a volumetric flask. The end point is determined by visually detecting the pink color of phenolphthalein.

***Answer*:**
Random Measurement Errors: Every measurement is a source of random error. However, we must identify those errors that have a significant effect on the final result. The effects on the final result are determined using significant figure rules. The concentration of the unknown acid is:

$$M_{unknown} = V_{titrant}M_{titrant}/V_{unknown} = 0.01567\ L(0.1042\ mol/L)/0.02500\ L = 0.4821\ M$$

Since only multiplications and divisions are involved, the number of significant figures in the final result is equal to the smallest number of significant figures of the terms in the calculation.

*Volumetric Glassware and Analytical Balance Measurements*: Standard volumetric glassware typically has a precision and accuracy of four significant figures. The accuracy and precision of mass measurements on an analytical balance are also typically to four significant figures (±0.0002). The expected precision in the final result, using the analytical balance and volumetric glassware, is four significant figures.

*Measurements that are Interpolated between Scale Markings*: The burette readings are not so precise. To determine the volume of titrant delivered, two readings are made. Each reading is recorded to the nearest 0.01 mL. However, visually estimating the volume to better than ±0.02 mL is difficult. Consequently the precision of the volume delivered by the burette is poorer than ±0.02 mL, since two readings are necessary. Correspondingly, the final unknown concentration is officially known to three significant figures. The conclusion is that the precision is determined primarily by the random error in the burette readings. The random error in the other volume and mass determinations are not consequential.

Systematic Measurement Errors: Every measurement is a potential source of systematic error. However, with thoughtful construction of the procedure many measurements can be discounted as significant sources of systematic error. So while the calibration of the glassware and the balance used in a titration experiment are technically sources of systematic error, these errors are easily avoided. The calibration of the balances is periodically checked using a registered calibration mass. Standard volumetric glassware is certified by the manufacturer through calibration against National Institute of Standards and Technology (NIST) traceable procedures. In a titration experiment the only significant systematic errors are in the purity of the reagents and the visual determination of the end point. The purity of the reagents also includes absorption of moisture from the ambient air. So reagents that are susceptible to atmospheric moisture absorption are usually kept in low humidity desiccators. In a titration, the primary systematic error is the endpoint determination. The difference between the **equivalence point** and the measured **end point** is called the titration error. A visual end point is always slightly beyond the equivalence point because of the necessity of seeing the color change by eye. The result is that the volume of titrant delivered is too large, giving a larger final concentration than the true value. The conclusion is that the accuracy is determined primarily by systematic error in the end point.

## Lab Report Section on Error Analysis

The discussion, above, gives the complete thought process for determining the most important errors in the experiment. The section in the lab report that presents your conclusions is disappointingly short, by comparison:

> The precision is dominated by the random error of the volume readings on the burette. The volumetric glassware contributes insignificant random error upon proper use. The accuracy is determined by the systematic error in the visual detection of the end point. A visual end point is always larger than the equivalence point, giving a higher final result than the true value.