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Mathematicians Writing

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In this article we report on part of a study of the epistemological perspectives of practicing research mathematicians. We explore the identities that mathematicians present to the world in their writing and the ways in which they represent the nature of mathematical activity. Analysis of 53 published research papers reveals substantial variations in these aspects of mathematicians' writing. The interpretation of these variations is supported by extracts from interviews with the mathematicians. We discuss the implications for students and for novice researchers beginning to write about their mathematical activity.

Key Words: College/university; Connections in mathematics; Discourse analysis; Language and mathematics; Linguistics; Writing/communication

In recent years, there has been increasing recognition in the mathematics education community of the social nature of mathematical activity and of the importance of communication within the practices of doing, teaching, and learning mathematics (e.g., Boaler, 1997; Burton, 1999b; Resnick, Levine, & Teasley, 1991; Steffe & Gale, 1995; Stephens, Waywood, Clarke, & Izard, 1993; and with a particular focus on equity, Secada, Fennema, & Adajian, 1995). Interest in mathematical language has gone beyond analysis of the mathematical symbol system and specialist vocabulary. The way in which language is used by teachers and students in classrooms is now a consideration as is its influence on the social identities and the views of mathematics that students construct (see, e.g., Pimm, 1984, 1987, on the implications of the use of *we;* Rowland, 1995, on the expression of uncertainty; and Gerofsky, 1996, on word problems). A considerable body of literature has appeared on writing to learn mathematics (see, e.g., Countryman, 1992). The language used in mathematical practices, both in and out of school, shapes the ways of being a mathematician and the conceptions of the nature of mathematical knowledge and learning that are possible within those practices.

Professional organizations have recognized the importance of engaging students in talking in the classroom, analyzing the nature of mathematical talk, and helping students to learn how to participate in oral mathematical discourse (Mathematical Association, 1987; National Council of Teachers of Mathematics, 1989). Although there is some interest in using writing as a means of learning (Connolly & Vilardi, 1989), less attention has been paid to the nature of written mathematical texts or to learning how to write mathematically (but see Morgan, 1998). Yet, writing plays

We are particularly grateful to our participants for their generosity of time and for providing papers and to the anonymous referees for their helpful comments on this article.

a crucial role in many mathematical practices. The stakes involved in producing mathematical texts that are seen to be acceptable are often high, both for students who are likely to be assessed on the basis of their written work and for professional mathematicians whose status within the community and even job security may depend on the quality (and publishability) of their writing. In the educational context, students at many levels find writing difficult and may not communicate their mathematical thinking effectively (see MacNamara & Roper, 1992, on reports of investigative work in secondary school; Alibert & Thomas, 1991, on proofs at the undergraduate level). To help students (and more experienced mathematicians) at all levels to become effective mathematical writers, educators need to understand more about the kinds of writing they may be asked to produce and the forms that are seen as being appropriate for specific purposes. We therefore consider investigating the nature of writing in mathematical practices to be important. Our focus is the "natural language" within which the symbolic or special vocabulary and structures particular to mathematics are embedded. Although sometimes seen to be peripheral to the main mathematical content, natural language serves in the construction of the identities of the author and reader and of the epistemological and ontological assumptions underlying the writing.

Much writing about "the language of mathematics" treats it as if it were a unitary object that is the same in all circumstances. This approach is taken particularly when, as is often the case, research focuses on discrete features of language such as algebraic notation or special items of vocabulary. In whole texts, however, enormous diversity exists. Just as varying social practices may be labeled as *mathematics* (including academic mathematics, school mathematics, recreational mathematics), various genres of text may be called *mathematical* (e.g., research paper, textbook, examination question and answer, puzzle). The variety of types of writing used by mathematicians has been discussed by Mousley and Marks (1991), and at a more general level, Richards (1991) identified at least four distinct "domains of discourse" and modes of argument associated with mathematics (research, inquiry, journal, and school math).

Most readers of this article can probably distinguish readily among extracts taken from, for example, an academic research paper, a primary school textbook, or the puzzle corner of a popular magazine. Not just the subject matter varies. The social functions that may be fulfilled by each of the texts also vary, including the construction of roles for both author and readers and of a relationship between them. Our current interest in the language of mathematical practices lies in understanding "what language is doing and being made to do by people in specific situations in order to make particular meanings" (Kress, 1993, p. 23). This understanding not only provides insight into "the social needs and the cultural values and meanings of its users" (p. 23) but also lies at the root of efforts to help mathematical writers to write more effectively, using mathematical language deliberately within their own specific situations to make the particular meanings that they intend. In this article, our focus is on the writing of research papers by professional mathematicians. By analyzing this writing, we hope to shed some light on the values and meanings of the practices in which these mathematicians are engaged. At the same time, however, the knowledge gained about the nature of the language used has potential to empower writers in this genre. Because the analytic tools used here (derived from Halliday's systemic-functional grammar, see Halliday, 1985; Morgan, 1996) may also be used for analyzing other mathematical genres, they may serve to support students' learning to write mathematically in a variety of practices.[1]

The fact that we can identify that a text belongs to a given mathematical genre—and that we can be surprised by a text that does not quite match our expectations—arises from the conventional use of particular linguistic structures. As novices try to gain acceptance and status within mathematical communities, they must produce texts that meet the conventional expectations of the gatekeepers within their fields. This requirement holds at all levels for students whose work is evaluated by their teachers and for new researchers seeking their first publications. Little attention has been paid, however, to the question of how novices are to learn what the conventions of the various mathematical genres are. Those advocating writing to learn mathematics appear generally to have assumed that students already know how to write or will pick up what they need to know through experience, even stating categorically, "We don't need to teach writing" (McIntosh, 1991, p. 423), in order to encourage mathematics teachers to try using writing in their classrooms. But, as Marks and Mousley pointed out, experience by itself may be too limited, both in quantity and in the variety encountered, to enable such natural development. "Writing is an *unnatural* act: it needs to be learned" (Reid as quoted in Marks & Mousley, 1990, p. 134). Marks and Mousley argued that to develop mathematical literacy, students should be led "to development of an explicit understanding of the role of language in specific mathematical contexts" (p. 133). As Galbraith and Rijlaarsdam (1999) have pointed out, "In order to learn to write pupils need to become aware of, and gain control of, the processes involved in producing text" (p. 102).

There is some debate in the domain of literacy education about whether students should be explicitly taught the characteristics of specific genres of writing (see, e.g., Reid, 1987). On the one hand, teaching the features of a genre is seen to be prescriptive and restrictive, curbing students' opportunities for creativity and self-expression (Dixon, 1987). On the other hand, it is seen as empowering, enabling students to participate in high-status forms of discourse. It is this latter perspective that we adopt. We believe that knowledge of the forms of language that are highly valued within mathematical discourses and of the effects that may be achieved by various linguistic choices would not just help writers to conform to conventional expectations but would also empower them to make informed choices to break the conventions in order to achieve deliberate effects, including to demonstrate creativity and to express their own personalities through their writing. Most

[1] See Morgan (1998) for an example of analysis of secondary school students' writing of reports of mathematical investigations and discussion of how this writing may be used to support learning to write in this genre.

of all, however, we adopt a pedagogic perspective on this debate because we believe that becoming critically aware, through analysis of text, of how it fashions and influences meaning can only enhance one's written communication.

The characteristics of genres are not fixed for all time but are always in the process of change, allowing possibilities for individual writers to participate in that change (Kress, 1993). Such possibilities depend, however, on knowledge of the conventional linguistic forms of the particular genre in question together with understanding of how these forms produce and are produced by the social relations, cultural values, and meanings of the community. We refer not just to knowledge of the formal characteristics of the language (*langue*, to use Saussure's, 1974, term) but, equally important, to knowledge of how the language is actually used (*parole*). In considering the genre of mathematics research papers, we examine the professional discourse of mathematicians, as represented in advice professional organizations (e.g., Gillman, 1987; Knuth, Larrabee, & Roberts, 1989; Krantz, 1997) have published for writers; such advice presents a picture of correct or good practice, contrasting it with errors or what the authors consider to be infelicitous usage. This picture provides knowledge of (current) convention but, because of its normative style, does not empower its readers to view the conventions critically, to choose between alternatives, or to choose to adapt or reject the conventional forms to achieve particular effects. For such empowerment, an approach to mathematical language is needed in which one addresses what *can* be written and what can be achieved instead of attempting to prescribe what *should* be done. In this article, in our analysis of the writing of mathematicians, we are trying not to say that one way of writing is better than any other but to provide some understanding of the variety of ways of writing available within the research-paper genre and the effects that different choices may have on the social relationships, values, and meanings constructed.

THE STUDY

Purpose

In this article, we look at the language used in 53 published mathematics research papers provided to us as part of a broader study undertaken in 1997. We had two aims in analyzing the language of these papers. First, we explored the mathematicians' perspectives as these might be represented in their writing—to identify what social positionings were possible and the extent to which different epistemological standpoints were apparent in the published work. Second, we attempted to describe some of the range of writing styles that may legitimately be available to professional mathematicians. In doing so, we had a particular interest in providing knowledge that might help students and novice mathematicians to make deliberate choices about the images of self and subject matter that they wish to construct in their writing.

Participants and Data Collection

Seventy research mathematicians, 35 women and 35 men, were interviewed for the broader study (Burton, 1999c). Because of their comparative rarity, the women were sought first through an appeal on electronic networks. Each woman's involvement depended upon both her willingness to be interviewed and her ability to find a male counterpart who also agreed to be interviewed. Of the female mathematicians, 17 were categorized as pure mathematicians, 10 were applied mathematicians, and 8 were statisticians. Of the male mathematicians, the numbers were 15, 13, and 7, respectively.

Procedure

The 53 papers we analyzed were obtained by a request, prior to the start of the interview, for a copy of a paper of the interviewee's choice. No constraints were put on the choice of papers submitted for this study. As may be seen in Table 1, a substantial number of these papers were written jointly with one or more other authors; we discuss the implications of coauthoring papers later in this article. These published papers were the focus of a discursive analysis utilizing epistemological perspectives raised within the interviews; when appropriate, our discussion is also informed by extracts from the interviews. We were interested in whether and how the variables of gender, status (as understood in the United Kingdom, i.e., lecturer, senior lecturer, reader, or professor), and the distinction among pure mathematics, applied mathematics, and statistics might interact with the epistemological categories identified in the main study. Thus these categories are used, when appropriate, as descriptors in the body of the article. This practice has enabled us to formulate some conjectures to which we return later. The nature of the study, the choice of participants, and the complexity of the data are unsuitable for quantitative analysis, but at times we have presented quantitative summaries to inform the discussion and to raise particular issues.

Table 1
Characteristics of the Sample of Papers

Field of mathematics	Single author (female/male)	Multiple authors (female/male)[a]	Totals
Pure	14 (7/7)	9 (6/3)	23
Applied	8 (4/4)	12 (5/7)	20
Statistics	2 (2/0)	8 (5/3)	10
Totals	24 (13/11)	29 (16/13)	53

[a]Only the gender of the participant in the study is included. The genders of coauthors not participating in the study have not been taken into account.

Framework for Analysis

In the case of academic papers submitted for publication, making a good impression on editors and reviewers depends on the appropriateness of the writing style as well as on the quality of the content of the paper. That writing style is important was pointed out by a male applied-mathematics lecturer: "You learn that you certainly do have to write all the letters in exactly the form the editor wants or else you won't get to referee those papers, and they won't referee yours." In Anderson's (1988) studies of the reading of academic scientific papers, he suggested that papers written in simpler language may be judged to be less valuable research and hence may be less likely to be accepted. This conclusion appears to contradict some of the advice on writing style offered to beginning academic mathematicians (e.g., Knuth et al., 1989; Steenrod, Halmos, Schiffer, & Dieudonné, 1973), which is that complex syntax should be avoided. Anderson's suggestion was also contradicted by the participants in the study, many of whom, when asked about criteria for publication, expressed strong feelings against obfuscating writing practices and claimed to place a high value on clarity and accessibility of writing. One (female reader) said, "I would like my papers to be understood and well written, and I would like the ideas to come across—the basic ideas to be clear, the insights, the understanding and the resolution, to come across." Another (male lecturer) pointed out, "Mathematical quality alone is not enough. I am very conscious of how the ideas are expressed. I like a story—a paper, a seminar are both stories. There is a beginning, a middle, and an end—so well written, clearly explained in a manner which is easy to follow and logical."

But, like all features that are subject to judgment, clarity is not absolute; it is relative to the context and, in particular, to the expectations and prior knowledge of readers. Moreover, there are some tensions between the ideal of clarity espoused in theory by mathematicians and the practices represented in their writing and in their relationships with editors and reviewers.

So how do beginners learn to write effectively? Is effective writing a matter of learning the expected norms and conforming to them, however much this conformity may restrict what may be said or even conflict with the individual's self-image? Is there even a single set of norms? A critically aware writer (Clark & Ivanic, 1997; Fairclough, 1992) knows that there is always a choice about which form of language to use and that each choice is likely to induce a different effect. The challenge for the beginning mathematical writer is to know what choices are available, which norms can be broken, and what will be achieved by a particular choice.

To understand the norms and the choices available, we used features in the interview data to examine the 53 research papers from two major perspectives: the interpersonal and the ideational (Halliday, 1973). Halliday argued that every text, as well as performing an *ideational* function—saying something about the nature of the world—also performs an *interpersonal* function—saying who the author is and expressing the relationships between author and audience and between author and subject matter.[2] Halliday's functional framework and tools, derived from his functional grammar (Halliday, 1985; see Morgan, 1996, for a fuller discussion of the application of these tools to mathematical texts), helped us to formulate questions and explore them for each of the papers.

Concerning the interpersonal function (*identity*), we tried to answer the following:

- How is the author's identity constructed as an *authority* in the field of research mathematics?
- To what extent and how is the author positioned as *a member of the community* of mathematicians?
- How is the author's relationship to the subject matter of mathematics constructed? In particular, to what extent does the author appear to claim ownership of the subject matter through *demarcation of knowledge or territory* within the field?

Concerning the ideational function (*focus*), we explored the following:

- What picture of the nature of mathematics and mathematical activity is presented? In particular, is the focus on mathematics as a *product of human mathematicians* or *an autonomous mathematical system*, or is it on *the processes of doing mathematics?*

APPLICATION OF THE FRAMEWORK TO EXAMINE RESEARCH PAPERS

Identity

Ask anyone what the characteristics of academic writing are, particularly in the sciences; the response is very likely to include the concept *impersonal.* Yet, according to Halliday's theory, every text says something about the author(s) and the reader(s) as well as about the subject matter. Our interest lies in challenging the assumed impersonality of mathematics texts and teasing out the ways that the personal and interpersonal materialize.

From our interview data, we found that academic writing was seen to be neutral and that its authority stemmed from the facts and arguments contained within it rather than from the personality or position of its author. This perception appeared to stem from *common knowledge,* in the Edwards and Mercer (1987) sense of a common referential framework and a set of shared meanings established within a community through its joint interactions and discourse. Often, members of scientific communities make use of their established sets of meanings without explicit statement or conscious awareness, and the idea that writing should be impersonal coincides with ideas of scientific truth and objectivity. In mathematics, in particular, the apparent absence of the author from the text fits with positivist epistemologies in which the mathematician's role is subordinate to that of the mathematics itself. Davis and Hersh (1981) described the writing of their "ideal mathematician":

[2] Halliday also identified a *textual* function—making the text into a meaningful message—that we do not consider in this article.

> His writing follows an unbreakable convention: to conceal any sign that the author or the intended reader is a human being. It gives the impression that, from the stated definitions, the desired results follow infallibly by a purely mechanical procedure. (p. 36)

Studies of the language of academic scientific papers (e.g., Master, 1991), which consistently show a high proportion of passive constructions, provide evidence for this impersonal quality. For example, in the following extract from one of the papers, the agency of the author is obscured and a view of the subject matter as autonomous, independent of human activity, is suggested by the author's use of passive constructions:[3]

> Now if the same procedure is followed for the solution of the terms proportional to $\Lambda^3 E$ in the perturbation quantities, solutions can be obtained for U_{10}, V_{10}, W_{10}, T_{10} and Ψ_{10}. These are found to be identical to the linear solutions (4.12) with f replaced by the amplitude f_4.

Here, as elsewhere, we are not suggesting that individual authors have made deliberate decisions about their forms of writing, but they are conforming to cultural norms that produce the effects we are indicating. At the same time, those advising writers on style tend to suggest that authors should avoid the passive voice, perhaps because it is thought to be difficult to read. For example, the authors of notes on *Mathematical Writing,* published by the Mathematical Association of America (MAA), stated, "The word 'we' is often useful to avoid passive voice." They advised that an author use *we* in such cases only when meaning to include the reader, not to replace *I*: "Think of a dialog between author and reader" (Knuth et al., 1989, p. 2).

We noted that this advice was *not* taken in most of the papers we inspected. In single-author papers *we* was frequently used to indicate the author alone; for example, a single-author paper contained the sentence "In section 2, we give the full compressible equations governing the flow of interest and the boundary conditions to be satisfied." In many other cases, for example, "We determine V_0 from the following relation . . .," whether *we* is being used to refer to the author or whether the reader is being invited to participate in the determining is unclear. Interpreting the meaning of *we* in particular instances is often problematic. We (the authors and you, our readers) can only speculate about whether any individual author intended to invite her reader to participate in the actions described in the text or merely intended to report the author's own actions. Rotman (1988) pointed to a common-knowledge assumption that to make sense of the text, readers must read mathematics actively, reperforming at least some of what has been done by the author. Rotman distinguished the "thinker" and "scribbler" roles that are constructed for the reader by the use of imperatives in mathematical texts. Imperatives (*consider, integrate*), although unambiguously involving the reader, may or may not include the author as well (implicitly, *let us consider*). The use of *we,* necessarily including the author, is ambiguous as to its intention to include the reader, particularly in the light of its use in cases in which it cannot logically do so (as when used in the past tense

[3] Note the effect on the reader of our own choice of the passive voice.

to describe the author's processes). This apparent inclusivity may, however, have important effects on individual readers. Pimm (1987) reported his own rather negative response to a text that made extensive use of *we:*

> The effect on me of reading this book was to emphasize that choices had been made, ostensibly on my behalf, without me being involved. The least that is required is my passive acquiescence in what follows. In accepting the provided goals and methods, I am persuaded to agree to the author's attempts to absorb me into the action. Am I therefore responsible, in part, for what happens? (pp. 72–73)

The sense of coercion or alienation that may result from a resistant reading of such texts seems to confirm the role that this indeterminate use of *we* plays in asserting the author's authority in relation to the reader.

The MAA advice continues: "In most technical writing, 'I' should be avoided, unless the author's persona is relevant"(Knuth et al., 1989, p. 2). The unwritten assumption here is that, in most cases, the author's persona is not relevant. This assumption is consistent with the commonsensical belief that the subject matter is neutral and that the use of such impersonal language reflects this neutrality. From our examination of the papers in our sample, we find that this rule, at least, is followed; *I* does not appear except in the most remarkable circumstances. In the 24 single-authored papers we examined, only 3 authors, 1 male and 2 females, used *I*. In no case in a paper by multiple authors was *I* used as it was by Mason (1982) to reflect the authors' "way of working, as well as the fusion that has taken place during writing" (p. xi). In our inspection of the 29 jointly authored papers, we carefully discriminated those authors who appeared to be using an agentic *we* (i.e., referring to themselves and their own actions) and those who seemed to be using *we* conventionally (as a means of avoiding the passive voice). In the former category were authors who, for example, appeared to refer to their own thought processes by reflecting, "We may ask if consistency is necessary for convergence, and for a certain class of difference schemes this [that consistency is necessary] is true." Such devices seem to introduce a degree of tentativeness and diminish the level of assertion. Agentic *we* was also used by those who explicitly described their own actions (*we* plus past tense): "In order to resolve the discrepancy we have carried out calculations on ... we have evaluated exactly...." The conventional use (often characterized by *we* plus present tense) was more common, for example, "We apply this well-known procedure to graphical Gaussian model selection."

Among our participants there was a stark difference between those who wrote personally, for example, using the personal *I* and a modifier such as *just* ("Rather than producing an exhaustive list of work on quantum gravity using Regge calculus (which can be found in ...), I shall just mention a few typical applications in different categories.") or making meta-comments on their own writing ("The word 'generic' is used widely in mathematics; we use it in the precise sense of [cited authors], although our description will be informal.") contrasted with those who used the conventional, impersonal language in which mathematics is stereotypically presented. We noticed that some authors used a discursive approach in their writing, avoiding the conventional barriers between author and reader found in the majority of papers. For example, here an author shares with the reader an affective response to the subject matter: "Unfortunately there is not a very systematic definition of integrability. There is no shortage of examples, or indeed definitions, but the equivalence and general validity of the definitions is far from established.... My preferred definition...." Very few authors used such personal or tentative language throughout as the general style, but many authors introduced personal comment to an otherwise conventional presentation. Because such authors were more often, although not exclusively, male, we wondered whether women found it more difficult to use a personal or tentative style. The process of socialization into the discipline positions women as not *quite* belonging (see Burton, 1999a), and, hence, perhaps they are less able to risk rejecting the use of the language that they themselves see as representing competition and power.

The explicit expression of agency through use of the active voice with personal pronouns is not the only way in which the author's persona enters into a text. Relations among author, reader, and subject matter are expressed in the modality of a text, that is, in the "indications of the degree of likelihood, probability, weight or authority the speaker attaches to the utterance" (Hodge & Kress, 1993, p. 9). Modality may be expressed linguistically through use of modal auxiliary verbs (*must, will, could,* etc.), adverbs (*certainly, possibly*), or adjectives (e.g., I am *sure* that....). The naive assumption might be that because mathematics is about certainty, mathematical writing would always have the same absolute modality. That is, all verbs would be in the present tense and would be unmodified; no adjectives or adverbs would signal the author's degree of certainty about the result. Yet many of our participants remarked upon and deplored the frequent practice of signaling high modality in mathematics papers by the use of terms such as *clearly, it is obvious that,* and *trivial.* In the interviews, the participants responded to a request to provide indicators of what makes a paper, either their own or one they refereed, publishable. Many of the women focused, although of course not exclusively, upon style of writing—was the paper "reader friendly"—and drew attention to different styles and reasons for using language in particular ways, some picking out *clearly* for specific attention. One female lecturer said

> It ought to be understandable to people, but somehow in pure maths that doesn't happen. I try to do things in simple words and explain things fully, and [I] get quite cross when referees say you could chop this down; it's obvious. If something is obvious, there is no point in saying so, and it just irritates other people—similarly, *clearly* and all those [words]. It is the ideas' being elegant, rather than the paper being short or concise.

Another recognized some dangers implicit in clarity:

> I think that scholarly articles are often written to impress people how clever you are. I try not to do that; I don't like it. If you explain something well, it can sound quite simple, but you don't want people to think it was easy, because it wasn't.

At the same time, many of our participants could, simultaneously, occupy quite conflicting philosophical positions during the interview, and we suppose, therefore, that their writing styles might not always be consistent with the content of their interviews. Speaking of the widespread use of words such as *clearly* and *it is obvious that* (words used to absolve authors from providing detail and to allow them to demonstrate their expertise), many of the participants said, "No *clearly*s." Then they remembered that they had provided us with a copy of one of their published papers, and potential embarrassment, or at least awareness, would cross their faces and they would say something like "Well, if I use *clearly*...."

From what we have seen, there is conflict and confusion among the conventional view of the impersonal nature of mathematical writing, the advice offered to writers, and the attitudes and practices of mathematicians. The beginning mathematician who needs to know what is possible and what is acceptable thus faces a problem.

In an attempt to make the available choices explicit, we turn now to look at the range and extent of differences in relation to the interpersonal functions performed by the forms of language used in our sample of papers. In the analysis that follows we have addressed three aspects of how the *identities* of the authors are constructed through their texts. These aspects reflect (a) the ways in which authors do or do not attempt to convince readers of their authority—positively (e.g., by assertion) or negatively (e.g., by tentativeness); (b) whether, and in what ways, they show identification with the community of which they are members; and (c) how they demarcate the intellectual territory and the knowledge to which they are laying claim. We deal with each of these aspects in turn.

Authority—Positive and negative: The first aspect of identity. One important aspect of the identities of authors as projected by their texts is the extent to which and the manner in which they claim to be authoritative within their community. If an author appears too tentative in his or her claims, less value might be placed on the results, whereas if one appears inappropriately self-assured, a reader might question the author's right to be so certain and even dismiss the work.

Terms such as *clearly* and *obvious* are relative to the individuals using them (implying that this information is obvious to me but may not be so obvious to you). Our interpretation that the use of such terms serves as a claim to authority on the part of the writer (implying that this derivation is clear to me and I do not need to explain it further because if it is not clear to you, that is your fault not mine) is reinforced by the interview data, as demonstrated in some of the earlier quotes. We believe that, whatever the author's intent, the extent or absence of such words is one of the interpersonal aspects of the writing that will influence the ways in which the readers of the text will construct an image of the author and will consequently judge the worth of the text itself. Of course, there is not a simple causal relationship between the presence of particular words in a text and a reader's response; the way in which an individual reader responds to a particular text will depend on a complex interaction between the text itself and the resources brought to bear on it by the reader. These resources arise from the individual reader's social and cultural history, including the reader's self-perceived degree of expertise in relation to the subject matter, current positioning within the discourses in which the text is read (Kress, 1989), perception of himself or herself as an authority, and position in relation to the author(s). Expressing an appropriate and acceptable degree of authority is likely to be important for beginning mathematical writers in order that they may be accepted as (properly positioned) members of the community; Ward (1996) pointed out that "the author must consider the tactics to use in order to convince an audience of his or her truth claims" (p. 40). We felt, therefore, that the forms and extent of claims to authority were significant enough to warrant examination.

Indicators of authority claims from the use of modifiers, such as *clearly, easily, of course,* and *immediately obvious,* and phrases, such as *without loss of generality, it suffices to consider the case,* and *the last stage is trivial,* all signal a gap in the argument (implying that there is no accompanying justification for the claim that generality is not lost) and hence signal to the readers that they should accept the author's claim. By using *well known,* the author claims the right to define the field and what is well known within it. Readers, in particular those defining the field of study differently from the author, might make different classifications of what is well known. Other indicators of authority claims include *for simplicity* and *for completeness.* The properties of simplicity and completeness are desirable within this discourse; thus the author is making a claim about the quality of her or his own work.

An important function of many of these phrases is to increase readability by removing unnecessary detail and to keep the paper to manageable length. Authors, however, in making decisions about what is unnecessary, explicitly exercise their authority in relation to both subject matter and reader.

Authority claims were sometimes made by denigrating the work of others: "The paper contains a great deal of notation, which, to our way of thinking, obscures the ideas. On occasion the author seems to assume the main point he is trying to prove." In this way, the author of this comment exercises his or her own right to determine what is of value in the field.

Although in the interview some participants expressed dislike of such authority claims and asserted that they would not make extensive use of them, authors of remarkably few texts among those we examined completely avoided such claims. Of authors who did avoid authority claims, four (3 females and 1 male) provided us with extremely impersonal texts with few indicators in any of our categories related to the construction of the identity of the authors. Three others (all by male authors) used unusually high numbers of indicators of negative authority, suggesting a possibly deliberate attempt to relate in a less authoritative way toward their readers. (Table 2 summarizes the distribution of numbers of positive and negative authority indicators.)

We analyzed the submitted papers to determine whether some authors either failed to exert such authority or even explicitly admitted a lack of knowledge or certainty. Signs of such negative authority included not only absence of the positive authoritative signs described above but also modifiers such as *difficult* or *not obvious* and comments such as "the integral in this expression can only be calculated numerically so there is no 'quick and easy' method of generation available,"

Table 2
Extent of Indicators of Positive and Negative Authority in Each Paper

Gender	Number of indicators of positive authority[a]				
	0	1	2–10	11–20	>20
Females	3	3	18	4	0
Males	4	1	11	5	4
All	7	4	29	9	4

	Number of indicators of negative authority[b]			
	0	1–3	4–6	>6
Females	11	9	9	1
Males	4	9	5	5
All	15	18	14	6

[a]Median = 4. [b]Median = 2.

all of which indicate that the author has encountered difficulties. The author is thus vulnerable to a reader who does not accept that difficulties exist. Comments such as "we think but have not yet shown," "plausible though unproved assumption," "the quantum stability is an open question," "it has to be admitted that our reanalyses were idealized in that . . . ," and "this identity is too crude a tool" expose the authors' failure to meet completely the standards demanded by the community and thus constitute a risk to their status. Sometimes authors transferred authority to others by saying, for example, "It is reassuring that the results presented here are similar to those [of X] . . . ," a statement that could result in a diminution of status for the author.

There were fewer indicators of negative authority than of positive authority claims, as can be seen from the distribution also shown in Table 2. There were, however, nine females and six males in whose papers we found more negative than positive indicators or an equal number of each. At the other end of the scale, substantially more female than male authors avoided taking the risk of including any explicit indicators of lack of authority. Although we have described negative authority in terms of vulnerability and risk for the author, the seriousness of this risk is, of course, relative to the author's preexisting status within the community. For example, a very eminent male professor of pure mathematics provided a paper that contained unusually high numbers of indicators of both positive (59) and negative (7) authority. His paper also appeared extreme in other categories discussed below, suggesting strong orientation toward the community (8 references to the community and 18 citations of other authors) and focusing extensively both on mathematics as an autonomous system (50 indicators) and on the human processes involved in mathematical activity (46 indicators). A high-status author can afford to state that a piece of mathematics is difficult in the confident expectation that others will not challenge this judgment (and may even question their own understanding of the mathematics if they do not perceive the difficulty themselves). For low-status authors, such statements would be likely to reinforce any existing doubts about the value of their work; such authors, therefore, may feel themselves better advised to construct a positive authoritative identity in their writing. As a female senior research officer said

> One of my collaborators likes to make things seem as sophisticated and complicated as possible, and I think that is a problem for us. If I write the paper, our result looks like a simple result, and if he writes it, it looks like something else completely. I remember giving a seminar once and being told afterwards that I had made it look far too easy.

The texts that we have examined in this article are located within the discourse Richards (1991) called "journal math," using in most cases "reconstructed logic" (p. 16). Solomon and O'Neill (1998) argued that this discourse, which makes use predominately of the "timeless" present tense and logical cohesive devices (e.g., *hence* or *so that*), is constitutive of mathematics itself. We agree that such linguistic forms are essential for expressing many mathematical meanings but deny a necessity that logical, rather than temporal, relations make a narrative form unavailable. On the contrary, we believe that a mathematical text can be seen as a narrative, albeit told within the different conventions of that genre. With McLaren (1995), we see narratives as providing "the discursive vehicles for transforming the burden of knowing to the act of telling" (p. 92). However, we also feel that to exclude the personal narrative of the "logic of discovery" (Richards, 1991, p. 15) neglects other important aspects of mathematics. In our sample, those papers that made the author's role in the production of the mathematics clear or constructed an active role for the reader were not any less mathematical than the others but indeed presented a broader view of mathematics, including the human mathematician in the picture. To include the activity of doing mathematics as well as the results and the reconstructed logic may be one way of making such texts more accessible to both writers and readers. But doing so also demonstrates that "science is not the supreme way of knowing simply because of its philosophical, ideational, or rhetorical supremacy, but because it is a powerful associational network containing a strong and expansive web of heterogeneous actants" (Ward, 1996, p. 138). Having included this comment, we do not wish to argue that there is one best or correct way of writing mathematical research papers. To do so would merely be to substitute one form of authority for another. Rather, we would like to see greater tolerance of diversity and, as important, awareness of the variety that already exists and is available for writers to use to express their own chosen mathematical and personal meanings.

Community membership: The second aspect of identity. In the research project of which this study forms a part, many of the participants spoke in similar positive terms to those of the lecturer who said, "I am an applied mathematician—a fluid dynamicist. My community provides me with colleagues to talk with and work with, a place to find new ideas." We were, therefore, interested to investigate how their writing demonstrated this identification and relationship with the community.

One feature that was striking in the interview data was the change in recent years from individualized to collaborative research (see Burton, 1999c). One participant referred to this change as a cultural shift in the discipline. All except 4 of the 70 participants claimed to be doing some collaborative work, so we examined the effect of collaboration in the 53 papers we analyzed and found that 24 of the papers had one author, 19 had two authors, and 3 had four authors.

Gender did not appear to be a differentiator (see Table 1); approximately 55% of both female and male participants gave us jointly authored papers. Of course, the choices of article were not random and did not necessarily reflect the authors' current work or thinking about how they preferred to work. Indeed, we believe it is likely that jointly authored papers are underrepresented in the sample. Three factors may have led to such underrepresentation. First, the interviews were individual so the participants may have assumed that it would be preferred or better, in some way, to offer a paper by a single author. Second, the content of the interviews dealt with epistemological questions about their mathematical thinking, how they understood mathematics, how they organized their work, the role of intuition and insight, and so on; again, these questions could have influenced participants to assume that a paper by a single author would be preferred. Finally, a few participants gave us more than one paper, reflecting different ways of working, different areas, changes from one period to another. In these cases, we chose a paper by a single author to contribute to this analysis.

Having identified community membership as an area of interest, we examined the papers with respect to the ways in which the authors positioned themselves in relation to the wider community of mathematicians. The first indicator was a simple count of the number of times that the author(s) made reference to the work of others (other-citations) and the number of times they cited their own work (or cited a coauthor). The other-citations ranged from 0 to 58, and self-citations ranged from 0 to 10, but the lengths of papers also varied considerably, as did their purposes, making comparisons difficult. Citations, of course, serve a wide range of functions (see, e.g., Cronin, 1984), but we have used them in this study simply as indicators that authors have located themselves in some (undetermined) way within a community of other authors. The highest number of self-citations (10) was made by a female. The highest number of other-citations (58) was made by a male.

We also explored the texts to find other instances in which the authors referred to the community of mathematicians to which they belonged and to see how frequently they made such references. Some remarks were nonspecific, referring in general terms to work ongoing: "The rich microstructure of cosmic strings is starting to receive considerable attention." In other remarks, authors cited particular persons, for example, "since the pioneering work of [X]"; some comments located the authors in relation to specific contributors to the field and to a broader community: "The problem is a difficult one and there have been numerous papers written on the subject including an excellent review by [Z]." As can be seen in Table 3, the number of such references ranged from 0 to 22. Gender did not appear to be a differentiator.

Table 3
Numbers of References to the Community

	References				
	0	1–4	5–9	10–15	>15
Participants	2	31	15	2	2[a]

[a]One of these participants made 17 references to the community, and the other made 22.

The majority of our authors are positioning themselves in general with respect to the community or citing a small number of appropriate references, but there is considerable variation in practice. We are not convinced that counting citations in this way has given us a clear indication of community identification or of the conventions of academic writing in mathematics. The content of the interviews was convincingly community related, but the published writing did not clearly reflect this fact.

Demarcation of territory and of knowledge: The third aspect of identity. The worth of a piece of research is likely to be judged not only by its internal validity but also by the extent of its contribution to the field of knowledge of which it claims to be a part. Indeed, the mathematicians interviewed were quick to invoke a number of words to describe categories of contributions, from *significant* or *important* to *interesting* and, dismissively, *trivial.* In the mathematicians' thinking, these categories appeared to form a hierarchy, of which the pinnacle was a *significant* contribution. Work that is derivative or that reviewers or editors judge to be uninteresting or trivial is unlikely to be published. A lecturer in applied mathematics said

> To decide whether work is important, I would ask, "What sort of application? How significant is the application? How significant is the result? Does it back up something someone else has done? Does it tie in very well with the experiments which could validate your model or method?" You could have some experiments that had been done elsewhere; you could do some nice maths matched up with it, which is satisfying in itself, but if it isn't going to help in pushing forward design, it might be nice in itself, but it isn't going to be so significant to the community.

The criteria in other areas of mathematics may be slightly different from those expressed by this lecturer, particularly in pure mathematics in which experiments are unlikely to be a means of validation. Whatever the field, however, authors are likely to be concerned with demonstrating the place their work may occupy within it. As well as establishing one's authority to speak within the community of mathematicians, one of the interpersonal functions that a research paper may thus fulfill is the demarcation of an author's claim to be entitled to speak about a particular area of knowledge. On examining the papers, we found that we could distinguish two types of such demarcation: the demarcation of territory and the demarcation of knowledge.

In demarcating territory, authors separated their own work from that of others in the field, identifying gaps in the community's knowledge and stating what remains to be investigated. In the papers we examined, authors not only located and made a claim for the unique contribution made by a paper (e.g., "One way in which our treatment differs from all previous treatments, is that we do not assume we start with an embedded fundamental domain.") but also laid a claim to areas of work not, as yet, published ("That is already a substantial programme of work ... which we hope to develop in future publications"). In some cases, claim was made on areas yet to be investigated, thus establishing a larger territory (and hence, perhaps, greater status for the author) not directly justified by the contents of the current research paper or ongoing research: "As an extension to this work we intend to investigate nuclear vibrational motion.... Future calculations are planned to employ a multistate approximation.... We also plan to carry out *ab initio* studies of photoionization cross sections." By making such statements, authors not only claim status for their current work but also attempt to ensure that they will be able to make further claims in the future (and that other workers in the field will find it necessary to cite their contributions).

As well as demarcating areas of their field of mathematics, authors also claimed status for the knowledge produced within the field and, in particular, for the value of their own results. In some cases, simple statements of ownership of results achieved this demarcation of knowledge, for example, "We establish a general multi-transversality theorem...." In other cases, more ambitious claims for the value (in terms of significance or interest) of these results were involved: "The algorithm presented here could lead to significant saving of time and complication."

The extent to which the papers included such demarcation of territory or knowledge varied substantially although only one paper contained none at all. Interestingly, we found little relationship between the extent of authority claims and the extent of demarcation in individual papers, suggesting that establishing the author's personal authority in relation to her or his readers and the author's right to recognition as the owner of an area of mathematics are two separate aspects of the interpersonal function of the text.

Focus

In the interviews we found great variability among the participants about the nature of the beast, mathematics. One mathematician said, "The only thing mathematicians can do is tell a good story, but those stories do uncover mathematical truths—mathematical truths are discovered." Another explained, "I see mathematics as a way of describing physical things using formulae that work, modeling the real world, and having a set of rules, a language, which allows you to do that."

Although both these mathematicians talked in terms of the communicative aspect of mathematics, the first, a pure mathematician, is telling a story whereas the second, an applied mathematician, is describing aspects of the world. Of course there are many cultural differences between pure and applied mathematicians (see Davis & Hersh, 1981, and Burton, 1999c), but from the perspective of mathematical writing, we were particularly taken by the difference between assertions of certainty made by most of those who were interviewed (58 of the 70) and the comments by a small number of participants (mostly statisticians), who deliberately contrasted their knowing with certainty:

> If I know something, it means I understand how a particular system is described mathematically and what the mathematical implications are.... Sometimes you never know if you know. You can do it by contradiction, by finding fault. You can never know it is right. You always have to live with uncertainty. (Professor of statistics)

Jane Hutton (1995) also affirmed that "statisticians ... are concerned with variation and error, and the resulting near impossibility of knowing anything with certainty" (p. 254).

We found another distinction between those in pure and those in applied mathematics. Applied mathematicians tended to emphasize utility: "You can do things with it; you can model real things; you can make predictions; you can compare experiments." They were mainly concerned about "describing physical things using formulae that work, modeling the real world." But even this very practical approach did not preclude this mathematician from acknowledging the sociocultural basis by "having a set of rules, a language, which allows you to do that." A pure mathematics lecturer gave an interesting example of a very different interpretation of the nature of mathematics:

> The definition, theorem, proof style is sometimes necessary to the health of mathematics, but it can be overprescriptive. People think that is what maths is whereas I think it is about filling in gaps, making the map. Maths isn't what ends up on the page. Maths is what happens in your head. I don't think maths is about proving theorems. It is one constituent, but maths is about mapping abstract ideas in your head and understanding how things relate.

In the light of these differing images of the nature of mathematical activity, we were interested to investigate the focus of the papers that we were analyzing. Within this category we explored two aspects of the writing: the representation of the source of new mathematical knowledge (whether human or from within the system of mathematics itself) and the extent to which the human processes of doing mathematics (as opposed to presenting the results of this activity) were apparent in the texts. The highly stylized way in which mathematical papers are often written cannot necessarily be reinterpreted as intent or as representative of particular positions. Indeed, as earlier extracts from the interviews have shown, experienced authors are aware of the expectations of their audience and of the ways in which they may need to adapt their writing to match these expectations. So although the writing of naive beginning writers might represent their beliefs more or less transparently, we cannot assume that the same will be true of more experienced authors.

Mathematics—Human product or autonomous system? The first aspect of focus. We looked for differences between authors who made the mathematics the object of their attention by focusing on the products (proofs, results, etc.) of their own activities, using phrases such as "our claim is proved," and those who appeared to give the mathematics a "life" of its own, indicated by the use of words that are generally associated with animate beings (e.g., "The prolonged good behaviour of the Hartree-Fock type wave function ... is probably due to....") and by the extensive use of clauses in which mathematical objects themselves acted as the subjects of material activity (as opposed to being operated upon by a human mathematician, for example, "The functors H and K induce a duality....").

In many of the papers, the mathematical objects were themselves the actors producing the results that were being described by the authors. Frequency of use of this style varied from 0 to 50 such examples. To try to demarcate differences, we looked closely at those papers in which authors used no such examples and those that had more than 20 examples. Two female statisticians avoided the use of this style entirely, choosing to focus upon humans as responsible for the mathematics. Both made references to mathematicians as producers and to the processes whereby the mathematics was produced (see next section). At the other end of the scale, all the papers with more than 20 examples of mathematical objects presented as actors came from male participants. There may, however, be a relationship between the extensive use of this style and the field of mathematics. In particular, the structure of the discourse of some areas of pure mathematics seems to lend itself to a focus on the autonomous activity of mathematical objects. A further result of the removal of humans from the writing is that the style then tends to become obscure and abstract; we have already referred to this practice as one way in which an author can seek to establish her or his authority (and see Davis & Hersh, 1981).

In some cases, authors may have presented the mathematics as a human product as a matter of conventional presentation, including the convention of marking the completion of a proof with a statement to that effect. However, in other cases of papers with this focus, the style was not solely conventional but was tempered with process comments—indicators of a focus on human activity as well as human product. Thus, alongside comments such as "We can obtain an algebraic equation," we found "We attempt to prove stability." A strong focus on mathematicians as producers sometimes accompanied strong acknowledgment of the community, leading us to wonder if these are connected, but our data are insufficient to test this hypothesis.

The processes of doing mathematics: The second aspect of focus. A further aspect of interest to us was the extent to which the focus of a paper was on the processes by which the authors arrived at the end product of theorems and proofs rather than solely on the products themselves. We looked for indicators of a focus on the human processes of doing mathematics, such as expressions of opinion or feelings, conjectures, questions, explanations for decisions (e.g., "This is a purely analytic proof which gives little insight into why...." or "The motivation for this paper was an attempt to build models").

The number of explicit mentions of human processes involved in doing mathematics ranged from 0 to 46 examples in a single paper. We might expect a complete absence of process comment to indicate a very conventionally written paper with a strong focus upon mathematicians as producers, but our data do not support this conjecture; some of these papers also used the language of negative authority, acknowledging the community but focusing on the knowledge product rather than the process of its production. Authors who used a high number of examples describing processes often appeared to be more strongly assertive of these processes than they were of their claims to knowledge. We particularly noted that some authors who made frequent reference to process also asked questions of the readers (e.g., "How do we know that...? And, if so, how can we find...?"), thereby involving them in the paper in what we felt to be a very user-friendly way. Because of the important role that humor can play in introducing the humanity of author and reader as they share the joke, we appreciated the ways in which two authors used humor: "The noose is a good place from which to hang these ideas"; "The details of this argument are thankfully hidden under the very general apparatus of G-structures."

CONCLUSIONS

During her interview, a female senior lecturer was particularly critical of mathematical writing:

> Mathematics is full of hidden assumptions. You have to read a paper at least twice to get the real agenda, and you have to go through a process of deduction to find out what on earth it is. Is it that the people who are doing it are blithely unconscious of the wider process or is it a process of mystification? I think sometimes they are only semiconscious of what they are doing and why.

Hersh (1989) also blamed poor mathematical communication on authors' failure to have learned how to write in prose. In spite of widespread agreement within the mathematics community that much mathematical writing at all levels is obscure and difficult to read, remarkably little guidance is available for beginning or experienced writers. This lack of guidance contrasts with the situation in science and technology, in which a substantial number of publications offer guidance on writing, apparently reflecting a recognition that many of those involved in these areas are likely not only to have underdeveloped skills in this area but also, at some point in their careers, to be involved in writing reports for nonspecialists, with the consequent need to focus on the clear communication of key ideas. The Mathematical Association of America has shown some interest in publishing guidance on mathematical writing (see Gillman, 1987; Knuth et al., 1989; Krantz, 1997), but the training of mathematicians does not appear to include any systematic attention to the development of writing skills. Moreover, the guidance that is available is largely algorithmic, sets of rules to be followed rather than ideas for developing critical awareness of the different effects of various forms and structures of language. As the mathematician quoted above suspects, many mathematical writers are likely to have little awareness of how their readers may respond to their writing. In particular, given the rule-based nature of the guidance that is available, authors are unlikely to be aware of the possibilities for variation and flexibility in style that we have found in our examination of these papers, despite the writings of authors such as Steven Ward, who pointed out that "rhetoric and literary invention are key communal ingredients for doing and conveying science...; if scientists and other knowledge producers can see how rhetoric and symbolization work in the writing process, they will be able to write better texts" (1996, p. 40).

However, apparently not all mathematicians are happy about the ways in which mathematics is presented publicly. A female reader asked that papers be "well written." She continued

> I get annoyed with some of my collaborators and a lot of the papers I am sent, which are definition, theorem, lemma, proof. That seems to me to be appallingly bad. It is the sort of thing that no one is ever going to want to read. I think it is important to grab the reader from the opening sentence. Not "Let A be a class of algebras such that...." Change it to "This paper opens a new chapter in duality theory."

We believe that the dominance of a style of writing that she, and many other participants, abhorred is probably the result of a combination of three factors: desire for feelings of safety, the quantity of published papers providing conventionally written models, and a nonreflective approach to writing. Yet there were substantial differences among the papers the participants gave us. "Differences between writers reflect not just differences in their cognitive skills but also the extent to which they share the goals, and understand the conventions, of the discourse community they are addressing" (Galbraith & Rijlaarsdam, 1999, p. 98).

We hope that we have demonstrated the diversity in the variables we have examined and the lack of universal conformity to commonly acknowledged conventions. Other strong influences on writing-style choices may include the authors' epistemological stances on mathematics as well as their beliefs about the expectations of their readers. Nonetheless, our analyses of the various aspects of identity demonstrate that a research paper does more than report research. We have shown how authors, through their writing, convey very particular perspectives on themselves as well as on their mathematics and on how they build their identities with respect to their communities.

If some mathematicians want to move away from obfuscation, mystification, and put-down toward clarity and straightforwardness without oversimplification, the knowledge that some authors attempt to follow these principles would provide them support. However, editors of journals and their chosen reviewers also have a role to play in emphasizing certain principles that, at present, do not seem to influence either those who submit papers for publication or their peers who review. Textbooks also play a role in formalizing the mathematical language that is accepted. During the past 10–15 years, in some social science writing, noticeable shifts have occurred away from the conventional, third person, objective writing toward a style that incorporates the *I* of the author and links the author's beliefs and values more clearly to the chosen structure and outcomes of the research. Some social science journals provide authors and reviewers with guidance about, for example, the use of non-gender-specific language. Providing similar guidance to mathematicians could be an important step in raising awareness of the power of language. We hope that this article may contribute to such a process.

For research mathematicians, writing (and, more important, being published) is a critical activity. Although it may be seen as secondary to the act of doing mathematics itself, through their writing, individual mathematicians establish their identities within the academic community and secure employment. Common knowledge about the nature and process of mathematical writing (as represented, for example, in the MAA guidance mentioned above) suggests that there is only one standard way of presenting one's research. Yet our examination of this relatively limited sample of published papers has shown considerable variation and, in a few cases, wholesale flouting of the recognized conventions. The conventions of mathematical writing are neither necessary nor natural consequences of the nature of the subject matter; they are rather "the product of current relations of power and discourse practices" (Clark & Ivanic, 1997, p. 14) within the community. Knowing the conventions and being able to use them may be one step toward establishing one's position; knowing how they work and how and when to break them to achieve a particular effect is, however, an important way to express and establish a more powerful position.

We see teaching and learning mathematics not as just filling students' heads with facts and skills but as inducting them into mathematical communities. An important part of learning to be mathematical, whether in the primary school or in the university, is learning to take part in the discourses of mathematics, becoming both a consumer and a producer of texts that are recognized as legitimately mathematical within one's community. Current practice in the training of mathematicians and in mathematics education more generally does not explicitly involve teaching and learning about mathematical writing. The novice may learn through using the existing models of published writing, through an apprenticeship of collaboration with more experienced writers, or through the often harsh process of peer review. None of these methods is designed to help learners to acquire the kind of knowledge about language that might enable them to be aware of what they might achieve by choosing to write in different ways. Moreover, although the rules governing what is and what is not considered acceptable writing within the community remain implicit, access is likely to be more difficult for those from social groupings (of class, gender, or ethnic group) whose linguistic resources do not match those of the dominant group. At the level of school mathematics, one of us (Morgan, 1998) has argued that teaching should address mathematical writing explicitly to make successful participation in mathematical practices more accessible for all students; the same need exists at more advanced levels. For such explicit attention to writing to be useful, teachers and students need to develop a mathematical-linguistic vocabulary that equips them with tools of thinking and speaking about the various forms of language available to them so that they can make informed, critical judgments about which forms to use. We see this acquisition of mathematical-linguistic vocabulary as being a necessary and crucial component of the educative process. The tools that we have offered here and used in the analysis of research papers provide a starting point for the development of such a vocabulary for those learning to write in this genre. A more general set of tools has been suggested by Morgan (1996); further work needs to be done, however, to characterize other mathematical genres and the choices available to writers within them, particularly in those genres in which producing appropriate text has important consequences for learners and novices. Meanwhile, mathematicians, teachers, and learners can embark upon the process of learning to read and write mathematics critically.

Writing, for both students and researchers, is not just about communicating mathematical subject matter. It is also about communicating with individual readers, including powerful gatekeepers such as examiners, reviewers, and editors. The writer needs to know how to write in ways that are likely to convince such readers that she or he has the authority to write on this topic, that the subject matter is important enough to be interesting, and that paying attention to what is being said is worthwhile. This need for knowledge about language is as great for learners of mathematics as it is for practitioners. Where, as in the United Kingdom, new developments in assessment demand that students present mathematical projects, extended essays, investigations, and so on, the students' experience of considering how and in what form to convey their meaning becomes even more important than in systems in which assessment relies on multiple-choice tests or tasks involving short responses.

We have seen that authors can write effectively and powerfully without subsuming their identities within the conventions. Indeed, the papers of some mathematicians with the highest status within the community used some of the least conventional language. In this exploratory study we have identified linguistic means for achieving various types of authority, significance, interest, and so on. But characterizing the various forms used by mathematical writers requires further research. This study could be a starting point for work with novice (and, indeed, experienced) researchers to develop their critical linguistic awareness—their knowledge of the forms of language that are available to them and their abilities to make effective choices among them. We hope that our study may help join potential mathematicians, learners of mathematics, to practitioners, those who are researching in the community of mathematicians.

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