MATHEMATICS AND STATISTICS

Chair, *Professor Fernando Gouvêa*
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The Department of Mathematics and Statistics offers courses for students who: (1) plan a career in an area of pure or applied mathematics, including statistics; (2) need mathematics as support for their chosen major; or (3) elect to take mathematics as part of their liberal arts education or to fulfill the area requirement in quantitative reasoning.

The department offers three programs: majors in mathematics and mathematical sciences and a minor in mathematics. The major in mathematical sciences is also offered with a concentration in statistics. Majors in mathematics and mathematical sciences can be taken with honors. In addition, there is an interdepartmental joint major in economics-mathematics.

Colby mathematics majors in recent years have entered graduate school to do advanced work in mathematics, statistics, biostatistics, engineering, economics, computer science, biomathematics, and the sciences. They also have used the major as a solid foundation for careers in teaching, law, medicine, banking, insurance, management, the computer industry, and other areas.

All incoming students who intend to enroll in mathematics courses in the fall semester are required to complete the mathematics placement questionnaire prior to registration.

Requirements for the Major in Mathematics

Completion of each of the following with a grade of C- or higher*: One year of calculus, Mathematics 253, 274, 333, 338, either 434 or 439, plus four additional courses selected from Mathematics 262 and any three- or four-credit mathematics courses numbered 300 or above (excluding 484). In exceptional cases, with the permission of the department, another 400-level course may be substituted for 434 or 439.

Although Mathematics 262 and 352 are not specifically required, the department strongly recommends that mathematics majors complete both courses.

Requirements for the Major in Mathematical Sciences

Completion of each of the following with a grade of C- or higher*: One year of calculus, Mathematics 253, 262, 274, Computer Science 151; one course (to establish an overall theme for the major) selected from Mathematics 311, 332, 372, 381, Computer Science 231; four additional three- or four-credit courses selected from mathematics courses numbered 200 or above (excluding Mathematics 484). With written permission of the advisor, one (or, in exceptional cases, two) of these courses may be replaced by a course with significant mathematical content from another department.

Requirements for the Major in Mathematical Sciences with a Concentration in Statistics

Completion of each of the following with a grade of C- or higher*: One year of calculus, Mathematics 212 or 231, 253, 262, 274, 381, 382, Computer Science 151; one course selected from Mathematics 306, 321, and 374; one additional three- or four-credit course selected from mathematics courses numbered 300 or above (excluding Mathematics 484). Students interested in pursuing a graduate degree in statistics are advised to take Mathematics 338.

Requirements for the Honors Program in Mathematics or Mathematical Sciences

An honors program is available for students majoring in mathematics and mathematical sciences who have a grade point average of at least 3.25 in all mathematics courses numbered 200 or higher and who complete an additional, preapproved program of independent study in the major (Mathematics 484) culminating in both a written paper and a colloquium presentation. Students who successfully complete the requirements and who receive recommendation of the department will graduate "With Honors in Mathematics" or "With Honors in Mathematical Sciences."

Requirements for the Minor in Mathematics

Six three- or four-credit mathematics courses, including completion of at least one semester of calculus, Mathematics 253, and at least one course at the 300 level or above.

The point scale for retention of the majors and minor applies to all courses in the majors/minors. No requirement for the majors

or minor may be taken satisfactory/unsatisfactory.

**Note: The requirement of a grade of C- or above for each course begins with the Class of 2015.*

Course Offerings

MA101f Calculus with Pre-calculus I Designed for students who enter Colby with insufficient algebra and pre-calculus background for the standard calculus sequence. It is expected that all students who complete Mathematics 101 will enroll in Mathematics 102 in the following January. The combination of 101 and 102 covers the same calculus material as Mathematics 121. Completion of 101 alone does not constitute completion of a College calculus course for any purpose; in particular, it does not qualify a student to take Mathematics 122 nor does it satisfy the quantitative reasoning requirement. First-year students must complete the mathematics placement questionnaire prior to registration. *Three credit hours.* SUKIENNIK

MA102j Calculus with Pre-calculus II A continuation of Mathematics 101. Successful completion of both Mathematics 101 and 102 is equivalent to completion of Mathematics 121. *Prerequisite:* Mathematics 101. *Three credit hours.* **Q** SUKIENNIK

[MA110] Statistical Thinking Statistics is the science of learning from data; it provides tools for understanding data and arguments based on data in many diverse fields. Students will learn to describe data in basic terms and to verbalize interpretations of it. Topics include graphical and numerical methods for summarizing data, methods of data collection, basic study design, introductory probability, confidence intervals, and statistical inference. Does not count toward any major or minor. Credit may be received for only one of Mathematics 110, 212, or 231. *Three credit hours.* **Q**

[MA111] Mathematics as a Liberal Art Mathematics is one of humanity's longest-running conversations. Its crucial role in the thought-world of medieval Europe can be seen in the fact that four of the original seven liberal arts were inherently mathematical. Today, mathematics is just as important, permeating our culture. Students will develop awareness of the historical and contemporary roles of mathematics so that they will better understand the nature of mathematics, will know what kinds of things mathematics does well, and will know when to ask for a mathematician's help with their intellectual work. Specific topics discussed will vary. *Four credit hours.* **Q**

MA121fs Single-Variable Calculus Calculus is the result of centuries of intellectual effort to understand and quantify change, such as the position of a moving object or the shape of a curve. Competent users of calculus understand its intellectual structure sufficiently to apply its ideas to a variety of intellectual pursuits. Topics include differential and integral calculus of one variable: limits and continuity; differentiation and its applications, antiderivatives, the definite integral and its applications; exponential, logarithmic, and trigonometric functions. First-year students must complete the mathematics placement questionnaire prior to registration. *Four credit hours.* **Q** FACULTY

MA122fs Series and Multi-Variable Calculus A continuation of Mathematics 121. Students will learn how to use infinite series, both to represent and to approximate functions, and will extend all of their skills from single-variable calculus to the multivariable setting. Topics: infinite series; vectors and analytic geometry in two and three dimensions; partial derivatives, differentials and the gradient; integration in two and three variables. *Prerequisite:* A course in single-variable calculus. First-year students must complete the mathematics placement questionnaire prior to registration. *Four credit hours.* **Q** FACULTY

MA161f Honors Calculus I The first in a two-course sequence that treats the material of Mathematics 121 and 122 with a focus on the intellectual structure behind the methods. Students will acquire a deep understanding of the theory and foundational facts of calculus, will be able to use the techniques in an intelligent manner, will understand and be able to explain the arguments that undergird those techniques, and will be able to construct original arguments of their own. Topics are presented as a deductive mathematical theory, with emphasis on concepts, theorems, and their proofs. May not be taken for credit if the student has earned credit for Mathematics 122. *Prerequisite:* One year of calculus in high school. First-year students must complete the mathematics placement questionnaire prior to registration. *Four credit hours.* **Q** MATHES

MA162s Honors Calculus II A continuation of Mathematics 161. Topics are essentially the same as for Mathematics 122, but they are presented as a deductive mathematical theory, with emphasis on concepts, theorems, and their proofs. May not be taken for credit if the student has earned credit for Mathematics 122. *Prerequisite:* Mathematics 161. *Four credit hours.* MATHES

MA194s Mathematics Seminar An opportunity to read and discuss audience-appropriate mathematical material in an informal setting with members of the mathematics faculty, away from problem sets and exams. Successful students will show improvement in reading comprehension of mathematical articles, will increase their knowledge and understanding of the scientific community and the specific ways of mathematicians and statisticians, and will become familiar with mathematical issues of the past and present not normally covered in other courses. May be repeated for additional credit. *Prerequisite:* Mathematics 102, 121, 122, or 161. *One credit hour.* MALMENDIER, SCOTT

MA212fs Introduction to Statistical Methods An exploration of statistical methods relevant to a broad array of scientific disciplines. Students will learn to properly collect data through sound experimental design and to present and interpret data in a meaningful way, making use of statistical computing packages. Topics include descriptive statistics, design of experiments, randomization, contingency tables, measures of association for categorical variables, confidence intervals, one- and two-sample

tests of hypotheses for means and proportions, analysis of variance, correlation/regression, and nonparametrics. To learn about multiple linear regression techniques students should take Mathematics 231. Credit can be received for only one of Mathematics 110, 212, or 231. *Four credit hours.* **Q** SCOTT

MA231fs Applied Statistics and Regression Analysis Regression modeling provides a way to interpret data and to gain insight about the processes and populations behind them. Extracting useful information requires careful consideration of context, critical thinking, and a sound understanding of fundamental statistical concepts. Students will explore tools, including statistical software, that help them make sense of data and will analyze it with quantitative outcomes. Topics include descriptive statistics, sampling theory, confidence intervals, one- and two-sample tests of hypotheses, correlation, simple linear regression, and multiple linear regression. Credit may be received for only one of Mathematics 110, 212, or 231. *Prerequisite:* Mathematics 102, 121, or 161. *Four credit hours.* **Q** O'BRIEN, SCOTT

MA253fs Linear Algebra Linear algebra is a crossroads where many important areas of mathematics meet, and it is the tool used to analyze the first approximation of complex systems. Students will learn to understand and use the language and theorems in both abstract and applied situations, gain insight into the nature of mathematical inquiry, and learn how to reason carefully and precisely about formally described situations. Topics include vectors and subspaces in $\mathbf{R}^n$, linear transformations, and matrices; systems of linear equations; abstract vector spaces and the theory of single linear transformation: change of basis, determinants, eigenvalues and eigenvectors, and diagonalization. *Prerequisite:* Mathematics 102, 121, 122, or 161. *Four credit hours.* BRETSCHER, LIVSHITS, MATHES, SUKIENNIK

MA262s Vector Calculus Develops ideas first seen in Mathematics 122 by applying the notions of derivative and integral to multi-variable vector-valued functions. The goal is to understand the high-dimensional versions of the fundamental theorem of calculus and to apply these theorems to specific scientific applications. Topics include parameterized curves and surfaces; gradient, divergence, and curl; change of variables and the Jacobian; line and surface integrals; conservative vector fields; Green's, Stokes's, and Gauss's theorems; applications. Previously offered as Mathematics 302. *Prerequisite:* Mathematics 122 or 162. *Four credit hours.* HOLLY

MA274fs Introduction to Abstract Mathematical Thought An introduction to fundamental mathematical techniques used in all upper-level mathematics courses: proofs, logical reasoning, and the axiomatic method. Topics include principles of mathematical logic and standard methods of direct and indirect proof, including mathematical induction; set-theoretic approach to functions and relations; the theory of infinite sets; elementary algebraic structures; and techniques from discrete mathematics. Credit can be received for only one of Mathematics 274 and 275. *Prerequisite:* Mathematics 102, 121, 122, or 161. Two semesters of calculus is recommended. *Four credit hours.* LAMBERT, MATHES

MA275fs Introduction to Topics in Abstract Mathematics Some students are sufficiently proficient with proofs and logic that they do not need to take Mathematics 274; this offers an alternative that focuses less on proof techniques and more on the set theory and related topics. The goal is to equip students to continue their study of mathematics. Topics include set-theoretic approach to functions and relations, the theory of infinite sets, elementary algebraic structures, and techniques from discrete mathematics. Credit can be received for only one of Mathematics 274 and 275. *Prerequisite:* Mathematics 161 and 162 and permission of the department. *Two credit hours.* MATHES

MA306s Topics in Epidemiology The purposes of epidemiological research are to discover the causes of disease, to advance and evaluate methods of disease prevention, and to aid in planning and evaluating the effectiveness of public health programs. Students will learn about the historical development of epidemiology, a cornerstone of public health practice. Through the use of statistical methods and software, they will explore the analytic methods commonly used to investigate the occurrence of disease. Topics include descriptive and analytic epidemiology; measures of disease occurrence and association; observational and experimental study designs; and interaction, confounding, and bias. Previously offered as Mathematics 398 (spring, 2010). *Prerequisite:* Mathematics 212, 231, or 382. *Four credit hours.* SCOTT

MA311fs Ordinary Differential Equations Differential equations allow us to deduce the long-term behavior of quantities from information about their short-term rates of change; for that reason they are the language of classical science. Students will learn to analyze concrete situations modeled by differential equations and to draw conclusions using equations, graphical techniques, and numerical methods. Topics include theory and solution methods of ordinary differential equations, linear differential equations, first-order linear systems, qualitative behavior of solutions, nonlinear dynamics, existence and uniqueness of solutions, and applications. *Prerequisite:* Mathematics 122 or 162, and 253. *Four credit hours.* HOLLY, MALMENDIER

MA313s Differential Geometry The study of curves and surfaces in three-dimensional space, with the primary focus being on the nature of "curvature" and the distinction between intrinsic and extrinsic geometry. Students will improve their spatial intuition and learn to move easily between general theorems and specific examples. Topics include curves: tangent, normal, and binormal vectors; curvature and torsion; the moving frame; surfaces: the first and second fundamental forms, sectional and Gaussian curvature, the *Theorema Egregium*, geodesics, parallel transport; and selected additional topics. *Prerequisite:* Mathematics 122 or 162, and 253, and 274 or 275. *Four credit hours.* MALMENDIER

[MA321] Applied Regression Modeling The influence of statistics has grown to affect all aspects of our lives, from health to business to public policy. Students will expand on their inferential statistical background and explore methods of modeling data through linear and nonlinear regression analysis. Through the use of statistical software, they will learn how to identify possible

models based on data visualization techniques, to validate assumptions required by such models, and to describe their limitations. Topics include multiple linear regression, multicollinearity, logistic regression, models for analyzing temporal data, model-building strategies, transformations, model validation. *Prerequisite:* Mathematics 212, 231, or 382. *Four credit hours.*

MA331f Topology Begins as the abstract mathematical study of the notions of proximity and continuity and then deploys these methods to understand interesting objects and spaces. Students will develop their ability to construct precise arguments and to explore concrete examples as instances of a general theory. Topics are selected at the discretion of the instructor from the areas of point-set, differential, and algebraic topology. *Prerequisite:* Mathematics 274 or 275. *Four credit hours.* MATHES

MA332f Numerical Analysis In practice, a solution to a problem might be impossible to obtain by classical methods of manipulating equations. Nonetheless, solutions can often be obtained by numerical methods, usually with the aid of a computer. Numerical analysis is the study of those numerical algorithms. Students will acquire the ability to use standard methods and mathematical software for solving the most common types of numerical problems and to analyze the speed and accuracy of the solutions. Topics include solution by numerical methods of linear and nonlinear equations, systems of equations, and differential equations; numerical integration; polynomial approximation; matrix inversion; error analysis. *Prerequisite:* Mathematics 122 or 162, and 253. *Four credit hours.* HOLLY

MA333f Abstract Algebra Simply called "algebra" by mathematicians, it is the study of abstract sets with operations and is fundamental in expressing and working in theoretical mathematics. An introduction to that language, to the motivating examples, and to some of the fundamental theorems. Students will develop their ability to discover and write formal arguments, explore the relationship between general theory and specific examples, and learn to recognize algebraic structures where they occur. Topics include groups, rings, and fields: definition, basic theorems, and important examples. *Prerequisite:* Mathematics 253, and 274 or 275. *Four credit hours.* GOUVEA

MA336s Mathematical Economics Listed as Economics 336. *Prerequisite:* Economics 224, Mathematics 253, and either Mathematics 122 or 162. *Four credit hours.* VULETIN

MA338s Real Analysis An exploration of the theory behind calculus, as well as its extension to more general settings. Students will learn to think carefully and clearly about limiting processes such as differentiation, integration, and summation of series and to interpret their knowledge in terms of the topology of metric spaces. They will develop the ability to read and to produce formal mathematical arguments, with particular attention to handling exceptional cases and delicate issues of convergence. Special focus on foundational issues: topology of metric spaces, continuity, differentiation, integration, infinite series. *Prerequisite:* Mathematics 122 or 162, and 274 or 275. *Four credit hours.* LIVSHITS

[MA352] Complex Variables The arithmetic and calculus of complex numbers and functions. The properties of analytic functions, including Cauchy's integral theorem and formula, representation by Laurent series, residues and poles, and the elementary functions. Offered in alternate years. *Prerequisite:* Mathematics 122 or 162, and 274 or 275. *Four credit hours.*

[MA357] Elementary Number Theory Number theory deals with questions about numbers, especially those related to prime numbers and factorization. It offers a wide array of problems that are easily stated and understood but that can be difficult to solve. Students will gain an understanding of the beauty that such problems offer as well as the persistence that is often necessary in tackling them, and they will strengthen their problem-solving and proof-writing skills. Topics include prime numbers and unique factorization; congruences, Fermat's Little Theorem, the Chinese Remainder Theorem, and RSA cryptography; quadratic residues, reciprocity, quadratic forms, and the Pell Equation. *Prerequisite:* Mathematics 102, 121, 131, or 161. Two semesters of calculus or Mathematics 253 is recommended. *Four credit hours.*

[MA372] Mathematical Modeling Applicable mathematics becomes applied mathematics when we construct a mathematical theory that models the world in a useful way. Students learn to do this using many different types of mathematical tools. Students will continue to develop their problem-solving skills and their ability to present mathematical models to others. Topics include application of mathematics to problems in a variety of areas; interpretation of existing mathematical models, analysis, and computer simulation; formulation and development of new mathematical models. *Prerequisite:* Mathematics 122 or 162, and 253. *Four credit hours.*

MA374f Design and Analysis of Experiments Students will learn how to identify potential sources of variation and plan experiments accordingly, paying attention to the desired comparisons. Statistical computing software will be used to perform analysis of variance and post-estimation techniques in a variety of experimental designs. Emphasis on statistical thinking and applications as well as the underlying mathematical structures and theory. Topics include completely randomized factorial designs, randomized block designs, Latin squares, factorial designs, and fractional factorial designs. Formerly offered as Mathematics 398A. *Prerequisite:* Mathematics 212, 231, or 382. *Four credit hours.* O'BRIEN

MA376s History of Mathematics The history of mathematics with emphasis on the interaction between mathematics, culture, and society. Writing-intensive and involving careful reading of original historical documents. By studying the mathematics of different times and cultures, students will deepen their own understanding of mathematics and develop a clearer idea of how society and mathematics influence each other. A survey of the history of mathematics is followed by a more careful tracing of the development of one theme or topic. Specific topics vary from year to year but often include the mathematics of non-Western cultures. *Prerequisite:* Mathematics 274 or 275. *Four credit hours.* **H** GOUVEA

[MA378] Introduction to the Theory of Computation Listed as Computer Science 378. *Prerequisite:* Computer Science 231 and either Mathematics 274 or 275. *Four credit hours.*

MA381f Mathematical Statistics I: Probability A mathematical introduction to probability theory, the foundation for commonly used inferential statistical techniques (covered in Mathematics 382). Students will learn the basic theorems of probability and computational techniques for finding probabilities associated with stochastic processes. Topics include axiomatic foundations, combinatorics, random variables, discrete and continuous probability distributions, special probability distributions, independence, conditional and marginal probability distributions, properties of expectations, moment generating functions, sampling distributions, weak and strong laws of large numbers, and the central limit theorem. *Prerequisite:* Mathematics 122 or 162. *Four credit hours.* O'BRIEN

MA382s Mathematical Statistics II: Inference Building on their background in probability theory, students explore inferential methods in statistics and learn how to evaluate different estimation techniques and hypothesis-testing methods. Students learn techniques for modeling the response of a continuous random variable using information from several variables using regression modeling. Topics include method of moments and maximum likelihood estimation, sample properties of estimators including sufficiency, consistency, and relative efficiency, Rao-Blackwell theorem, tests of hypotheses, confidence intervals, linear models, and analysis of variance. Although applications are discussed, the emphasis is on theory. *Prerequisite:* Mathematics 381. *Four credit hours.* O'BRIEN

[MA391] Problem-Solving Seminar Mathematicians solve problems. This seminar on problem solving is designed for students of all levels. The focus is on mathematical puzzles and curiosity-driven mathematics. The goal is to explore systematic ways in which nonstandard problems can be approached. Facts and strategies presented will be of value to both pure and applied pursuits. Topics will vary. May be repeated for credit. *One credit hour.*

[MA411] Partial Differential Equations Applying the methods of differential equations to a multi-variable setting involving both time and space generates a whole new theory, which is at the core of much scientific computation, mathematical physics, and several other areas of applied mathematics. An introduction to the main ideas of that theory, preparing students for further work in applied mathematics. Topics include linear and nonlinear partial differential equations, systems; initial value problems, boundary value problems; analytic and numerical methods of solution; applications. *Prerequisite:* Mathematics 253 and 311. *Four credit hours.*

MA434s Topics in Abstract Algebra One semester's exposure to algebra is not sufficient for further work in mathematics, so this is a continuation of Mathematics 333. Students will further develop their ability to speak the language of and use the methods of algebra through the study of one particular algebraic theory. Improving one's written and oral communication of mathematics is an integral part of the course. Topics will vary from year to year. May be repeated for credit with permission of instructor. *Prerequisite:* Mathematics 333. *Four credit hours.* LIVSHITS

MA439f Topics in Real Analysis A sequel to Mathematics 338. Students will deepen their understanding of analysis through the exploration of more-advanced topics and will sharpen their ability to read, analyze, construct, and present proofs. Improving one's written and oral communication of mathematics is an integral part of the course. Topics will vary from year to year. May be repeated for credit with permission of instructor. *Prerequisite:* Mathematics 338. *Four credit hours.* GOUVEA

MA484s Honors Independent Study The independent study component of the honors program in mathematics. Cannot be counted toward the major or minor. *Prerequisite:* Permission of the instructor and admission to the honors program. *Three or four credit hours.* FACULTY

MA491f, 492s Independent Study Independent study in an area of mathematics of particular interest to the student. *Prerequisite:* Permission of the instructor. *One to four credit hours.* FACULTY